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Abstract

The aim of this paper is to share part of an ongoing study in which we are interested in introducing a Gallery Walk (GW) as an instructional strategy to contemplate in the classroom, in the context of preservice teacher training for elementary education (6-12 years old), to promote students' mathematical knowledge and skills, through problem solving abilities. In this study we intend, in particular, to identify the strategies used by students when solving challenging tasks with multiple approaches, using a GW, as well as characterize their reaction during their engagement in the GW as a teaching and learning strategy. A qualitative and interpretive study, with an exploratory approach, was adopted and the collected data included classroom observations and written productions. The results allowed to identify the strategies used by the participants and to verify the potential of the GW in the quality of the written productions and discussions, which proved to be more effective than in more traditional discussions, allowing to increase the repertoire of solving strategies of each student and communication and collaborative skills; it had a positive effect on the participants' achievements and it was an enjoyable and rewarding experience for all of them.

Introduction

We are living in a complex and rapidly changing world, in which it will be very difficult to keep up without solid knowledge and skills. Thus, teachers should use strategies, inside and outside the classroom, that meet the different types of thinking displayed by students, confronting them with tasks, with multiple solutions, that challenge them to see outside of the box, and motivate them to learn and collaborate with each other (Leikin, 2009; Vale & Barbosa, 2020a). Learning requires an active and reflective engagement on the part of the student in solving meaningful, diverse and challenging tasks (NCTM, 2014; Vale & Barbosa, 2015). So, an effective teaching approach implies adequate instructional strategies, where the orchestration of productive discussions takes place, giving learners opportunities to communicate, reason, be creative, think critically, solve problems, make decisions and understand mathematical ideas (e.g. Vale & Barbosa, 2020a). In this perspective many educational organizations (e.g. CCR, 2015; WEF, 2016) recommend that school must prepare students in the 21st century to be proficient in four skills known as the 4 Cs: Critical thinking (including problem solving) - Make informed decisions or judgements, to achieve the best solution; Communication - Understand and share ideas, thoughts and solutions with others; Collaboration – Provide opportunities for working together to make decisions in favor of a common goal; Creativity – Provide opportunities for new and efficient approaches. In this context, the Gallery Walk (GW) (Fosnot & Jacob, 2010) emerges as an instructional strategy to contemplate in classroom practices which allows students, through collaborative work, to solve diverse mathematical tasks in a creative way, present and discuss their solutions in posters, located around the classroom, to encourage students to examine the posters of their peers. This strategy perspectives learning in an active way and emerges from the experiences and interactions of the learners among their intellectual, social and physical dimensions (e.g. Nesin, 2012; Prince, 2004). On the other hand, many studies recommend that learning requires movement, as an active body incites the brain, making students more engaged, which contributes to a better performance (Clark & Paivio, 1991). It is also important to highlight the relevance of the posters, during a GW, which are tools that enable visualization to foster students' learning as a key component of cognition.

In this sense, teacher training should promote an insight into the nature of mathematics and its teaching, enabling (future) teachers to have different teaching and learning experiences that they are expected to use with their own students. Taking those ideas into account, and after a brief theoretical contextualization, a qualitative study with an exploratory approach (Erickson, 1986) was carried out to introduce a GW as an instructional strategy developed within elementary preservice teacher training for basic education (6-12 years old). Beyond mathematical knowledge, we intend, through a GW, to promote students' problem solving skills, in particular, to identify the strategies used by students when solving tasks with multiple solutions, as well as characterize their reaction during their engagement in this strategy.

