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Metacognitive Skills and Problem-Solving

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Abstract

The purpose of this study is to investigate the metacognitive strategies that middle school students used in the process of solving problems individually. The study group consisted of 37 middle school students in the eighth grade. The students were asked one non-routine word problem, and their written answers were collected. After solving the problem, the students filled out a self-monitoring questionnaire that requested them to reflect retrospectively on the metacognitive strategies they employed for the problem. In order to obtain detailed data, semi-structured interviews were conducted with six students, three of whom gave correct responses and the other three gave wrong answers to the problem. The data was analyzed through the model of metacognitive activity during problem-solving. The results showed that metacognitive skills have a significant effect on students' problem-solving success. The study found that students with high metacognitive skills tend to solve the problem correctly by using appropriate strategies, mathematical notations and logical reasons. The results also revealed that students with poor metacognitive skills have difficulties in understanding the problem, selecting appropriate strategies, and finding correct answer. Although the students thought the opposite, it was concluded that their habits of checking, detecting, and correcting mistakes in their solutions were poor.

Introduction

Metacognition is what one knows about his/her own thinking ways and how he /she monitors and regulates his/her thinking processes while engaging in a task (Goos, Galbraith & Renshaw, 2000). The concept of metacognition is defined by many researchers (Biryukov, 2003; Goh, 2008; Hecker, 1998; Iwai, 2011; Koriat, 2007; Salam, Misu, Rahim, Hindaryatiningsih & Ghani, 2020; Wells, 2009). For instance, to Koriat (2007) metacognition means what people know about cognition, their cognitive processes and how they use metacognition knowledge to adjust their information processes and behaviors. Metacognition is a thinking practice of the mind (Wells, 2009) or thinking about thinking (Iwai, 2011). Flavell (1976) defines metacognition as "being aware of one's own cognitive processes or knowledge of any subject" (p. 232). This definition highlights two significant aspects, which are knowledge about cognition and regulation of cognition. Knowledge about cognition includes declarative knowledge, procedural knowledge, and conditional knowledge (Schraw, 1998). Declarative knowledge refers to knowledge about an individual's self, strategies, and the factors that influence his/her performance (Schraw & Moshman, 1995; Schraw, 1998). Procedural knowledge is

awareness of how to do things. (Schraw & Moshman, 1995). It includes knowing how to use which strategy (Schraw, 1998). Conditional knowledge refers to knowing why and when to employ cognitive activities (Schraw & Moshman, 1995). It allows students to adapt their knowledge to changing situations (Schraw, 1998). Regulation of cognition consists of metacognitive skills such as planning, monitoring, control and evaluation of cognitive processes (Ader, 2019; Whitebread et al., 2009). Planning involves choosing appropriate strategies for the task; monitoring is one's awareness of his/her own performance while engaging in a task. Control and evaluation mean reviewing the whole process (Schraw & Moshman, 1995).

Educators emphasize the significance of the “active and constructive process of sense-making, understanding and problem solving in a community of learners” (De Corte et al. 2011, p. 155). Various countries such as the US (National Council of Teachers of Mathematics [NCTM], 2000) and Turkey (Ministry of National Education [MoNE], 2018) have introduced metacognitive skills as an essential component of mathematics education (Ader, 2019). Based on the continuous changes in mathematics curricula, teachers should offer experiences for reasoning and problem-solving (Ader, 2019). Metacognitive skills play a critical role in monitoring and regulating cognitive processes (Chan & Mansoor, 2007) and help students to understand when, why, where, and how to use their own knowledge to solve problems successfully (Carr & Jessup, 1995). On the other hand, students who are not able to notice the errors they have made in solving problems, monitor what they have done, use appropriate strategies or explain their solutions are not good at mathematics (Carlson & Bloom, 2005; Lucangeli & Cabrele, 2006). Therefore, metacognition is one of the critical issues in problem-solving (Lester, 1994). Problem-solving involves not only cognitive strategies but also metacognitive skills and it is more than just implementing strategies to solve the problems. In fact, metacognitive skills are related to problem-solving (Ader, 2013; Mayer, 1998), and students who have these skills can decide whether a problem is sensible, analyze the consistency between strategies and solutions, determine the correctness of the answer, and recognize their errors. However, since analysis and observation of metacognitive skills are difficult, this issue has not been sufficiently investigated by mathematics educators.

