ISSN: 2148-9955

International Journal of Research in Education and Science (IJRES)

www.ijres.net

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To cite this article:

Abu Mokh, R., Othman, A., & Shahbari, J.A. (2019). Mistakes made by students with logical connectives when solving equations and inequalities, and how teachers assess these mistakes. *International Journal of Research in Education and Science (IJRES), 5*(2), 421-428.

International Journal of Research in Education and Science
Volume 5, Issue 2, Summer 2019 ISSN: 2148-9955

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Article Info

Article History

Received:
22 September 2018

Accepted:
04 January 2019

Keywords

Logical connectives
Algebra
Inequalities
Practicing mathematics teachers

Abstract

The aim of this study is to examine the mistakes made by students in the use of logical connectives while simplifying algebraic equations and inequalities, and the extent to which teachers are aware of these mistakes and how they assess them. The study was conducted among 50 ninth grade students and 63 practicing teachers of mathematics. The data was collected from two questionnaires: a questionnaire for students comprising items containing inequalities or equations, and a questionnaire for teachers comprising students' solutions of different items. The data was analyzed according to interpretative theory. The findings identify common mistakes in the way students use the mathematical logical connectives OR, AND, IF … THEN when manipulating algebraic expressions. The findings further indicate that teachers were not aware of the errors made by students in working with the mathematical logical connectives, where the most common mistake identified among students was ignoring the logical connective entirely. Moreover, the findings indicated that teachers may assess solutions as correct even when they use mathematical logical connectives wrongly.

Introduction

Understanding how to simplify and solve equations and inequalities is a topic that has been emphasized by the National Council of Teachers of Mathematics (National Council of Teachers of Mathematics [NCTM], 2000). It is considered to be one of the basic mathematical procedures studied at secondary school level, involving a variety of basic algebraic and arithmetic skills such as performing rote operations with algebraic symbols, or applying the quadratic formula (Li, 2007). However, this important topic of solving equations and inequalities is considered to be difficult for students (Cai & Moyer, 2008). Various studies have focused on the difficulties experienced by students when simplifying algebraic equations and inequalities, or manipulating algebraic expressions (e.g. El-khateeb, 2016; Samuel, Mulenga & Angel, 2016; Poon & Leung, 2010).

The research has examined a variety of aspects, such as the students' understanding of the equals sign, their ability to solve equations (Knuth, Stephens, McNeil, & Alibali, 2006) and the strategies that students use to solve inequalities (Tsamir & Almog, 2001). However, researchers have not so far paid significant attention to how teachers understand equations and equation solving (Doerr, 2004). More specifically, there is a need for closer examination of both how students use logical connectives and how teachers assess their students' use of logical connectives. It is this need that gave rise to the current study, which examines the mistakes students make when using logical connectives, and how teachers respond to these mistakes.

Solving Equations and Inequalities

By 'algebraic equations', we simply mean the typical types of equations (linear, quadratic, exponential, rational, etc.) introduced in secondary school algebra curricula. An inequality is a mathematical sentence built from expressions using one or more of the symbols ($<$, $>$, $\leq$ or $\geq$) to compare two quantities (El-khateeb, 2016 p. 124). To solve an equation means to find the numerical values that the unknown quantity can take that make the equation a true statement. The solution of equations and inequalities is considered to be an important topic in the study of algebra, in particular for the study of function properties and applications. These require students to be aware of and to understand methods for finding the solution set for each inequality and equation (El-khateeb, 2016).

Students experience a range of difficulties when solving equations and inequalities, such as: inadequate understanding of the meaning of the equals sign when solving equations (Knuth et al. 2006); inadequate understanding of how to manipulate algebraic expressions and statements (Samuel et al., 2016); lack of symbolic understanding of variables and coefficients within an equation (Kilpatrick & Izsak, 2008); and changing the direction of the inequality when multiplying by a negative number or due to other conceptual errors (El-khateeb, 2016).