The Classroom and the Tasks

In a traditional classroom, what some refer to as 'Triple X' teaching: 'exposition, examples, exercises (Evans & Swan, 2014), and which still prevails in many classrooms, the teacher uses tasks to introduce a new concept, or procedure for a given concept, then students practice that knowledge using similar tasks, mostly of low cognitive demand. It is predetermined what all students should do to solve each task, previously presented by the teacher, using the same technique and concepts. It is unlikely that students will manifest difficulties and surprise the teacher, since the tasks are proposed, immediately, after the students are taught the content and the procedure, thus applying and practicing what the teacher showed. This type of class does not meet the expectations of today's students, that is, nowadays people are no longer successful in life and work only for what they know, reproducing content knowledge, but for what they manage to do with what they know. However, there has been a change from this type of classroom, focused on teaching, to an exploratory classroom, focused on learning, where students learn by doing, understanding, analyzing, and discussing multiple approaches to a problem task.

In an exploratory classroom, we use an inquiry-based learning approach, a term that refers to classroom practices in which students understand, pose questions, explore and discuss. They are encouraged to use different approaches and strategies to solve the same non-routine task, the role of the teacher is less dispensing knowledge and transitions to one of facilitating learning, where students are invited to use methods and ask unforeseen questions that can put the teacher in an uncomfortable situation (Maaß & Doorman, 2013). The role of the teacher is more demanding, since students may use surprising solution-methods and unpredicted difficulties may arise. In this type of class, the tasks that teachers select are fundamental and characterize their work (Smith & Stein, 1998). The use of high cognitive tasks is advocated, involving complex problems or procedures, in particular multiple-solutions tasks, as they allow more students to solve the proposed tasks and can contribute to the development of flexibility, one of the characteristics of creativity, which requires a change in the way we "see" the situations, opposing fixation. The intuitions that lead to short and elegant problem solutions are called Aha! experiences (Liljedahl, 2016; NCTM; 2014, Vale, Pimentel & Barbosa, 2018). We can use tasks with multiple solutions in classroom through two approaches: inviting students to solve each task in more than one way; or wait the occurrence of different approaches naturally within the classroom. In each case, the different approaches should be discussed (e.g. Evans & Swan, 2014; Levav-Waynber & Leikin, 2010; NCTM, 2014; Silver, 1997). We defend the first approach, because it is richer for the discussions, otherwise we risk that only the most common solutions connected to the concepts involved in the task appear and normally students became comfortable when they find a way to solve a task, not looking for another way that could be more intuitive and easier (Barbosa, Vale & Palhares, 2012; Vale & Barbosa, 2018a; Vale & Barbosa, 2018b; Vale et al., 2018).

Liljedahl (2016) has defended the idea that the classroom must provide students *Aha! Experiences*. That is, to establish opportunities where students think individually, but also think collectively, learning together and building knowledge and understanding through action and discussion (Lildejahl, 2016). This implies that the teacher has a repertoire of tasks, with an adequate degree of challenge, to propose to students which can appeal a solution through an insight or an Aha! experience. And here we defend that visual solutions are a privileged field to provide students with this type of experience (e.g. Presmeg, 2014; Vale, 2017; Vale, Pimentel & Barbosa, 2018). We believe that there are tasks with certain characteristics, more geometric or of visual nature, that stimulate students to develop intuition allowing them to have an Aha! experience.

