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Preservice Teachers' Reflections on Learning Number Bases and Teaching Base Ten

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Abstract

This study aimed to reveal the solutions of pre-service teachers to contextual problems related to number bases other than the base ten. Furthermore, it was intended to investigate which issues pre-service teachers discussed about their learning in the problem-solving process, including context, and about student learning in their future teaching practices. To achieve this, task-based interviews were held with 12 pre-service primary school teachers. During semi-structured one-on-one task-based interviews, some questions were asked to reveal pre-service teachers' way of thinking about the problems and their reflection on the learning. In this context, data were obtained from the audio recordings and the students' written responses. Besides, researchers' notes are used as supporting documents for the validation of the data collection process. The main findings of the study revealed that context is a facilitator for preservice teachers in understanding number bases other than base ten. They gained insight and provided instructional explanations on how to teach the student by understanding the different number bases. Furthermore, context enabled preservice teachers to reflect on how to teach better by considering their own learning processes.

Introduction

In educational programs designed and implemented by many countries to train pre-service primary school teachers (hereafter referred to as "pre-service teachers," or "PSTs"), there are both content courses and pedagogical courses, though in varying proportions. Teacher training programs aim to enable their students to acquire content and pedagogical knowledge. In addition to this, as an intersection of these two domains, courses aiming to support pedagogical content knowledge are also exist.

As an important focus of pedagogical content knowledge, PSTs must master the curriculum they will teach, develop a correct understanding of the program's concepts, and reflect their learning processes in their teaching processes. There are various approaches to create a link between the curriculum that PSTs learn and the curriculum they will teach. This objective can be more effectively accomplished by designing learning processes through which content knowledge and content-specific pedagogical knowledge are associated. PSTs' re-learning regarding school mathematics can be accomplished by revisiting topics taught at those grades since this re-learning supports them to reconstruct their mathematics knowledge and become more aware of mathematics practices (Pournara &

Adler, 2021). In this way, raising their awareness of their learning would provide an opportunity to gain empathy for their future students' learning processes. Besides, getting familiar with reflection early on as a potential tool for professional development would provide useful information for them (Mulryan-Kyne, 2020). Thus, PSTs should be given time to reflect on students' mathematics learning (Kilic & Dogan, 2021).

One of the main areas where PSTs should transfer their learning into teaching is "Numbers and Operations." Numbers and operations are the most dominant learning areas of the Primary Education Mathematics Program (Verschaffel et al., 2007). However, PSTs do not show competence (e.g., Hill et al., 2008). PSTs themselves state that they have difficulties learning some primary school subjects (Akyüz, 2016). Even though they carry out the procedures, they have difficulty explaining the process conceptually (Thanheiser, 2010). In this context, involving PSTs in a learning process associated with the mathematics they will teach and providing the opportunity to re-learn the subjects and concepts they are obliged to teach in primary school may contribute to their Mathematical Knowledge for Teaching (MKT) (Hoover et al., 2016). This study focuses on the solutions of pre-service teachers to contextual problems related to number bases other than the base ten. Furthermore, investigating the issues pre-service teachers discussed about their learning in the problem-solving process, including context, and about student learning in their future teaching practices is subject of this research.

Theoretical Underpinnings

Re-learning mathematics involves extending, replacing, or reorganizing one's knowledge (Zaskis, 2011). Learning something for the first time is different from learning it again. Re-learning supports looking at the learned subject as part of a bigger concept (Zaskis, 2011). PSTs are already familiar with operations in base ten, which are taught at primary school. During re-learning concepts, this situation may prevent them from thoroughly investigating concepts in whole numbers (Hopkins & Cady, 2007).From this point of view, arithmetic operations in alternate bases can develop PSTs' mathematical knowledge. (Spence, 2010). Fasteen (2015) defined an alternate base number system as a positional number system based on a number other than ten. For example, instead of having a ones place, tens place, and hundreds place for any number in base eight, there are ones place, 8s place, and 64s place. For example, 123eight refers to 1 group of sixty-four, 2 groups of eight, and 3 ones.