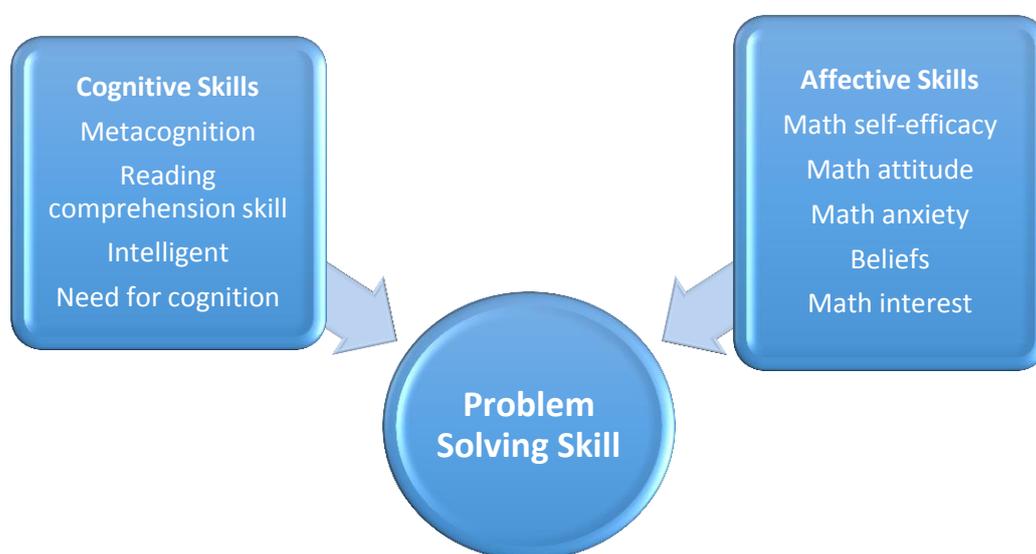


Figure 1. Cognitive and Emotional Variables that Affect Problem-solving (Ozturk, Akkan & Kaplan, 2020)

The variables affecting problem-solving skills can be considered in two categories as cognitive and affective skills (Ozturk et al., 2020). Belet and Yasar (2007) stated that the cognitive variable is about reading comprehension skills like identifying the main idea and auxiliary ideas of the text, finding the cause-effect relations in the text, and predicting the meanings of the unknown words in the text. The students who feel unable to solve problems will spend less time on the problems and, thus, solving problems will be challenging for them. Affective skills like self-efficacy perception are significant for problem-solving (Hoffman & Schraw, 2009). Briefly, cognitive skills (metacognition, reading comprehension skill, intelligent, need for cognition) and affective skills (mathematics self-efficacy, mathematics attitude, mathematics anxiety, beliefs, mathematics interest) affect problem-solving skills together as seen in Figure 1 (Ozturk et al., 2020).

There are various studies on the criticality of metacognition in the success of problem-solving (Artzt & Armour-Thomas, 1992; Pennequin, Sorel, & Mainguy, 2010; Swanson, 1990). For instance, Kuhn (2000) emphasized the influence of the development of metacognition in early years on higher-order thinking processes since it provided better cognitive skills. Students better performed the tasks in learning mathematics and solving problems (Mevarech & Fridkin, 2006). Eggen and Kauchak (2001) stated that successful students are those who are aware of the times when they act strategically or not as learning becomes effective when it is accomplished consciously. Metacognition helps students to carry out the steps of problem-solving and manage this process (Sevgi & Cagliköse, 2019).

To solve problems, there is a need not only for carrying out the stages of problem-solving but also for following, regulating, and controlling these stages consciously. Therefore, metacognition is an essential element of the problem-solving process (Schoenfeld, 2005). Although various studies have revealed the positive relationship between metacognition and success in mathematics (Kitsantas, 2002; Rysz, 2004), some have not supported these findings (Ader, 2004; Hong & Peng, 2008; Kuyper et al., 2000). This contradiction also demonstrates the need for further research on the relationship between metacognition and mathematics education.

The international exams such as Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) demonstrated that students were not good at solving mathematics problems (OECD 2004, 2007). While solving mathematics problems, students need to understand the given knowledge in the problem, represent the problem by using the knowledge appropriately, generate a plan to solve the problem, and make the necessary calculations. Therefore, it is difficult to follow these steps for many students, especially for those who do not have sufficient content knowledge (Verschaffel et al., 2000).

Students' poor performance in problem-solving is not only due to their lack of mathematics knowledge but also due to their lack of awareness of when and how to do, namely, metacognitive skills (Altun & Arslan, 2006; Garofalo & Lester, 1985; Mayer, 1998; Özsoy & Ataman, 2009; Pintrich, 2002; Schoenfeld, 1985). Many studies emphasize the critical role of metacognition in problem-solving and mathematics education (Desoete, Roeyers, & Buysse, 2001; Garcia, Rodríguez, González-Castro, González-Pienda, & Torrance, 2015; Jacobse & Harskamp, 2012; Mayer, 1998; Mevarech, 1999; Verschaffel, Greer & De Corte, 2000). However, there are deficiencies in understanding metacognition and its effects on being a good problem solver (Asik, 2015).