A Visual Approach to the Tasks

Students often show preferences with regard to the way they communicate and the way they understand the information they receive and synthesize, which presuppose that the teacher considers the existence of a diversity of learning styles in the classroom. It has been a common practice in math classes that all students must be exposed to the same mathematical content at the same time and in the same way. However, teachers must be aware that students may have different learning styles (e.g. Krutetskii, 1976; Presmeg, 2014) and may have different preferences in relation to mathematical communication, which can be a difficulty in understanding mathematical ideas, especially when using a single form of communication. According to Krutetskii (1976) students can manifest two modes of thinking: verbal-logical and visual-pictorial. He adds that it is the balance between these two ways of thinking which determines how an individual operates on mathematical ideas, so students can be placed in a continuum with regard to their preference for thinking. In consequence, mathematical educators (e.g. Borromeo-Ferri, 2012; Krutetskii, 1976; Lowrie & Clements, 2001; Presmeg, 1986, 2014; Vale & Barbosa, 2017; Vale et al., 2018) consider three types of students depending on their thinking preferences and methods used in mathematical problem solving: (a) *Non visuals*, Verbalizers (analytical) - those students who have a preference for the use of non-visual solution methods, preferring to use verbal-logical modes of thinking, which involve algebraic, numeric, and verbal representations, even with problems that would yield to a relatively simple way to solve through a visual approach; b) *Visuals,* Visualizers (geometric) - those students who have a preference for the use of visual solution methods, preferring the use of visual-pictorial schemes, which involve graphic representations (i.e., figures, diagrams, pictures), even when problems are easily solved by analytical means. They have preference for an extensive use of visual methods to solve a mathematical problem that can be solved either by visual or non-visual methods; and (c) *Harmonic* (mixed or integrated) - those students who have no specific preference by either verbal-logical or visual-pictorial thinking. They have an integrated thinking style because they combine analytical and visual reasoning.

The important role of geometric thinking in mathematics is widely recognized, therefore, it is necessary to reinforce teaching and learning with processes that allow students to develop their intuition and spatial perception (e.g. Arcavi, 2003; Jones, 2001). In this context, visualization stands out as a fundamental component of mathematical reasoning (e.g. Jones, 2001; Vale & Barbosa, 2017) with strong connections with geometry, however it contributes significantly to learning in other fields of mathematics. Despite these recommendations, it should be noted that visualization is not always developed in the classroom and there are also students who do not show a predisposition to use it. This situation is also reflected in future teachers. We argue, therefore, that, for this purpose, learning mathematics should include practices that lead students to think visually and develop this ability using experiences that require this type of thinking, beginning with the future teachers' preparation. Visual solutions are understood as the way in which mathematical information is presented and/or processed in the initial approach or during the solution of a problem. Visual solutions are considered to include visual methods that involve visual images, i.e., the solution includes the use of different visual representations (e.g. figures, drawings, diagrams, graphs) as an essential part of the solution method. On the contrary, non-visual solutions use methods that do not depend on visual representations as an essential part to arrive to the solution, using others, such as numerical, algebraic and verbal representations (e.g. Presmeg, 1986; Vale & Barbosa, 2017; Vale et al., 2018). Visual representations are of particular importance in the mathematics classroom, helping students to advance their understanding of mathematical concepts and procedures, which allow them to make sense of the mathematical content involved in the problems and become engaged in the mathematical discourse (e.g. Arcavi, 2003; Kruteskii, 1976; NCTM, 2014).

Some studies (e.g. Clements & Del Campo, 1989; Vale & Barbosa, 2017; Vale et al., 2018) show that students taught in a visual way tend to learn to use visual methods. Depending on the solvers' problems and thinking patterns, visual reasoning may be more efficient (or less efficient) than a reasoning that employs more verbal / non-visual modes of thinking. In this sense, the activity of "seeing" is something that can be created, developed, learned. Students often incline to use algebraic information processing paths to the detriment of visual ones, even when the algebraic methods are more complex - a trend that often leads to disastrous results, because these students do not have enough mathematical knowledge to do a complete analysis of the problem situations (e.g. Eisenberg & Dreyfus, 1991).

According with the previously expressed ideas, we can summarize by saying that an effective teaching of mathematics engages students of a whole class in solving and discussing challenging tasks that promote mathematical reasoning and allow multiple approaches. We defend the use of visual images as an important support for all kinds of problems, even those where the visual component is not evident. So, teachers should orchestrate productive discussions around tasks that appeal to the development of the 4 Cs skills of students. In this context we propose the use of a Gallery Walk as a powerful instructional strategy in the classroom.