Logical Connectives

A proposition is "a sentence that is either TRUE or FALSE (but not both)" (Remsing, 2005 p. 2). A sentence that contains a finite number of variables and becomes a proposition when specific values are substituted for the variables is called a predicate (or open sentence) (Remsing, 2005 p. 15). The symbols $\neg$, $\wedge$, $\vee$, $\Rightarrow$and $\Leftrightarrow$ are called propositional connectives. Any sentence built up by application of these connectives has a truth value that depends on the truth values of the constituent sentences (Mendelson, 2009 p. 3). The translation from a natural language statement to formal logic is seen as difficult for students, and using one connective in place of another is one of the major errors made by students (Barker-Plummer, Cox, Dale, & Etchemendy, 2008). In discussing these difficulties, Strannegård, Ulfsbäcker, Hedqvist and Gärling (2010) point out that in English, "or" does not always correspond to the connective $\vee$, since "or" sometimes translates into an exclusive OR and sometimes into an inclusive OR. Similarly, the English construction "if... then..." does not always correspond to the connective $\rightarrow$. An ability to understand logical connectives and set operations is vital, meaning that students need systematic treatment of these connectives (Dreyfus & Eisenberg, 1985).

Mathematical Content Knowledge and Pedagogical Knowledge Held by Teachers

Content knowledge (CK) includes the structure of knowledge, facts, theories, and principles in the field (Shulman, 1986). Mathematical content knowledge includes common content knowledge and specialized content knowledge (Ball, Thames & Phelps, 2008). The former relates to the content of the curriculum, concepts, procedures, and the ability to read and write concepts and notions correctly. Specialized content knowledge refers to the knowledge and skills unique to teaching (Delaney, Ball, Hill, Schilling, & Zopf, 2008). It includes an understanding of mathematical structures, which enables tasks to be tackled that require significant mathematical resources (Ball et al., 2008).

Pedagogical content knowledge (PCK) is the knowledge needed to make subject matter accessible to students (Shulman, 1986), and combines an understanding of both content and pedagogy (Ball, Lubienski, & Mewborn, 2001). It comprises an awareness of student difficulties and misconceptions relating to the concepts being taught, an understanding of different methods used in specific or representative taught content, and an understanding of the teaching methods that make learning easy or difficult (Shulman, 1986). Ball et al. (2008) separate mathematical pedagogical content knowledge into two subcategories: Knowledge of content and teaching, and knowledge of content and students. The former combines knowledge about teaching with knowledge about mathematics. Teachers need to be aware of how to design instructions, the various representations of the concept being explained, and how to evaluate these representations (Ball et al., 2008). The latter is a type of pedagogical content knowledge that involves an understanding of students, including awareness of how students think about, know, and learn the specific mathematical content (Hill, Ball & Schilling, 2008).

According to the definitions above, we can see that the MCK and MPCK held by teachers relating to solving equations and inequalities should include: mathematical procedures, algorithms, routines, skills, conceptual understanding, and procedural knowledge. A study of how mathematics teachers understand these issues could provide a better understanding of their teaching practices and influences on what and how their students learn.

Questions Posed by the Study

To what extent do ninth grade students have difficulties with logical connectives? What are their major difficulties?
How do teachers respond to the mistakes students make when using logical connectives? How do they assess mistakes made by students?

Method

Participants

The study was conducted among 50 ninth grade students and 63 practicing teachers of mathematics. The students came from two ninth grade classes. The students in each class had different levels of mathematical achievement, but each class had average performance based on the national mathematical exams. The practicing teachers were all ninth grade mathematics teachers and participated on a voluntary basis. A description of the participants' background is displayed in Table 1.

Table 1. Distribution of the participants according to background variables

Variable	Categories	Practicing teachers
Gender	Female	84%
	Male	16%
Level of mathematics at high school	Basic	1.6%
	Intermediate	42.8%
	Advanced	46%
Teacher training institution	College	74.6%
	University	20%
Teaching experience	1-5 years	60.3%
	6-10 years	19%
	11-15 years	6.3%
	More than 15 years	12.7%
Current school	Elementary	44.9%
	High school	55.1%

Data Source

The data was collected from two sources: a questionnaire for students, and a questionnaire for teachers.