Metacognition and Problem Solving

There have been various definitions of a problem in literature. Some define a problem as the exercises that need to engage in using new mathematical knowledge and techniques, while the others describe it as complex tasks in which the solver encounters obstacles (Schoenfeld, 1992). Problem-solving is an essential part of mathematics education and continues to carry its importance (Evans, 2012). Thus, one should know how to deal with a problem to understand mathematics (Van De Walle, Karp, & Bay-Williams, 2010). It is a process that necessitates comprehension, visualization, analysis, association, generalization, and reasoning (Garafola & Lester, 1985). Polya (1985) defines problem-solving in four phases:

- (a) understanding the problem,
- (b) making a plan,
- (c) carrying out the plan, and
- (d) checking out the solution.

These phases are parallel with the planning, monitoring, control and evaluation skills in metacognition (Ader, 2019; Whitebread et al., 2009).

Problem-solving requires not only knowledge and cognitive skills but also metacognitive skills, when and how to use knowledge and cognitive resources (Mayer, 1998). Cognitive skills assist in understanding the task and using strategies for solution, whereas metacognitive skills help to regulate the problem-solving process and make decisions (Goos, et al., 2000). Lucangeli and Cornoldi (1997) emphasized the vital role of metacognition in mathematics education. For example, in the early stages, problem-solving such as the representation of a problem and making a plan to solve the problem includes metacognition (Desoete & Veenman, 2006). After the solution, its accuracy is checked. Therefore, problem-solving requires using cognitive skills to know what and how to do and controlling the process. Such practices refer to metacognition and increase problem-solving achievement by enabling students to represent the problem mathematically and try various strategies on it (Davidson & Sternberg, 1998).

Problem-solving helps to construct new mathematical knowledge, which is named doing mathematics (Van De Walle, Karp, & Bay-Williams, 2010). However, students have difficulty in problem-solving which requires regulating their learning (Fuchs et al., 2003) and using specific strategies (Montegue, 2008). Desoete (2007) has suggested that teachers should give importance to metacognition for developing students' problem-solving skills. Students should be aware of their cognitive resources and control how to use them (Lucangeli & Cornoldi, 1997).

Metacognition helps students use the knowledge and strategies effectively (Lucangeli et al., 1998) and overcome the challenges in mathematics (Carr & Jessup, 1995). The development of metacognition begins in early childhood and becomes a more complicated form (Veenman, 2011). The frequency of using metacognitive skills and their effectiveness increase in time (Van der Stel & Veenman, 2010). Goos et al. (2000) identified behaviors that should be observed during problem-solving among students with metacognitive skills.

Table 1. A model of Metacognitive Activity during Problem-solving (Goos et al., 2000)

Problem Solving Behaviors	Metacognitive Skills (Monitoring/Regulation)
<i>Before solving the problem:</i> Reading the problem more than once, trying to understand the problem, trying to restate the problem, checking the familiarity with a similar problem before, identifying the given information in the problem and considering which ways should be used to solve the problem.	Assess knowledge, understanding of problem and strategy appropriateness
<i>As solving the problem:</i> Checking the solution step by step, returning the beginning if a mistake was made, rereading the problem to control whether the solution steps were still true, determining whether they got closer to the solution, reviewing the solution and trying a different approach.	"Red Flag": Error detection Assess strategy execution Correct errors "Red Flag": Lack of progress Assess progress towards goal Assess understanding of problem and strategy appropriateness Change strategy
<i>After solving the problem:</i> Checking the correctness of the calculations, looking back over the solution to control the consistency between what the problem asked and what was done, examining whether the answer was sensible and thinking on different solution ways.	"Red Flag": Anomalous result Assess strategy appropriateness and execution Assess result for accuracy and sense

The notation of “red flags” is defined as warning signals of “the need for a pause or some backtracking while remedial action is taken” (Goos et al., 2000, p. 3). It alerts the lack of a mechanism in metacognitive process. Red flags are classified as “Error detection”, “Lack of progress” and “Anomalous result”. Error detection facilitates checking and correcting the errors as solving the problem. Lack of progress leads to reanalyzing the problem to reassess strategy appropriateness and decide whether to progress or change the strategy. Therefore, students reassess their understanding of the problem to reach new information and determine a new strategy. Anomalous result prompts an assessment of whether the solution is correct or meaningful. If the solution does not make sense or answer the problem, anomalous result leads students to check the calculation and strategy execution. This study used this model to evaluate students’ metacognitive skills during the problem-solving process (Goos et al., 2000).