A Gallery Walk: An Active Instructional Strategy

Most of the failures in mathematics are originated in the affective environment that is created, a fact that can seriously compromise the students' initial expectations and motivations (e.g. Hannula, 2001). In addition to intellectual engagement, it is necessary to consider social and physical engagement, in the mathematics class, which characterizes active learning, considered as “any instructional method that engages students in the learning process” (Prince, 2004, p.223). In this learning process, activities carried out in the classroom are those with which the students engage themselves. The focus is entirely on the student and the activity he/she develops, as opposed to a more traditional approach, as mentioned above. Social interactions, reflected in collaborative work and collective discussions, facilitate sharing of ideas, the development of mathematical meanings and the construction of knowledge, and it is up to the teacher to foster this sense of community (NCTM, 2014; Vale, 2019; Vale & Barbosa, 2020b). On the other hand, movement improves levels of attention and understanding of students, because they are active individuals, who build, modify and integrate ideas interacting with the physical world, materials and colleagues. Traditionally, movement has a prominent place in school through moments such as lunchtime or physical education classes. However, this approach must be rethought and movement must be incorporated as an instruction strategy in the classroom, bringing renewed energy to classroom discussions (e.g. Braun et al., 2017; Jensen, 2011). Active learning is grounded in the socio-constructivist learning theory, and advocates a classroom practice that involves students in activities such as speaking, listening, writing, discussing, reflecting on subjects through problem solving, in small groups, performing simulations or hands-on activities. This type of classes should include time for collaborative work, foster different forms of communication and contemplate freedom of movement. Thus, we can conclude that learning emerges from experiences and effective interactions between the intellectual, social and physical dimensions (e.g. Edwards, 2015; Nesin, 2012).

There are a variety of active learning strategies that can be used in the classroom, we propose a GW adapted from an idea of Fosnot and Jacob (2010). We consider a GW (Vale & Barbosa, 2018a; Vale & Barbosa, 2018b) as an instructional holistic strategy that allows students to solve multiple-solution tasks collaboratively, in small groups, displayed in posters around the classroom (or outside), in a similar perspective to that used by artists when they expose their works in a gallery. Students will have the opportunity to give and receive written/oral feedback about their posters in a "non-threatening" way, contributing for their learning. Finally, a collective discussion is promoted, during which each group has the opportunity to show their poster again and clarify aspects of their work. They acquire new knowledge and / or make that knowledge that is being worked on more robust, and in the particular case of problem solving, when contacting with different solutions, they increase their repertoire of problem solving strategies (Vale & Barbosa, 2020a). The organization of a GW leads students to get out of their chairs and get them actively involved with the mathematical ideas of their colleagues. For many students, moving can help them feel more motivated to get involved in classroom work. Thus, a GW is a strategy that falls within the scope of active learning as it promotes cognitive, social and physical engagement by students.

A GW has numerous advantages and is adaptable to different objectives and contents. A GW can be intended for a fifteen minutes moment, as an ice breaker, at the beginning of a class, or for some hours, or even for a bigger length of time. We can use written comments/feedback in post-its or composed reports. The tasks used can be a question to answer, a definition to analyze, an image to complete, a task to design, a problem to solve, the synthesis of an idea or a concept. It can be used to develop the 4C's skills of the students, providing opportunities for cooperative learning and discussion, to enhance communication skills and creativity, or to extend problem solving skills, promoting learners' discussion and higher-order thinking skills, sharing ideas and gives students the chance to move around the classroom, getting them out of their chairs, interrupting the lethargy that sometimes results from being seated for long periods of time, mainly if students are very young. Another advantage of a GW, that is not object of analysis in this article, it's its contribution for self and peer formative assessment as an essential component of learning that encourages reflection on performance. A GW is a powerful resource for learning because it connects learners to each other and learners to the mathematical topics or skills in a number of interesting and interactive ways, putting the emphasis on challenging tasks with multiple solutions. This means that it can readily be used to enhance conceptual and contextual skills, but not procedural and algorithmic skills. It is not particularly useful with tasks with a low cognitive level of demand, as exercises and procedures without connections. We can point out a disadvantage of a GW; teachers must be aware that it takes time to implement a GW, in particular students may spend too much time merely on the construction and presentation of their poster.