While PSTs work with other bases, they can develop their learning in many subjects, such as numbers and number relationships (Yackel et al., 2007). Moreover, while engaging in learning about different bases, they can experience a process similar to what primary school children learn for the first time (McClain, 2003). Since "effective teaching of mathematics requires that practicing teachers focus simultaneously on mathematics and children's ways of reasoning about mathematics" (Phillip & Thanheiser, 2019, p. 391), PSTs can reflect on how students learn in primary school by analyzing their learning processes. In other words, they can empathize with the primary school student in learning the new concept of the base ten system and related concepts (Cady et al., 2008; Thanheiser, 2009). Furthermore, PSTs continue their routine thinking processes toward base-ten systems, a subject they are familiar with.Thus, PSTs should be supported in developing different solution strategies (Tirosh, 2000), think of themselves as learners, and construct future teaching practices (Ell et al., 2012).

When students receive appropriate support, they become the inventors of a set of strategies for multi-digit calculations. According to van Putten, Brom-Snijders, & Beishuizen (2005), appropriate support is an opportunity to study contextual word problems to develop strategies rather than practicing procedures. Contextual problems make it easier to develop PSTs when learning in different number bases other than base ten. For example, the box problem can be a facilitator in understanding the concept of trading. In this story, one packet represents six pencils, and one box represents thirty-six pencils. That is 123six, which means one box, two packets, and three pencils.

PSTs should be allowed to re-learn problem-solving (Son & Lee, 2021). Dealing with the context of problems can help understand student thinking (Jacobson et al., 2018). PSTs re-learning concepts via contextual problems and thinking reflectively about their learning processes can provide an opportunity to make inferences about how to teach these concepts. In previous research studies, findings indicated that PSTs explore place value understanding, multidigit addition, and subtraction of whole numbers by using alternate bases. McClain (2003) stated that PSTs were able to explore different strategies that could be used by their future students. In addition to this, contextual problems are found to facilitate the understanding of place value, addition, and subtraction. In the candy factory problem, eight candies fit in a roll, and eight rolls fit in a box. Similarly, Yackel et al. (2007) focused on PST's number sense in the candy factory context. Thus, contextual problems were used to develop conceptual understanding instead of meaningless algorithms.

In light of these, in this study, Mathematical Knowledge for Teaching (MKT) was adopted as the theoretical framework. MKT can be defined as "a kind of depth and detail that goes well beyond what is needed to carry out the algorithm reliably" (Ball et al., 2005, p. 21). MKT has an important place in the description of what a mathematics teacher should know (Scheiner et al., 2019). It includes explaining how students can understand the basics and showing why this kind of practice works (Ball et al., 2005). Beyond teachers' or preservice teachers' understanding of how their knowledge is developed, MKT focuses on using this knowledge in teaching (Ball et al., 2008).

The mind develops a need-oriented thinking structure that includes flexibility and enhances the student's knowledge by focusing on his or her learning. Thus, a starting point for the studies to be carried out by teachers and preservice teachers to develop MKT may be to evaluate this data to realize their current concepts and to create a responsible action plan for developing their understanding of mathematics. Thinking about "What is this?" and "How did you learn?" and "How will you teach?" and "Why?" questions, as discussed in this study, can promote reflective thinking and help reveal the current situation of PSTs in their mathematical knowledge for teaching particular subjects.

Purpose of the Study

This study aimed to reveal the solutions of pre-service teachers to contextual problems related to number bases other than the base ten. Furthermore, it was intended to investigate which issues pre-service teachers discussed their learning in the problem-solving process, including context, and about student learning in their future teaching practices. The following stages were designed to increase prospective teachers' mathematical knowledge for teaching with a focus on "base ten and other bases." Thus, this research aimed to find answers to the following research questions: What are the solution processes of pre-service teachers for problems including the context of other number bases other than base ten?, and What are pre-service teachers' reflections on their learning processes and future students' learning?

Method

Design of the Study

This study aimed to reveal how pre-service teachers solve problems that contain context and number bases other than base ten, their reflections on the solution processes, and their reflections on their future teaching experiences. The emphasis in this study is on how the preservice teachers will relate their learning processes to the curriculum content they will teach. In this regard, the task-based interview was used in this study as an approach that supports the emergence of the participants' existing knowledge as well as their developing knowledge and enables them to observe and perceive the problem-solving process (Goldin, 2000). Task-based interviews are used to present a concrete example of how preservice teachers can develop a conceptual understanding that will facilitate them in teaching mathematics within the context of a mathematics course given at the undergraduate level. The general view of the design of the study is seen in Figure 1.