Questionnaire for Students

The questionnaire comprised ten items containing existence statements and algebraic simplifications requiring the use of propositional connectives. Some examples from the questionnaire:

If $-1 \le x \le 4$ then

If $-4 \le x \le 3$ then $x^2 \le 30$

Solve the equation $(x^2-3x+2)^2 + (x^2-6x+5)^2=0$

Solve the equation $(x^2-3x+2)^2 \cdot (x^2-6x+5)^2=0$

Solve the inequality $x^2 > -1$

Questionnaire for Teachers

The questionnaire comprised ten items. Each item contained a question involving existence statements and algebraic simplifications requiring the use of propositional connectives, followed by different student solutions. The teachers were asked to score the solutions on a scale of 0 to 10 and explain their decisions. Some examples from the questionnaire:

1) When the question "solve the inequality $x^2 > 1$" is given to ninth grade students we receive the following eight answers:
   $x^2>1 \Leftrightarrow x> \pm 1$
   $x^2>1 \Leftrightarrow -1>x>1$
   $x^2 >1 \Leftrightarrow x>1$ also $x>-1$

$x^2>1 \Leftrightarrow x>1$ or $x<-1$
$x^2>1 \Leftrightarrow x>1$ and $x<-1$
$x^2>1 \Leftrightarrow x>1, x<-1$
$x^2>1 \Leftrightarrow x\neq \pm1$ and $x\neq0$
$x^2>1 \Leftrightarrow x\neq1$ and $x\neq-1$ and $x\neq0$

2) Below are shown solutions given by students for the equation $[(x^2-7x+12)^2 + (x^2-4x+3)^2=0]$:
$(x^2-7x+12)^2 + (x^2-4x+3)^2=0$
$(x^2-7x+12)^2+(x^2-4x+3)^2=0\Leftrightarrow$
$x^2-7x+12=0, x^2-4x+3=0$
$(x-3)(x-4), (x-3)(x-1)$
$x=3, x=4, \quad x=3, x=1$

Data Analyses

Analysis of the data obtained from the student questionnaire was conducted in two phases. In the first phase, we analyzed the students' solutions, classifying them into four categories: correct answer, correct algebraic manipulation but incomplete explanation of the answers (OR, AND, IF), wrong answer — mistakes in algebraic manipulation, not solved. In the second phase, we focused on the second category, looking at the mistakes made using logical connectives. The data was analyzed using the constant comparison method (Glaser & Strauss, 1967). We identified, grouped and categorized types of mistakes using the logical connectives, derived categories from the data set and then compared with the rest of the data set. For each error category, we calculated how frequently it occurred in the students' solutions. The data obtained from the teacher questionnaire was used to calculate the mean scores for each item, and the frequency that full scores were awarded by the teachers analyzed against the different lengths of teaching experience.

Findings

We begin by presenting the findings relating to the difficulties experienced by students when solving equations and inequalities. Our focus is on difficulties using logical connectives. We then present the findings obtained from the questionnaire for teachers; in particular, how the teachers assessed the students' solutions.

Students' Difficulties When Solving Equations and Inequalities

The solutions of equations and inequalities given by the students can be divided into four categories: correct answer; correct algebraic manipulation but incomplete explanations of the answers (OR, AND, IF), wrong answer, not solved. Table 2 shows the distribution of solution categories across the 10 items over all 50 students.