An important component of a GW is the posters to be displayed. Posters are connected with the nature of the knowledge used in our high-tech daily lives, based largely on images full of information. They encourage students to reflect on their learning during their collaborative work, and enable them to show their learning and to learn from other students' ideas. Posters can readily be used to enhance conceptual and contextual skills, but not procedural skills. Posters require succinctness and so students have to select the most important aspects in their work, and display them in an attractive way to have an impact on readers. This helps to develop communication skills and to have synthetic reasoning (Berry & Houston, 1995; Francek, 2006; Zevenbergen, 1999).

Methodology

This work is part of an M&M project (Mathematics in movement - move your mind) in the context of active mathematics. The teacher instruction and the tasks used are important in our work, in which we privilege multiple solution tasks that include visual approaches, because in a class we have students with different ways of thinking, either because of their personal styles of thinking or because of the kind of instruction they were exposed to. In this manuscript we describe a particular study whose aim was to characterize the strategies used by the students (future teachers) when solving tasks with different approaches, in particular to identify the visual ones, and to characterize the reaction during their engagement in the GW strategy.

Setting

The teaching experience described here was developed with fourteen students, future teachers of elementary education, that attended an initial training/instruction course, who attended a curricular unit of Didactics of Mathematics in which a GW was implemented as a teaching and learning strategy in solving problems that address different contents (e.g. areas, fractions), in order to contradict the view of mathematics as a subject that should be taught using more traditional methods, making the mathematics class more active. We adopted an instruction where we use an exploratory approach grounded in an inquiry based learning approach, where the teacher's classroom practices support students through carefully chosen tasks and learning and teaching strategies, promoting struggle and challenging them, using active and inquiry methods of learning. In this context students pose questions, explore, solve, create, move their body and their minds, and evaluate ideas. We followed a qualitative and interpretive study (Erickson, 1986), with an exploratory approach where data were collected in a holistic, descriptive and interpretative way and included classroom observations, written productions about the proposed tasks (solutions, posters, comments of students to the posters) and a written report. At the end of the GW students wrote a short report about the experience carried out, focusing on the following aspects: How did they feel about solving the proposed tasks in a Gallery Walk environment? Will it be possible to use this strategy at any level / theme of teaching? What advantages do they identify in this approach in terms of learning and attitudes? Data analysis was inductive, grounded on the different productions of the future teachers, on the objectives of the study and on the reviewed literature, identifying two main categories: the different solutions and the reactions manifested by future teachers to the GW.

The GW used allows the development of the 4 C's skills that implies the orchestration of productive discussions, giving learners opportunities to communicate, collaborate, reason, be creative, think critically, solve problems, make decisions and understand mathematical ideas (e.g. Vale & Barbosa, 2020a). In this study, where students were organized to think, speak, listen, write and move, we adopted the following steps and procedures during the GW (Vale & Barbosa, 2018a; 2020a) (see Figure 1): 1) *Solving tasks* – students in group solve the same proposed tasks, with multiple solutions, that were two problems; 2) *Construction of posters* – students discussed among themselves how to display their solutions in the poster, in this stage they pushed for their originality and aesthetic sense; 3) *Presentation and observation of posters* – posters were affixed in the walls of the classroom for observation; 4) *Analysis and elaboration of comments* - Each student went through the different posters to analyze the different solutions and after their evaluation, wrote, in post-its, their personal comments, doubts, questions, possible errors, etc. While students discuss their colleagues' solutions, we circulate around the classroom, evaluating students' observations and discussions; 5) *Group discussion*– After this round, the students take their own poster and analyze the contents of the post-its, making a small report; 6) *Collective discussion* –with all the posters fixed again in the wall, the groups orally present the solutions and respond to the comments of their peers. This moment serves for us to highlight some of the ideas expressed, making connections between the different approaches, making syntheses, clarifying doubts and errors and provoking. This provides also an excellent opportunity to give feedback to the content of each poster, which all students already knew, grounded on the work displayed, commented and discussed and through what we saw and heard.