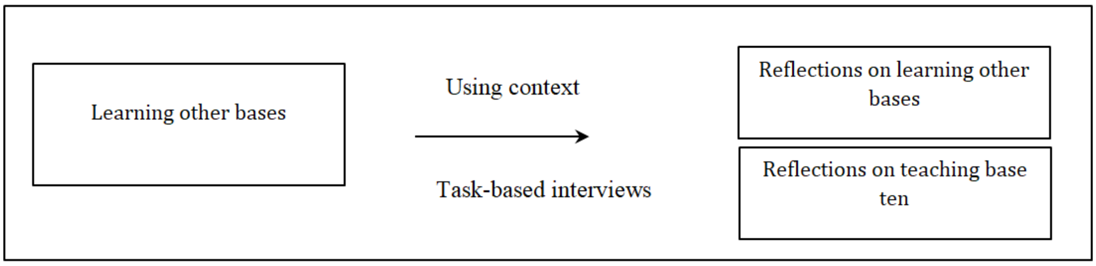

Figure 1. Design of the Current Study in General

Participants

This study's participants are preservice primary teachers. For the task-based interview (Goldin, 2000) process, a sample consisting of 12 students was selected. In line with the aims and philosophy of the university, mathematics and mathematics education courses are included in the Primary School Education Program. PSTs are expected to be open to reflecting the knowledge they learned in mathematics lessons on the mathematics they will teach, which is the general approach in this context. All participants are girls. Although this is not a purposeful selection, it is like this because there are predominantly girls in the program. This research study process was reviewed and approved by the Human Subjects Ethics Committee (HREC) of TED University with the approval number 2020/10.

Data Collection Tools

Task-based interview sessions are usually semi-structured in mathematics education (Maher & Sigler, 2020), and in such interviews, recordings such as audio or video are used to analyze the verbal expressions of the participants later and sometimes written answers and field notes to support them. (Mejía-Ramos & Weber, 2020). In this study, the "Different Bases Problem Form" developed by the researchers was used to reveal the PSTs' learning processes and reflective thoughts on numbers bases other than base ten, such as base eight, base five, etc. Various literature sources and former practical experiences of researchers as teachers were used in creating the questions in the form, and it was decided to include different question types in three parts. In the first part, two questions aimed to reveal PSTs' current knowledge on different number bases. An example of a question found in Part A is given in Figure 2.

Part A- Question 1. Express the number 1236 to base 2. Justify your answer.

Figure 2. Sample Question from Part A

The second part of the form included five questions requiring simple operations with numbers on different bases. An example of the questions in Part B is presented in Figure 3.

Part B- Question 5. 320 _____ - 42 _____ = 256 _____ on what base should the subtraction occur to be correct?

Figure 3. Sample Question from Part B

In the third part of the form, PSTs' learning processes were supported by the main context problem. The problem consisted of four sub-problems, each of which was designed in such a way as to promote PSTs' learning the concepts in problem situations associated with real life. Figure 4 depicts the main question stem in this section as well as two sample sub-problems related to this question.

Part C- Question1. A pencil maker; It sells 6 pens in a package and 6 pack pens in a box.
a. Buying 1 box, 2 packages, 3 pens, and how many pens did Esra get?
...
d. Since this pen maker has to produce 600 items in a week, express the total number of items to be produced in the remaining days as boxes / packages / pieces.

Figure 4. Sample Question from Part C

After the formulation of the questions, two academics who are experts in mathematics education were asked to express their opinions on the suitability of the questions in the form in terms of mathematical accuracy, language, and comprehensibility. In this direction, the wording of a question was corrected in the second version of the form with the comments received. Besides, two different questions were rewritten as part of a single question. After the revision of the form with the expert opinion, a pilot study was conducted with a PST who was not included in the study sample.

Task-Based Interview Process

During the study process, PSTs were provided with problems and exercises related to different number bases in task-based interviews (Goldin, 1997). For this, (1) questions are posed, (2) heuristic suggestions are provided when it is not spontaneous, (3) heuristic questions are used for guidance, and (4) metacognitive questions are provided (Goldin, 1997). The interview protocol was semi-structured, "allowing for modifications depending on the judgment of the researcher" (Maher & Sigley, 2020, p. 821). Some questions were asked during the solution processes to reveal the ways of thinking based on questions when necessary. Likewise, general questions about the study and the same form were asked by the researcher to PSTs at the end of the study. In this context, data were gathered from unstructured interviews' voice recordings and written responses. The data were collected during the periods when PSTs were available. The interviews lasted for about 30 minutes, and during the interviews, voice recordings were obtained with the PSTs' consent.