NCTM (2000) emphasizes the importance of problem-solving from elementary level to higher grades. Students fail in problem-solving due to the lack of understanding, choosing appropriate strategies, regulating the solution,

monitoring calculations, and controlling the whole process (Victor, 2004). These skills present the essential role of metacognition in problem-solving. Problem-solving is one of the complex processes that necessitate metacognitive skills in addition to cognitive resources. These skills are vital to problem-solving because they help identify the problem, determine the appropriate strategy, monitor the effectiveness of the chosen strategy, control the accuracy of the solution and regulate the problem-solving process (Sternberg & Hedlund, 2002). Researchers have revealed that students who suffer from a lack of success in mathematics do not use cognitive and metacognitive strategies in an effective way in problem-solving (Wilson & Clarke, 2004). Better problem solvers tend to represent the problem considering all the information, whereas poor problem solvers mainly obtain mathematical operations by only focusing on the keywords (Kramarski et al., 2002). The students who have high metacognitive skills are more successful in problem-solving (Desoete, 2008; Hollingworth & McLoughlin, 2005; Schoenfeld, 1985). Thus, investigating students' metacognitive skills is important to improve mathematics education (Livingston, 2006). Despite the increasing focus on the role of metacognition in problem-solving, there is a dearth of knowledge about the nature of students' metacognitive skills and the use of these skills in problem-solving (Suriyon, Inprasitha & Sangaroon, 2013).

The current study aimed to examine students' metacognitive skills in the problem-solving process, the differences between students who use metacognitive skills and those who do not, and their effects on the success of problem-solving. Most studies in the past did not seem to attend to non-routine problems in problem-solving (Lee et al., 2012) and the focus of this study was to examine students' metacognitive skills rather than evaluate their performance in problem-solving. In this vein, the following research question guided this study with one non-routine problem: *How are middle school students' metacognitive skills related to problem-solving?*

Method

Research Design

Case study is a qualitative approach in which the researcher investigates a case or cases in detail through rich data collection with multiple resources of information such as observation, interviews, and documents (Creswell, 2007). In this study, the case study design was used to reach a depth of understanding of the students' metacognitive skills in problem-solving process.

Participants

The study participants consisted of 37 students in the eighth grade in a middle school in the Eastern Anatolia Region of Turkey. The students were determined by the typical sampling technic to represent the eighth classes. They also had average achievement scores in mathematics. Although metacognitive knowledge begins to develop in early ages, the appropriate use of metacognitive skills seems to start at the ages of 11-12 (Garcia et al., 2015). Besides, mathematics teaching programs introduce metacognitive skills as an important component of mathematics education (Ader, 2019). Thus, the focus was on the eighth grade students' metacognitive skills in solving problems since they are able to use these skills.

Data Collection

One non-routine problem, including complexity without depending on a specific mathematical knowledge, was asked to the students, and their written answers were collected. The process of problem-solving took 20 to 30 minutes in total.

Domino Problem: Cem, who is very curious about playing dominoes, wants to find out how many dominoes there are. There are many options for this.

In the game Dominoes, all dominoes are divided into two groups, and the numbers 0-6 in each half are indicated by dots. Each domino contains a pair of numbers, and there are only one of these pairs in the whole set. How many dominoes are there in the whole set?

After solving the problem, the students filled out a self-monitoring questionnaire that requested them to reflect retrospectively on the metacognitive strategies they employed for the problem (Goos et al., 2000). It enabled the researchers to examine how students dealt with metacognitive “red flags” indicating the need for regulation during monitoring. The original form of the questionnaire was developed by Fortunato et al. (1991) for the seventh grade students. Goos et al. (2000) rearranged the questionnaire for older students. The questionnaire consisted of 21 statements under four titles: 1) “Before you started to solve the problem”, 2) “As you worked on the problem”, 3) “After you finished working on the problem”, and 4) “Ways of working on the problem”. The first section includes six items related to reading and understanding the problem and determining a solution strategy. The second section lists five items about using the solution strategy and monitoring the solution process. The third refers to four items on demonstrating the accuracy of the solution. The fourth section lists six solution ways to learn which ways were used by students. There are three options to mark *Yes*, *No*, or *Unsure* for each item.