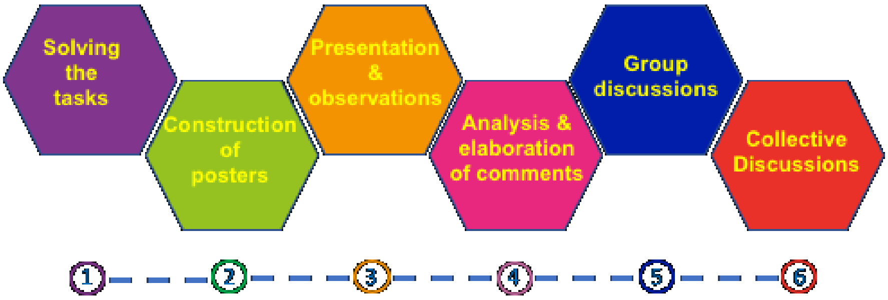

Figure 1. Steps of the Used GW

One aspect that we want to note is that all the dynamics of a GW is developed/ happens around the creation of posters. Posters encourage students to share and reflect on learning during collaborative work, and enable them to learn from other students' ideas (Vale & Barbosa, 2020a, 2020b; Zevenbergen, 1999). Teachers should promote the use of posters as an important tool that enable visualization to foster students' learning as a key component of cognition (Clark & Paivio, 1991; Vale & Barbosa, 2020a, 2020b).

Two previous notes in this work that we think are important to promote an effective learning by students: One is that we encourage students to solve each task in more than one way, allowing multiple strategies to arise within each group, because we believe that it stimulates students' creativity through divergent thinking, by provoking flexibility of thought and, in that sense, we are expecting that a visual solution appears; The other has to do with the fact that all students have to solve the same tasks, this implies a productive engagement during the analysis of the posters, because, as they have already solved and discussed the solution in small groups, they are more motivated to validate the solutions proposed by the peers, identifying errors, or appreciating a different approach.

The Participants along the GW

In this item, we characterize the participation of the students along the GW having as starting point one of the tasks used (see Figure 2) (Vale, 2019; Vale & Barbosa, 2020a).

The area of the square is 4 cm^2. The points P and Q are midpoints of two opposite sides. Given the conditions of the figure, what is the area of the shaded part of the figure? Present more than one process of resolution.	

Figure 2. One of the Proposed Tasks during the GW

The teacher proposed the tasks to the students who solved them in small groups, of 2/3 elements. They started by addressing each of the problems individually, and when they reached a solution or had doubts, they began to discuss possible strategies to arrive to the solution and try to identify which one was more expeditious and interesting (Figure 3). Solutions with different mathematical knowledge emerged and then they were concerned with finding an adequate solution to the level of the students for which they are being trained. The fact that they worked collaboratively facilitated the exchange of ideas and decision-making, in particular about the most effective strategy(s), with some groups presenting more than one solution, especially in the first problem.

After completing the tasks, they started to discuss what would be the best way to present them to their colleagues, so they started to plan the structure of the poster that they would have to display in the room. As a group, they decided which strategy to present, which were the most appropriate representations, how to write the text, among other aspects. There was some concern on the part of the students with the content of the poster and if it would be clear to those who consulted it, realizing the relevance of written communication in mathematics (see Figure 3).