During the task-based interview, some questions were asked by one of the researchers to provide standardization and reveal PSTs’ reflective thoughts. For example, after PSTs answered the question in Section B of the form based on the solution, the following questions were asked: "What difficulties could you encounter when you thought you were an elementary school student?" Similarly, in Section C, the researcher asked the following question: "If you were an elementary school student, by which method would it be easier for you to learn addition?" Apart from these, the researcher tried to reveal the reflective thoughts of the PSTs with other necessary questions in the process of problem task-based interviews.

Data Analysis

The recordings, accompanied by transcripts, researchers’ notes, and participants’ solutions, were utilized to analyze the data. The data obtained during one-on-one task-based interviews were examined in depth by the two researchers. Accordingly, codes and themes were created. The entire data set was coded twice by both researchers independently, four times in total. The codings on which the researchers reached consensus or disagreement were re-examined, and the codes were reconciled. In this process, the level of agreement in coding between the two researchers was found to be 0.96. Researcher notes are used to validate and support the data collected during task-based interviews.

In the data analysis, the answers given to the questions by PSTs, their reflective thoughts about the methods and processes they used while solving these questions, their predictions about how primary students will learn, and their solution suggestions for anticipated problems were handled separately. In the creation of these categories, both the questions asked during the semi-structured task-based interviews and the classification of the answers given by the PSTs in the solution and reflection processes were used. The findings are presented under these four main categories of data analysis. The first general category included the section where PSTs evaluate this process based on their own learning process. Under the heading of solution strategies and processes, their reflections regarding difficulties in their learning processes and their attitude toward the problem were presented. In the third category, considering their future students' learning processes, the difficulties that PSTs may experience as teachers were examined, and in the fourth category, the methods of overcoming these difficulties were examined. Besides, the PSTs' perspectives on learning in context (by comparing the operation questions used in the scale with the operation questions given in context) were presented throughout the categories.

Results

Solution Strategies and Processes

PSTs added and subtracted natural numbers with different bases, both as a standard algorithm and in the context of a scale. PSTs had many difficulties while adding and subtracting natural numbers on different bases. One of the most common difficulties has been forgetting the carryover, adding to a different place besides (the digit on the right), or subtraction with carrying over. Since PSTs had grown accustomed to working in base ten when performing addition and subtraction, they expected the sum of the two numbers to be ten to carry over. For this reason, they had difficulty with addition and subtraction when carrying over.

One PST stated that the addition and subtraction processes with different bases are confusing and complicated. PST1 displayed the answer to the question, "So here I take it, and here I translate it from base ten to base eight." (The student states that the student added seven and two in the base ten and then converted it to the base eight.) Here is how it remained: (This demonstrates that the student received nine and one as a remainder while carrying over.) Figure 5 shows the PST1 solution of addition with carryover.

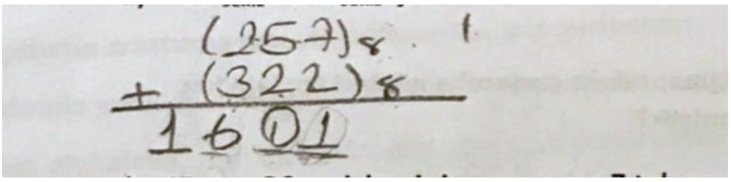

Figure 5. Solution of Addition with Carryover (PST1)

One PST stated that the addition and subtraction processes with different bases are confusing and complicated. PST1 displayed the answer to the question, "So here I take it, and here I translate it from base ten to base eight." (The student states that the student added seven and two to the base ten and then converted it to the base eight.) Here is how it remained: (This demonstrates that the student received nine and one as a remainder while carrying over.) Figure 5 shows the PST1 solution of addition with carryover.

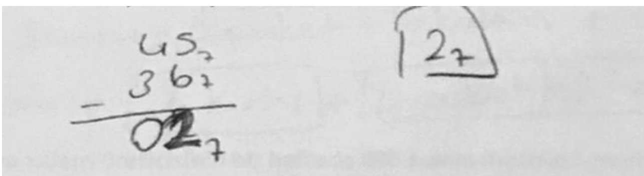

Figure 6. Solution of Subtraction with Carryover (PST4)

In this solution, PST4 first carried over to subtract six from five and then subtracted six from fifteen. PST4 had to take it in hand and subtract six out of twelve to get six. Some PSTs said they had trouble adding and subtracting because they didn't understand the big picture. Another stated reason was rediscovering numbers on a different base. Students indicated that rediscovering numbers on different bases is complicated for them. PST5 made a statement on this issue: "Well, when it is basically like that, for example, I always do addition and subtraction in base ten. While working in the base, I believe I do addition and subtraction in base ten. That's why I'm perplexed" (PST 5).