In order to obtain detailed data, semi-structured interviews were conducted with six students, three of whom gave correct responses and the other three gave wrong answers to the problems. The interview questions were prepared based on problem-solving behaviors and metacognitive skills that are expected in solving problems by the researchers. The recordings of the interviews approximately took twenty minutes for each student. The following questions were asked to the students to get more detailed information about their problem-solving process and interpret these behaviors in terms of metacognitive skills:

1. Have you ever encountered a problem like this before?
2. What did you understand when you read the problem for the first time? Can you express it in your own words?
3. What was given in the problem? What was asked you to find out?
4. What kind of ways did you choose to solve the problem?
5. Was the way that you chose to solve the problem helpful to reach the solution?
6. How did you check whether your solution was correct?
7. Do you think that the problem can be solved in another way?

Data Analysis

The content analysis technique was used to analyze the qualitative data. The students' solutions, the questionnaire data, and the transcripts of interviews were analyzed through the model of metacognitive activity during problem-solving (Goos et al., 2000) in Table 1. To ensure the reliability and validity of the findings, direct quotations from the interviews, the students' written solutions, and questionnaire responses were presented. In addition, two researchers (the authors conducting this study) separately analyzed and coded the data by using the model. The comparison of the independently-analyzed codes yielded the inter-coder agreement as 88%. The researchers reached a consensus after discussing the remaining 12% of the coding list.

Results

The Questionnaire Data

The students' solution attempts and their answers to the questionnaire were analyzed. The strategies used by the students were classified in certain ways. The solution strategies the students used were grouped into three categories: 1) random trial and error, 2) ordered listing, and 3) systematic listing. These categories were linked to the outcomes of their problem-solving activity – no answer, randomly writing the pairs, writing some of the pairs, wrong answer with the right strategy, or correct answer.

As seen in Table 2, one student using only the trial-and-error strategy could not answer the question. With the same strategy, one student wrote random pairs. The most preferred strategy among the students was making a systematic list ($n = 22$). Sixteen students reached the correct result. Although many students preferred ordered listing ($n = 13$), only three of them could reach the correct result.

Table 2. Solution Strategies and Outcomes

STRATEGY	Numbers of Students					Total
	No Answer	Randomly writing the pairs	Writing some of the pairs	Wrong answer with the right strategy	Correct answer	
Random trial & error	1	1	-	-	-	2
Ordered listing	-	-	3	7	3	13
Systematic listing	-	-	4	2	16	22
TOTAL	1	1	7	9	19	37

A high rate of Yes responses was reported for the Self-Monitoring Questionnaire statements referring to metacognitive strategies. For the four statements that might prompt initial recognition of the metacognitive “red flags” were investigated deeply. Response rates for these statements are shown in Table 3. The items with the most “yes” answers were about error detection and anomalous result. In the lack of progress item, the answer “yes” was given by a little less than half of the students (40%). This situation reveals that although the students were competent enough to detect mistakes and anomalous results, they lacked the skill of making progress.

Table 3. Questionnaire Responses to Metacognitive “Red Flag” Statements

“Red Flag”	Questionnaire Statement	Percentage of Students					
		Responding Yes		Responding No		Responding Unsure	
		N	%	N	%	N	%
		Error detection	I checked my work step by step as I went through the problem.	26	70	9	24
Lack of progress	I asked myself whether I was getting any closer to a solution.	15	40	15	40	7	19
Anomalous result	I checked my calculations to make sure they were correct.	28	76	7	19	2	5
	I asked myself whether my answer made sense.	24	65	9	24	4	11

It is very difficult to speak of successful self-regulation by just considering that students made simple progress. The places where students found it difficult to change their strategies or mistakes in their written work offer signs of self-control. The four items labeled “red flag” can be taken into consideration to make an early interpretation of students’ metacognitive levels. However, it would be incomplete to conclude their metacognitive activities by considering the questionnaire answers only. Therefore, semi-structured interviews were needed.

The Interview Data

Semi-structured interviews were conducted with six students. Some of their answers were as follows:

Students were asked what they understood when they read the problem and whether they could express the problem in their own words;

It was seen that all the students interviewed were able to express the problem in their own sentences. They also stated;

- o “I’ve read the problem several times, I wanted to write the dominoes one by one.”
- o “I’ve realized that the dominoes were between 0 and 6, and it was not the opposite.”

All of the students stated that they understood the problem when they read it, and they could express the problem in their own words. This situation shows that the students who could reach the correct solution did not have any difficulties in understanding the problem.

Students were asked whether they had ever encountered such a problem;

Most students stated that they had never encountered this kind of problem before. Only one student said;

- o “I saw problems like that before, but they were easier such as probability questions about flipping a coin.”