Table 2. Distribution of solution categories over all students

Category	Distribution	Examples from students' solutions
Correct answer	12%	$x^2 > 1$ $x>1$ or $x< -1$
Correct algebraic manipulation but incomplete explanations of the answers (OR, AND, IF)	38%	$(x^2-3x+2)^2 + (x^2-6x+5)^2=0$ $(x^2-3x+2)^2+(x^2-6x+5)^2=0\Leftrightarrow$ $x^2-3x+2=0, x^2-6x+5=0$ $(x-1)(x-2), (x-5)(x-1)$ $x=1, x=2, \quad x=5, x=1$
Wrong answer — mistakes in algebraic manipulation	32%	$\frac{1}{2} \leq \frac{x+1}{2} + x + 3 \leq 11$ $1 \leq x+1+2x+6 \leq 22$ $1\leq 3x+7 \leq 22$ $1\leq 3x \leq 15$ $1\leq x \leq 5$
Not solved	18%	-

The findings shown in Table 2 make it clear that difficulties with the logical connective represent the most common difficulty experienced by students when solving equations and inequalities. The findings indicate that more than one-third of the errors made by the students occurred when manipulating the logical connective. Focusing on students' mistakes when using the logical connective, we were able to distinguish five types of mistake: (1) Ignoring the logical connective: the students did not write any logical connective while carrying out the algebraic manipulation; (2) Ignoring the logical connective and replacing it with a comma: the students inserted commas where a logical connective was needed; (3) Replacing the logical connective "OR" with "AND", or *vice versa*: the students did not distinguish the correct logical connective words; (4) Replacing the logical connective "AND" with "ALSO": the students used the word "ALSO" as a logical connective; (5) Incorrect interpretation of the logical connective: the students used a logical connective word, but chose the wrong interpretation for the word as they fail to correctly understand the meaning of e.g. "AND" or "OR", and thus obtained the wrong final answer. Table 3 shows the distribution of these mistakes.

Table 3. Distribution of mistakes in mathematical logical connectives

Category	Students in category	Examples from student solutions
Ignoring the logical connective	64%	$\frac{1}{2} \le \frac{x+1}{2} + x + 3 \le 11$ $1 \le x+1+2x+6 \le 22$ $1 \le 3x+7 \quad 3x+7 \le 22$ $-6 \le 3x \qquad 3x \le 15$ $-2 \le x, x \le 5$
Ignoring the logical connective and replacing it with a comma	12 %	$(x^2-3x+2)^2 + (x^2-6x+5)^2=0$ $(x^2-3x+2)^2+(x^2-6x+5)^2=0 \Leftrightarrow$ $x^2-3x+2=0,\ x^2-6x+5=0$ (x-1) (x-2) ,(x-5)(x-1) x=1, x=2, x=5, x=1
Replacing the logical connective "OR" with "AND", or *vice versa*	2%	$X^2>1$ $X^2=1$ $X=\pm1$ x>1 and x<−1
Replacing the logical connective "AND" with "ALSO"	10%	$(x^2-3x+2)^2 + (x^2-6x+5)^2=0$ $x^2-3x+2=0$ also $x^2-6x+5=0$ (x-1)(x-2)=0 also (x-1)(x-5)=0 $x_1=1,\ x_2=2$ also $x_1=1,\ x_2=5$ $x_1=1,\ x_2=2$ also $x_2=5$
Incorrect interpretation of the logical connective	12%	$(x^2-3x+2)^2 + (x^2-6x+5)^2=0$ $(x^2-3x+2)^2+(x^2-6x+5)^2=0 \Leftrightarrow$ $x^2-3x+2=0$ and $x^2-6x+5=0$ (x-1) (x-2)=0 and (x-5)(x-1)=0 $x_1=1\ x_2=2$ and $x_3=5\ x_4=1$ The answer is x= 1,2,5

Table 3 shows that almost two-thirds of students ignored the logical connective: they reached the fully-simplified equation or inequality, but did not connect the parts, and thus did not successfully reach a final answer.