Here, PSTs emphasized that students understood carrying. However, PSTs had difficulties with the operation of different bases, which students had just learned. PST10 defined it as a habit of trading in base ten and explained it as follows: "When I think conceptually, I compare numbers in base ten. When I compare numbers on different bases, some points are not connected. We are used to it like this: I went to the neighbor, transformed a tenner into unity, and got used to taking it as ten. You know, when we subtract in the base six, we do so not from ten but by adding six and subtracting from six. In this subject, the PSTs also stated that dividing multi-digit numbers that are not on the scale was difficult. For example, PST11 stated this difficulty as follows: "There is no such example here. When I have to divide multi-digit numbers, I have a hard time with it. So I generally have difficulty with the division." (PST11).

While explaining their learning processes, most of them showed a positive attitude towards problems, but a few were prejudiced and explained that they were not good at solving problems. An example of a negative attitude towards problems is as follows: "I have difficulty with such problems." (PST1). As another example, one of the PSTs said that the student could solve this problem despite normally having a negative attitude toward problems: "Normally, I have more difficulties with problems. However, since this is the first time I have seen it, this is the part I know. It was easier to do the part I knew." (PST2).

Self-reflection on Learning Processes

PSTs shared their thoughts on the effect of context on learning, considering the four operational problems and solution processes given within the context given in the scale. Some of the PSTs stated that the context embodied the formal side of mathematics. They think that this situation enables the learner to make sense of the four operations conceptually. In this problem given in context, PSTs worked on base 6. However, since they do not know that they are working on base 6, they solve the problem according to the given context. Moreover, when they complete the process, they realize that breaking or getting it means breaking boxes or creating boxes from packages. Figure 7 shows the solution of PST6.

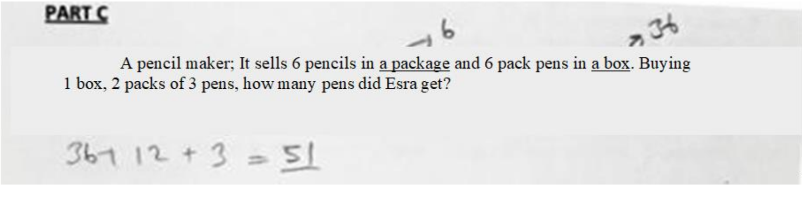

Figure 7. Solution of the Problem (PST6)

Thinking that the use of context in this situation has a positive effect, PST6 explained the situation that future students would experience if they learned four operations in context and their positive effects: "My students would experience enlightenment like me. It would help with carryover. Mathematics is abstract for them because, for them, it is like this apple-pear example. We are simplifying it." (PST6). PST2, who thinks similarly to PST6, stated that teaching in context is more permanent, saying, "So it is better as it was given in part C (in context) because it makes concrete in my mind." Call it six packs or something. This way will be more permanent." (PST2).

For PST10, who thinks that context makes learning easier besides embodying mathematics, first context and then theoretical knowledge should be given based on students' learning processes. "They would learn more easily by giving it in context. It would be better to warm up with a little practice and then theoretically, because it learns without realizing it in context" (PST 10). In addition to these, some PSTs think that using context is more logical. PST9 explains this issue as follows: "So it would be better if I first gave the algorithm and then reinforced it with problems in context. Just providing the algorithm leads to memorization. We can first explain the rules in context and then reinforce them with an algorithm." (PST9).

Predictions Related to Students' Learning Problems

PSTs evaluated the use of context in teaching, reflecting their learning processes to teach better. They predicted where their students might have difficulties in the future by considering their learning processes. PSTs stated that the basis of the difficulties students experience while doing operations might be related to the fact that they do not understand place value conceptually. PSTs think that they will have difficulty with addition and subtraction when there is a missing part in understanding place value. PST3 explains students' thoughts on this issue as follows: "I am confused about place value. For example, which one is tens, and so on. They can also mix things up. They should learn place value very well. Otherwise, they can mix them." (PST3). PST10 states that students may make mechanical mistakes because they do not make sense of the place value. For example, it is like writing a two-digit number on a digit, and it is explained as follows: "Again, if there is zero, if there is a subtraction with carryover, for example, if the student takes 10 from the side and writes ten to that digit like 10, then the student can say that the two-digit number has turned into a three-digit." (PST10). PST11 suggests that their difficulty with place values is because they memorize them instead of understanding them conceptually. Moreover, she explains as follows: "They struggle because they do not see in a concrete way why they use carryover, that is, why this place is decreasing or another one increasing, they do not understand why this step is getting it from the neighbor."