None of the students stated that they had encountered such a problem before. Therefore, it is possible to say that the domino problem was a new and unusual situation for them. This provides an excellent opportunity to examine students' metacognitive skills. It also shows that students were not familiar with non-routine problems.

Students were asked what was given in the problem and what was wanted from them to find;

Students' answers showed that almost all students understood what was given and what was asked them to find out in the problem. They were able to express the problem correctly in their own words.

Most of the students were able to explain what was given and wanted in the problem in their own words. It shows that they were able to understand the concepts in the problem and interpret them. This situation is a positive indicator of students' metacognitive skills.

Students were asked what pathway they chose to solve the problem, they stated;

- o "I wrote the numbers on the dominoes one by one."
- o "I tried to draw it visually."
- o "I separated one domino from the rest and eliminated the others. I counted the remaining dominoes and found the result."
- o "I paid attention to whether the numbers were the same."

When the students elaborated on the ways they solved the problem, they explained what they specifically did during the solution. Even the students who could not reach the correct conclusion made very reasonable explanations. However, it is seen that being able to explain the given and required concepts in the problem was not enough to solve it correctly. Selecting an appropriate strategy and correctly formulating the mathematical problem were not easy for students. This shows that although the students' metacognitive skills were high, they had mathematical deficiencies.

When asked if this pathway made it closer to the solution, whether the path they chose worked or not;

The students stated that the chosen path worked, and therefore, they did not have to select another way.

Whether or not the students reached the correct result, they did not need to try a different way, thinking that the path they chose was correct. This situation is a negative indicator of metacognitive skills.

Students were asked how they checked the solution, whether it was correct or not, they stated;

- o "I counted them all over again."
- o "I looked at the upper and lower numbers of dominoes. It was correct if there was no other piece of the same domino."

It was seen that the students did not need to check whether there was a missing domino or not, and in general, they preferred to write all the pairs and eliminate the same ones.

Since the students believed that their solutions were correct, they did not doubt their results. While some of

them checked the numbers by making a cursory examination, some only checked the first and last numbers. Not thinking about the alternative solution possibility to the problem is a negative indicator of metacognitive skills.

Students were asked whether the problem could be solved in any other way;

Most of the students stated that this problem could not be solved in any other way. This showed that students tended to believe that the problem had a single solution.

Similar to the previous questions, the students thought that the problem had no other solutions than their own. The students' assumption that the problem can only have one solution shows that they do not sufficiently use their metacognitive skills.

The Problem Solutions Data

In this section, the examples from the students' solutions to the non-routine problem were given, and their metacognitive skills in problem-solving process were explained. The students solved the problem by using the method of systematic list-making and reached the correct result (see Figure 2). They wrote the possible situations for zero, one, two, three, four, and five, respectively. It is seen that they indicated all situations clearly. These solutions reveal that the students understood the problem, recognized the concepts by using correct mathematical notations to express the mathematical ideas, selected an appropriate strategy and used it effectively. The solutions reflect logical sense, the correctness of the procedure, and the students' high metacognitive skills for this problem.