How Teachers Assessed Students' Mistakes When Using Mathematical Logical Connectives

The findings revealed that teachers assessed various answers with mistakes in the logical connectives as complete answers and awarded full scores. For the items where logical connectives where used wrongly, the mean score given by the teachers ranged between 5.26 points and 9 points. Table 4 shows different items containing equations and inequalities, example student solutions using the logical connective wrongly, and the percentage of teachers who marked each solution as correct with 10 points (full score).

Table 4. Teachers assessed solutions with mistakes as correct solution

Level of algebraic manipulation	Example item	Student solutions	Assessed as correct solution (full score)
Simple algebraic manipulations	$x^2>1$	$x^2>1 \Leftrightarrow$ x>1 also x<-1	8%
		$x^2>1 \Leftrightarrow$ x>1, x<-1	28.6%
		$x^2>1 \Leftrightarrow$ x>1 and x<-1	22.2%
More complicated algebraic manipulations	$\left(x^2-3x+2\right)^2 \cdot \left(x^2-6x+5\right)^2 = 0$	$\left(x^2-3x+2\right)^2 \cdot \left(x^2-6x+5\right)^2 = 0$ $\Leftrightarrow x^2-3x+2=0$, $x^2-6x+5=0$ (x-2)(x-1)=0, (x-1)(x-5)=0 $\Leftrightarrow$ x=2, x=1, x=1, x=5	44.4%
	$(x-2)\left(x^2-4x-22\right)=x^2-4$	$(x-2)\left(x^2-4x-22\right)=x^2-4$ $x-2/(x-2)\left(x^2-4x-22\right)=x^2-4$ $(x\|2-4x-22)=\frac{x^2-4}{x-2}=$ $\frac{(x-2)(x+2)}{x-2}=x+2$ $x^2-5x-24=\frac{5\pm\sqrt{25+96}}{2}$ $=\frac{5\pm 11}{2}=8,-3$	17.5%

The findings indicate that some teachers treated solutions which had mistakes in the use of the logical connective as correct (these solutions considered as wrong according to the exam system). The level of algebraic manipulation appears to affect how the teachers assess the solution. Based on the findings in Table 4, it seems that teachers ignore the logical connective when the item needs more algebraic manipulation. For example, for the fourth item, ($x+7=\sqrt{4x-4}+\sqrt{9x-9}+\sqrt{x-1}$), 63.5% of teachers awarded a full score. By contrast, when the same mistake was made with the logical connective in the first item ($x^2>1$), only one-fifth of the teachers awarded a full score. The findings indicate that the length of experience a teacher has affects how they assess their students' mistakes in use of the logical connective when solving equations and inequalities. Figure 1 shows the distribution of teachers with different lengths of teaching experience (1-5 years; 6-10 years; 11-15 years and above 16 years) and their assessment of students' mistakes in using the logical connective.