PSTs think that students may have difficulty adding and subtracting with carryover concerning the place value, similar to how they proceed themselves. PST3 gave the following example as a challenge for students: "For example, while trying to subtract six from five, by saying that it is not possible because five is not bigger than six. They may forget about carryover." (PST3). She expressed that since they had the problem of subtracting the larger number from the smaller number, other students may also experience it in the future. "I think we are adding, but it remains in our hands, or students may have difficulty with the rest. In carrying over, I mean that something would have been difficult for me. I mean, I have difficulty when I subtract when the upper number is smaller than the number below." (PST3).

Solutions Offered for Anticipated Problems

One of the solutions to the difficulties is visualization. PST2 thought of visualization as embodying a concept. She explained the reason as follows: "So actually, teachers should teach by showing figures to embody. I think they think very broadly because their imaginations are powerful. Nevertheless, if they see it, they can make sense of it more easily. It should be embodied a little more in mathematics." (PST2). In line with this thought, PST5 explained why it is crucial to visualize the problem as follows: "Let me say that it is valid for every field; for example, when something is shown directly, everybody has something in mind. If it is not used, then it is forgotten. The student learns more easily if it is embodied" (PST5).

On the other hand, one of the PSTs stated that students could not make sense of the problem without drawing when the context was given. For example, PST11 stated that "I think learning operations with algorithms would be easier because I could understand the problem rather than drawing it. We learn by memorization. We are doing something, but we do not know why we are doing it." (PST11). Besides visualization, another solution for obstacles is a real-life example. Some PSTs think that the difficulties encountered can be overcome by connecting learning with daily life and asking the question in context. PST1 explains this situation as follows: "So maybe we link it with daily life. Here eight chocolates represent one package. Perhaps we can ask this way: "How many packages should you buy to take eight chocolates?" (PST1). According to PSTs' views, making connections with related topics can also help overcome those difficulties. PST2 suggested: "So things might sound strange. They need to learn place value well" (PST2).

Finally, the PSTs thought about how they should teach this subject based on their learning processes. PST2 explains her thoughts as follows: "It would be easier for students to first embody the operation in context and then switch to the operation algorithm." (PST2). In a parallel vein, PST4 added the statement: "I first describe it as a box and two packages, so I teach in context. Then I put them together. Finally, I teach the algorithm of operations" (PST4).

Discussion

This study shows that pre-service teachers had some learning deficiencies related to base ten and other bases within the broader learning area "Numbers and Operations" which occupies an important place in the primary mathematics curriculum. In this respect, the study overlaps with the literature's findings (e.g., Hill et al., 2008). One of the essential aspects of this study is that it was carried out with first-year PSTs. PSTs may develop an awareness of the mathematics they will teach as early as their first year of undergraduate studies, laying the groundwork for future increases in their awareness of other subjects. Besides, detecting the shortcomings in their knowledge at an early stage can set an example and motivate them to overcome their possible deficiencies in other areas (Mulryan-Kyne, 2020).

Another significant aspect of the research is the role of task-based interviews and reflection in bridging the gap between learning and teaching. Reflecting on their subject matter knowledge of mathematics allowed the PSTs to reflect on points they had not thought about before. Reflection is a professional development tool that can be used continuously to enhance knowledge and its development (Sellars, 2014). Pre-service teachers' encounter with reflective thinking in more active classrooms may lead them to see it as a professional development tool (Mulryan-Kyne, 2020). Thus, PSTs should have time to reflect on students' understanding of mathematics (Kilic & Dogan, 2021).

The questions posed to the PSTs in the semi-structured task-based interviews encourage them to be more reflective and to consider the transition from learning to teaching and considerations from their learning to teach. Using certain guiding questions during the reflection process renders reflective thinking more suitable for its purpose (Shanahan & Tochelli, 2014). In this context, it can be argued that conducting task-based interviews in a semi-structured way, thus allowing different guiding questions where necessary, promotes reflection of PSTs.