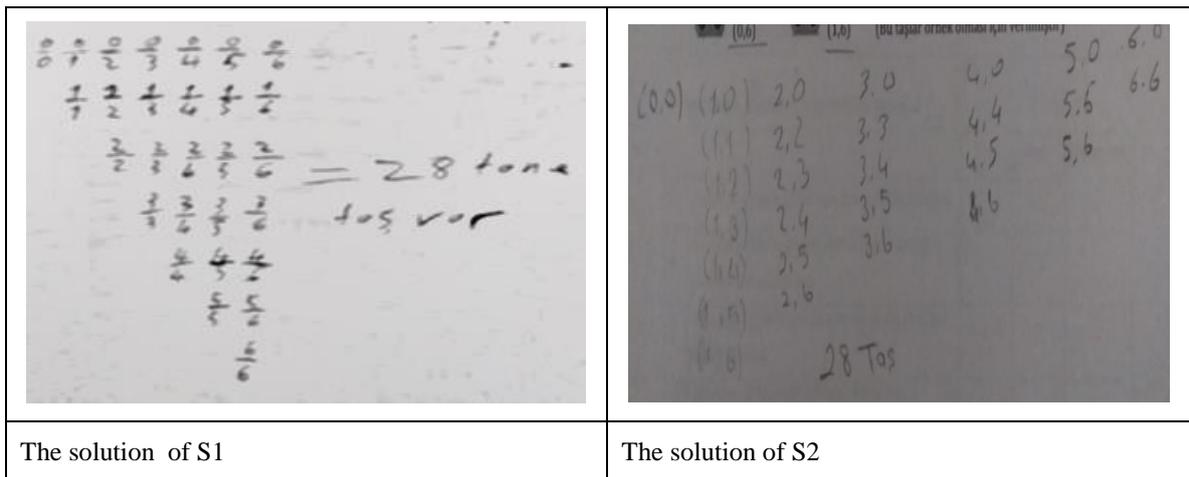


Figure 2. The Samples of Correct Solutions of S1 and S2

It is seen that the students tried to solve the problem by using the ordered listing method. They listed all the situations and eliminated the inappropriate ones. However, they made a mistake while listing the situations and reached the wrong answer (see Figure 3). These solutions refer to the lack of metacognitive skills. It can be said that they partially understood the problem and partly used correct terms to reach the solution. Nevertheless, they could not accomplish to formulate the mathematical problem because of the ineffectiveness of the solution method and the lack of checking the calculation.

<p>The solution of S3</p>	<p>The solution of S4</p>

Figure 3. The Samples of Incorrect Solutions of S3 and S4

The samples (see Figure 4) illustrate that both students' solutions did not reflect logical sense and appropriate strategy use. It is seen that S5 did not use correct mathematical expressions or symbols to present the problem and consequently could not complete the calculation correctly. It is not clear why s/he followed this kind of solution procedure. It seems irrelevant to the concepts given in the problem. On the other hand, the explanation of S6 as "There are 21 dominoes. I counted them one by one." is not enough to understand how s/he reached this answer. S6 did not present a mathematical and logical solution. Thus, both students' irrelevant solutions revealed that they could neither understand the problem, choose an appropriate strategy nor give reasons for the steps in their procedures. This situation refers to their weak metacognitive skills in terms of mathematical knowledge, thinking strategies, and skills.

<p>The solution of S5</p>	<p>The solution of S6</p>

Figure 4. The Samples of Irrelevant Solutions of S5 and S6

Discussion

The aim of the study was to examine students' metacognitive skills in the problem-solving process, the differences between students who used metacognitive skills and those who did not, and their effects on the success of problem-solving. Various studies have shown that individuals with a high level of metacognitive awareness are much better at understanding what they read and consequently using what they understand in problem-solving (Aydemir & Kubanc, 2014; Balci, 2007; Dogan, 2013; Gelen, 2003; Gumus, 1997; Hassan, 2003; Johnson, 2002; Kiremitci, 2011; Mayer, 1998; Ozbilgin, 1993; Ozturk et al., 2020; Salam et al., 2020; Tohir, 2019; Vadhan & Stander, 1994; Van der Walt & Maree, 2007). The students' written responses to the

problem, and the questionnaire, and semi-structured interview records revealed that using metacognitive skills was associated with problem-solving and positively affected problem-solving results. These findings are consistent with other studies in the literature (Arsuk & Memnun, 2020; Balci, 2007; Kaplan, Duran, & Bas, 2016).

Students who could use metacognitive skills gave correct answers to the problem and utilized various solution strategies through their awareness of the problem's requirements and goals. They checked the correctness of the solution. It is important for students to follow and control their own processes consistently while solving problems in terms of metacognitive skills (Asik, 2015; De Bruin & Van Gog, 2012; Koriat, 2012; Pieschl, Stahl, Murray, & Bromme, 2012). Many studies in literature have revealed that mathematics is related to the reading skill which is required for individuals to follow the process and make progress in a controlled manner (Asik, 2015; Fite, 2002). Therefore, in this study, students' inability to follow the process and control their strategies may be due to their shallow reading and lack of efforts to understand the question in depth. As Cakiroglu (2007) has expressed, being aware of the problem is only possible when students have fully understood the problem.

Students who could not use metacognitive skills gave wrong answers to the problem and had difficulties in understanding it, determining solution strategies, recognizing and correcting their mistakes. Most of the students were not successful in solving non-routine problems. As Schoenfeld (2006) states, problem-solving has a complex and relational structure that requires a careful consideration of different variables. It is not possible to talk about specific solutions and strategies that will solve all problems. In order to make sense of and solve the complex structure in questions, individuals should think in detail on the problems and utilize metacognition (Asik, 2015; Lucangeli & Cornoldi, 1997). This study showed that students' answers to the questionnaire and their suggestions for problem-solving were not clear and articulate enough. In their study with the seventh grade students to examine the effects of problem-solving activities on metacognitive skills, Kramarski, Mavarech, and Arami (2002) concluded that only a small number of students who solved the problems controlled the answers they found. Vergnaud (1991), Brun et al. (1994) and Sandır, Ubuz, and Argun (2002) have stated that students had difficulties because they could not comprehensively assign meanings to the concepts that form the basis of the problems.