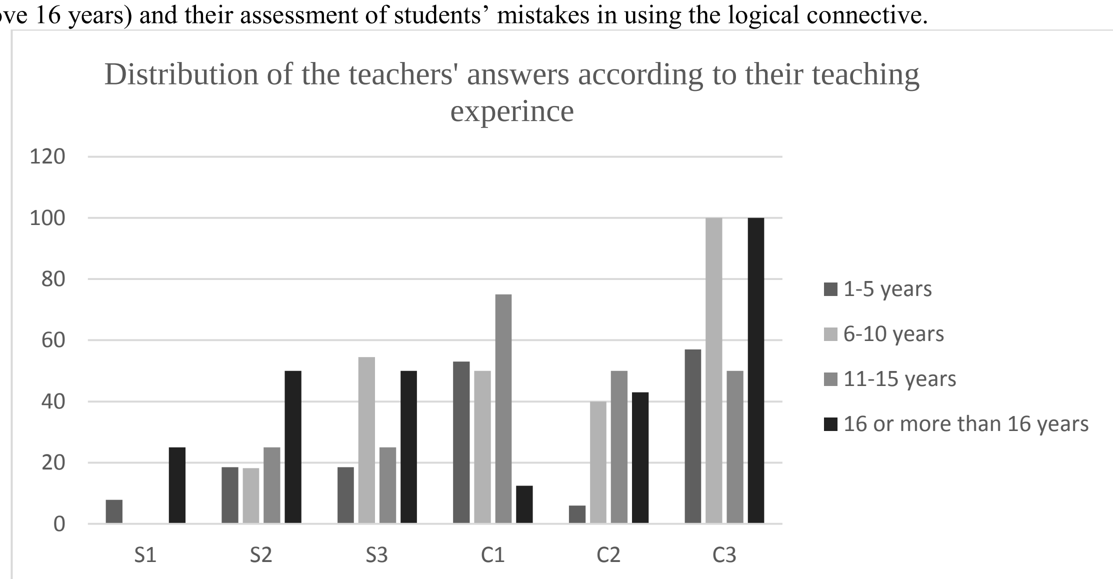

Figure 1. Distribution of teachers' assessment according to teaching experience
(S1, S2 and S3: Needed a simple algebraic manipulation)
(C1, C2 and C3: Needed a complicated algebraic manipulation)

Figure 1 shows that teachers with more teaching experience mostly ignored students' mistakes using logical connectives, while teachers with fewer years of experience (1-5) paid more attention to the mistakes in the logical connectives and did not award full scores.

Discussion

This study looked at the mistakes students make when using and writing logical connectives during the solution of algebraic equations and inequalities, and examined teachers' assessment and awareness of these mistakes. The main findings indicate that students successfully manipulate the algebraic expressions but have difficulties in applying the logical connective correctly. The findings furthermore reveal that teachers assess a range of answers with mistakes in the logical connectives as complete answers and award full scores.

The study's findings relating to the mistakes students make when solving algebraic equations and inequalities indicate that most of the problems are related to the logical connectives. The different types of mistakes associated with the logical connectives suggest that students did not consider the final answers, and did not evaluate whether their solution found the correct answers (Vaiyavutjamai & Clements, 2006). Focusing on the types of mistakes associated with logical connectives, it is evident that the most common mistake was to ignore the logical connective – almost two third of students made this error. The fact that students ignore the logical connective "OR" is supported by Almog and Ilany's study (2012), which was conducted among students in grade 12. Similar difficulties were reported by Tsamir and Almog (2001), who applied the square root property to equalities to reveal how the logical connectives were being ignored. Students applying the square-root property to e.g. $x^2 > 81$ would provide the solution $x > \pm 9$, instead of $x < -9$ or $x > 9$. The category of errors which occurred the least among the students was the use of the wrong connecting word between the expressions in the inequality — e.g. using "OR" in the solution where they should have used "AND", or *vice versa*. Neimark (1970) describes this mistake as interpreting the set union (A or B) as a set intersect (A and B).

The findings from the student questionnaires provide support for the recommendation made by El-khateeb (2016) that teachers must explain and discuss the meaning of the logical connective, e.g. by clarifying the meaning of the word (OR) when writing out the solution set. However, when evaluating how teachers assess their students' solutions of algebraic equations and inequalities, the study found that teachers did not appear to be aware of the mistakes being made by the students when manipulating the logical connectives. Teachers assessed solutions which made incorrect use of the logical connective as complete solutions and awarded full scores, particularly for the more complicated questions. This phenomenon was more commonly exhibited among teachers with longer teaching experience. Li's (2007) study highlighted three topic areas in solving equations where teachers' understanding of the mathematical subject matter should be strengthened: the balancing method, the concept of equivalent equations, and the properties of linear equations in their general forms. We now propose addition of a fourth topic, emphasizing the logical connective.

Conclusion

We believe that teachers' responses to the mistakes in using logical connectives and students' difficulties in this topic are related. The recommendation from this study is therefore is to strengthen the presentation of logical connectives to students, and at the same time enhance teachers' awareness of the importance of writing and using the logical connective words correctly. We recommend working with both teachers and students, because teachers' teaching knowledge and their students' understanding are tied together.

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