In the study, a process was designed for the PSTs to transfer their learning about different bases and their operations, which is a subject they were not acquainted with at all, to the base ten subjects that they already learned in primary school. In this fiction, it was observed that teacher candidates tried to benefit from their previous learning in aspects such as base ten and problem-solving. Involvement in such an experience encouraged them to potentially avoid unpleasant learning experiences that their students might have in the future. Furthermore, despite having a background, difficulties appear to have helped pre-service teachers empathize with primary school students (Cady et al., 2008; Thanheiser, 2009). Relearning is different from new learning (Zaskis, 2011). The use of reflection as a metacognitive tool in this study facilitated the process of looking at PSTs' teaching in this way.

The primary focus of PSTs, even after graduation, is to adhere to the lesson plan and complete the course material rather than provide quality education (Barnhart & van Es 2015). This preliminary concern for PSTs in the first year of prioritizing student learning is considered crucial. However, it is recommended that longitudinal studies be conducted on whether the same pre-service teachers preserve this awareness in further teaching planning.

When the findings of the part of the PSTs showing their performance regarding subject matter knowledge were examined, it was revealed that the majority of the PSTs discovered that solving problems by creating context is more beneficial for both their learning process and that of students'. This is consistent with the previous literature (e.g., Philipp et al., 2002). On the other hand, a few students argued that the context was more confusing and stated that the procedural questions given directly without context would contribute to a better understanding of the subject. This may be related to students' past learning experiences and their perspectives on and experiences with problem-solving. In this context, pre-service teachers' learning experiences and attitudes about certain teaching subjects and processes can be measured in advance in other studies.

With the method implemented in this study, students initially learn four operations concretely and in context. Then, they learn the theoretical part and the more algorithmic part. Students believe that learning with this method will be more permanent since teaching moves from concrete to abstract. Besides, PSTs think that the difficulties (e.g., carryover) students will experience originate from incomplete learning about place value. Therefore, they realized how fundamental the place value issue is. They also determined that it is indispensable to make sense of the number. This may give a clue that they will pay more attention while planning their teaching about place value.

Conclusion

PSTs need to command the curriculum they will teach, develop the correct understanding of the program's concepts, and reflect on their learning processes by reviewing it. Within this study's scope, it is intended that preservice teachers' understanding of different number bases will be a reflective learning experience for the concepts of base ten and place value they will teach. While the PSTs experienced how primary school students will learn a new base system through their learning processes, they also discovered the facilitating role of contextually created problems in providing this. Thus, this study has shown that contextualization can be used to develop pre-service teachers' knowledge about place value and number bases. Besides, reflection can act as a facilitator for how PSTs can transfer what they learn about other bases to their future teaching. This study provides the PSTs with the opportunity to review information about the knowledge base of base ten, dealing with different bases. The development of PSTs' MKT can be supported by enabling them to re-learn for different subjects in other studies.

Note

A part of this study was presented as an oral presentation at the International Conference on Research in Education and Science (ICRES 2023) Conference

References

Akyüz, D. (2016). Bir öğretmen adayının çözüm stratejileri: Sayıları sekizlik tabanda yeniden keşfetme [Solution Strategies of a Teacher Candidate: Rediscovering Numbers in Base Eight]. *İnönü Üniversitesi Eğitim Fakültesi Dergisi, 17*(2), 199-216. https://doi.org/10.17679/iuefd.17283573.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 29*(3), 14-17, 20-22, 43-46. http://hdl.handle.net/2027.42/65072.

Barnhart, T. & van Es, E. (2015). Studying teacher noticing: Examining the relationship among pre-service science teachers' ability to attend, analyze and respond to student thinking. *Teaching and Teacher Education, 45*, 83–93. https://doi.org/10.1016/j.tate.2014.09.005.

Cady, J. A., Hopkins, T. M., & Hodges, T. E. (2008). Lesson study as professional development for mathematics teacher educators. *Inquiry into Mathematics Teacher Education, 5*, 119-129.

Ell, F., Hill, M. & Grudnoff, L. (2012). Finding out more about teacher candidates' prior knowledge: Implications for teacher educators. *Asia-Pacific Journal of Teacher Education*, *40*(1), 55–65. https://doi.org/10.1080/1359866x.2011.643760.

Fasteen, J. I. (2015). *An Investigation of the Role of Alternate Numeration Systems in Preservice Teacher Mathematics Content Courses* (Unpublished Doctoral Dissertation, Portland State University).

Goldin, G.A. (1997). Observing mathematical problem solving through task-based interviews. *Journal for Research in Mathematics Education, 9,* 140-177. https://doi.org/10.2307/749946.