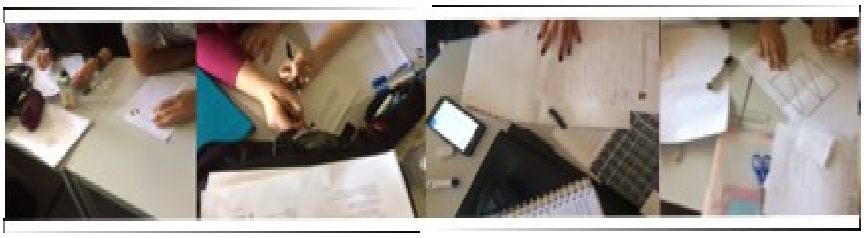

Figure 3. Students Solving the Tasks and Constructing the Posters

After the poster construction, they were displayed in the wall around the classroom, similar to what happens in an art gallery. Each student freely walked the space, carefully observing the contents of the various posters. Individually and autonomously, they made the comments that they considered relevant and also asked questions about aspects that raised doubts. This feedback, in the form of comments and questions, was written in anonymous post-its that were associated with the posters presented (see Figure 4).

Figure 4. Analysis and Comments of the Posters

This step was followed by the analysis of the comments and questions associated with each poster. The groups collected their poster, with their post-its, and read the feedback given by colleagues. They discussed among themselves the relevance of the comments, what they could improve in their work, how to improve the explanation of their reasoning and, in some cases, they detected errors that had gone unnoticed. The feedback given by the peers was a positive contribution to promote reflection on the work done (see Figure 4).

Finally, after each group analyzed the comments and questions raised and decided whether they wanted to clarify some aspect of the poster or even rectify any step, the collective discussion began. The posters were posted in a central area of the classroom, and each group had the opportunity to resume their work, explaining the impact of feedback from colleagues, thus being able to explain some aspects that had become less clear and correct some errors. This discussion was mediated by the teacher responsible for the unit course, who, in addition to what has already been mentioned, was also careful to focus on the diversity of strategies used. Although in this phase the role of the teacher was more interventionist, in the previous phases the action was limited to observing all the work, guarantee the engagement of the students and support whenever requested. During the collective discussion phase associated with the gallery walk, the opportunity arose to analyze in more detail each of the solutions presented by the different groups and answer the questions raised.

This task proposed to the students, preservice teachers, can be used with their own future elementary pupils. It is an easy and common task in elementary mathematics schools and they didn't present any difficulties with its solution. They used different approaches: only visual or only analytical; and visual solutions complemented with analytical solutions. In Figure 5, in the first poster we can see the most common solution used by the different groups. As it was asked the "area" of the shaded part of the square, students automatically used formulas of the square and the area of the triangle, following the traditional way to solve this kind of problem. We classified it as an analytical solution. These more traditional approaches were chosen, because it's the one they're most comfortable with. However, two groups presented a solution, which we consider to be of a visual nature, that uses the part-whole idea (see Figure 5, 2nd and 3rd posters). Here we can say that students had an Aha! experience, because they were able to think with flexibility, induced by the question of presenting another way of solving the problem. On the other hand, they managed to establish connections between two important mathematical topics: area and part-whole fractions. We can say that these students offered also an original solution. These solutions surprised the rest of the colleagues who, after analyzing this approach, found it simpler and more elegant.