It has been observed that some students do not know what and why they are doing while solving the problems. According to Gourgey (1998), one of the characteristics of individuals with advanced metacognitive control is that they can transfer their previous experiences and knowledge to subsequent processes while solving problems. In this study, it is seen that the students lacked this skill. Aksu (1984) (as cited in Sevgi & Caglikose, 2019) has revealed that after solving a problem and finding a solution, many students generally ignore the stage of checking their results. Generally, students focus on getting an answer without considering its accuracy. Since they cannot establish a logical connection between their solutions and the problem, they cannot detect their mistakes, and therefore they cannot correct their mistakes. According to Sengul and Yildiz (2013), one of the reasons why students have difficulties in problem-solving is that they turn directly to the process without answering the why questions or questioning the problem. Ozturk, Akkan and Kaplan (2020) emphasize that students' reading comprehension skills are positively and significantly related to their problem-solving skills.

Gelen (2003) states that students generally do not know how to use a strategy in solving a problem or why they choose that strategy after deciding on one. On the other hand, if students know but cannot explain the learning strategy, this shows that the students' metacognitive skills are deficient. Such situations highlight the importance of teacher guidance in developing students' metacognitive skills (Aydemir & Kubanc, 2014; Desoete, 2007; Gelen, 2003; Ho, 2004; Mayer, 1998). In line with the previous research studies, it can be said that these students also determined problem-solving strategies according to unnecessary details in the problem. Ozsoy's study (2007) has revealed that metacognitive skills have a significant effect on students' problem-solving success, and students with high metacognitive levels are more successful. Similarly, Young (2010) has examined the relationship between metacognition and academic achievement and concluded that there is a strong relationship between students' mathematics achievement and metacognition in the process of problem-solving. Likewise, Sevgi and Caglikose (2019) have showed in their study with sixth-grade students that students who use metacognitive skills are more successful in problem-solving processes.

Branigan and Donaldson (2020) have investigated how pupils were engaged with structured thinking activities throughout a school year by conducting an in-depth case study, examining factors that facilitated and/or inhibited pupil metacognition. Their observational data indicates that teacher-pupil interactions are essential for revealing pupil metacognition. According to Branigan and Donaldson (2020), teachers play an important role in encouraging pupils to elaborate on the descriptions of their own thinking and learning, particularly when pupils' initial responses are broad or superficial. Tohir (2019) has stated that there are several factors affecting the creative thinking skills and metacognition level, such as understanding the information of the problem, compiling an appropriate strategy, mastery of the chosen strategy, skills of answer elaboration, and a tendency to rely on the memorization or imitations based on the previous or discussed solutions.

Conclusion

The results show that metacognitive skills were related to problem-solving and they influenced problem-solving outcomes positively. Students who were able to use metacognitive skills gave correct answers to the problem, and they were able to recognize problem requirements and objectives, use various solution strategies and check the correctness of the solution. Students who were not able to use metacognitive skills could not solve the problem successfully and they had difficulty in understanding the problem, determining solution strategies, recognizing their errors, and correcting them. Most of the students also were not good at solving the non-routine problem. Their responses to the questionnaire and explanations about the solutions were not clear and insightful enough. This showed their lack of awareness of what and why they did while solving problems. They generally focused on obtaining an answer without considering its correctness. Since they were not able to establish a logical connection between their solutions and the problem, they could not notice where they made mistakes. Thus, they mostly gave wrong answers to the problem. They were in a tendency to think that there is only one solution to a problem, and they were not familiar with non-routine problems. Although they believed that they controlled and corrected their mistakes while solving the problem, it was found that they did not tend to check, detect, or correct mistakes in their solutions.

Recommendations

This study investigated the metacognitive strategies of 37 middle school students while solving one non-routine problem. The students' solutions to the problem, the data of the questionnaire and interviews were analyzed. Thus, future studies with experimental research designs and more participants need to be conducted. Long-term studies can be carried out to examine the development of students' metacognitive skills because a person's metacognitive level develops with age and problem-solving practices. This study can be replicated with more students at different grades (primary, middle, and high school), and the change in metacognitive skills can be investigated. The number of non-routine problems can also be increased to gain in-depth insights into the students' metacognitive strategies. Routine problems can be asked to the students as well as the non-routine problems to determine whether the students' metacognitive skills differ depending on the problem types. Considering the results of this study, various measures can be taken to develop metacognitive skills in middle school students. For instance, activities in the teaching process can be planned in a way that requires students to use metacognitive strategies, and students can be encouraged to use these skills. The teaching process can be designed in such a way that middle school students can encounter non-routine problems more often.

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