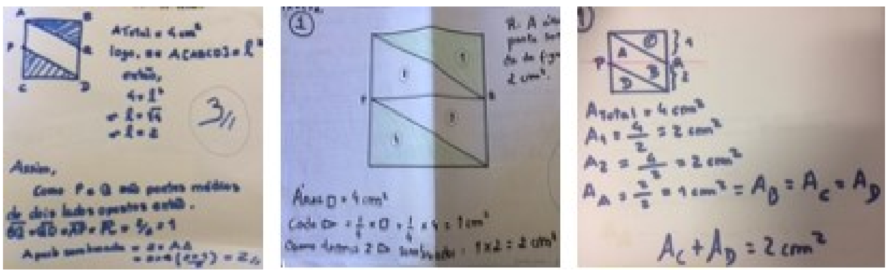

Figure 5. Types of Solutions to the Proposed Tasks

Their main difficulty was how to organize the solution in the poster so that their colleagues understood it, since they had never made a poster in mathematics. After some attempts and group discussions they got a well-organized poster and easy to follow. Overall, the students reacted positively to this experience, showing interest and motivation. In personal terms, they recognized the contribution of a GW to their own learning and, as future teachers, to mathematical learning at the levels of education at which they will teach. These evidences were identified in the comments made by the participants throughout the experience and in the report written by them. Here are some ideas expressed by the students: “an approach that is more enriching than the usual approach of solving individually a task and after only one or two students show how they thought”; “Helps to share ideas”; "Gives the opportunity to the most timid and least confident students to participate without fear of reprisals"; “It allows to observe other strategies and other reasoning and asking colleagues about aspects that have not been so clarified, helping to develop critical thinking”; “It is a good teaching and learning strategy to motivate students to participate in solving mathematical tasks, improving their performance”; “Making the poster makes students realize that it is sometimes difficult to put ideas on paper clearly”; “Breaks with monotonous routines”; “Promotes a free movement”.

Concluding Remarks

Throughout this article, we intended to defend that the selection of tasks used in learning mathematics is crucial, but the actions used to explore them are also important (NCTM, 2014). Tasks must be challenging and with multiple solutions (e.g. Leikin, 2009; Smith & Stein, 1998; Vale et al., 2018) desirably involving various concepts and representations, giving students the opportunity to show flexibility of thinking reaching the different ways of thinking (e.g. Kruteskii, 1976; Leikin, 2009; Silver, 1997), meeting their learning preferences, from the most analytical to the most visual (e.g. Presmeg, 2014; Silver, 1997; Vale et al., 2018), provoking productive discussions (e.g. Evans & Swan, 2014; Liljedhahl, 2016; NCTM, 2014; Smith & Stein, 1998) through an active learning (e.g. Edwards, 2015; Prince, 2004) using a GW (Vale, 2019; Vale & Barbosa, 2018a).

The students’ productions allowed to identify the strategies used by the participants and to verify that, although they continued to choose routinized formulas and procedures, visual solutions also appeared. The GW allowed students to be engaged in peer solutions and classroom discussions, clarifying doubts and increasing the individual repertoire of solution strategies. We observed, and they also stated, that students are more comfortable to discuss their work in this informal setting than in a more formal traditional mode. Students reacted positively to the GW by expressing interest, motivation and recognition of its importance in mathematical learning at any level and it was an enjoyable and rewarding experience for all of them. They also valued this strategy because, by solving the same task, they were able to engage in the analysis and discussions in a more depth the solutions presented by their colleagues. A GW, as a teaching and learning strategy, involved students in the solutions of their peers, when they had to comment on them through post-its and in discussions with each other, allowing them to increase their repertoire of strategies more effectively. than in traditional discussions. We can conclude that the GW allowed to identify different types of engagement by the participants: intellectual (in solving tasks), social (in interactions in small and large groups) and physical (in free movement in the classroom). Preparing posters and a GW was a different and novel experience for these students, future mathematics teachers, and thus provided an innovation which added interest to their learning but we also could verify the potential of the GW for a more effective teaching of mathematics which served as a reference for these future teachers. It was an opportunity for them to contact with other (teaching) learning contexts and also an opportunity to work together.

To close, this work is being carried out with elementary preservice teachers, and the results about the reaction and performance are very similar to the results in other studies developed by us with elementary school pupils (aged 6-12). One interpretation could be that preservice teachers along their compulsory schooling had no contact with similar experiences (GW, visual approaches, multiple solutions). So, teacher education programs should provide opportunities, to experience didactic approaches during the instruction that stimulate preservice teachers' knowledge, solving the same tasks and using the same teaching and learning principles that they are expected to use with their own future students (e.g. Cooney & Krainer, 1996; Sullivan, Clarke & Clarke, 2013; Vale & Barbosa, 2020b; Vale et al., 2018).

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