Goldin, G (2000). A scientific perspective on structures, task-based interviews in mathematics education research. In: Lesh, R, & Kelly A, E. (eds) *Research design in mathematics and science education* (pp.517–545). Hillsdale.

Hill, H. C., Ball, D. L. & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372-400.https://doi.org/10.5951/jresematheduc.39.4.0372.

Hopkins, T. M. & Cady. J. A. (2007). What is the value of @*#: Deepening Teachers' Understanding of Place Value? *Teaching Children Mathematics, 13*(8), 434-437. https://doi.org/10.5951/TCM.13.8.0434.

Hoover, M., Mosvold, R., Ball, D. L. & Lai, Y. (2016). Making Progress on Mathematical Knowledge for Teaching," *The Mathematics Enthusiast, 13*(1), 3-34. https://doi.org/10.54870/1551-3440.1363

Jacobson, E., Lobato, J. & Orrill, C. H. (2018). Middle School Teachers' Use of Mathematics to Make Sense of Student Solutions to Proportional Reasoning Problems. *International Journal of Science and Mathematics Education, 16,* 1541-1559. https://doi.org/ 10.1007/s10763-017-9845-z.

Kilic, H. & Dogan, O. (2021). Preservice Mathematics Teachers' Noticing in Action and in Reflection. *International Journal of Science and Mathematics Education.* https://doi.org/10.1007/s10763-020-10141-2.

Maher, C. A. & Sigley, R (2020). Task-Based Interviews in Mathematics Education. In Lerman, S. (Ed.), *Encyclopedia of Mathematics Education,* 821-824. https://doi.org/10.1007/978-3-030-15789-0_147.

McClain, K. (2003). Supporting preservice teachers' understanding of place value and multidigit arithmetic. *Mathematical Thinking and Learning, 5*(4), 281-306. https://doi.org/10.1207/S15327833MTL0504_03.

Mejía-Ramos, J. P. & Weber, K. (2020). Using task-based interviews to generate hypotheses about mathematical practice: mathematics education research on mathematicians' use of examples in proof-related activities. *ZDM, 52*, 1099-1112. https://doi.org/10.1007/s11858-020-01170-w.

Mulryan-Kyne, C. (2020): Supporting reflection and reflective practice in an initial teacher education programme: an exploratory study, *European Journal of Teacher Education*. https://doi.org/10.1080/02619768.2020.1793946.

Phillip, R. A. & Thanheiser, E. (2019). Seeing Mathematics through the Lens of Children's Mathematical Thinking: Perspective on the Development of Mathematical Knowledge for Teaching. In *International Handbook of Mathematics Teacher Education: Volume 1* (pp. 391-417). Brill Sense.

Pournara, C. & Adler, J. (2021). Revisiting School Mathematics in Pre-service Secondary Teacher Education: Purposes, Opportunities, and Challenges. *International Journal of Science and Mathematics Education*. https://doi.org/10.1007/s10763-021-10150-9.

Sellars, M. (2014). Reflective Practice for Teachers.Sage.

Shanahan, L. E. & Tochelli, A.L. (2014). Examining the Use of Video Study Groups for Developing Literacy Pedagogical Content Knowledge of Critical Elements of Strategy Instruction with Elementary Teachers. *Literacy Research and Instruction, 53*(1), 1–24. https://doi.org/10.1080/19388071.2013.827764.

Son, J-W. & Lee, M. Y. (2021). Exploring the Relationship Between Preservice Teachers' Conceptions of Problem Solving and Their Problem-Solving Performances. *International Journal of Science and*

Mathematics Education. https://doi.org/10.1007/s10763-019-10045-w.

Spence, D. J. (2010). Extending K-8 Mathematics Concepts in Alternate Bases. In Proceedings of the Annual Meeting of the Georgia Association of Mathematics Teacher Educators (Vol. 4, No. 1, pp. 21-26).

Thanheiser, E. (2009). Preservice elementary school teachers‘ conceptions of multidigit whole numbers. Journal for *Research in Mathematics Education, 40*(3), 251-281. https://doi.org/10.5951/jresematheduc.40.3.0251.

Thanheiser, E. (2010). Investigating further preservice teachers’ conceptions of multidigit whole numbers: Refining a framework. *Educational Studies in Mathematics, 75*(3), 241–251. https://doi.org/10.1007/s10649-010-9252-7.

Tirosh, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions. Journal for Research in Mathematics Education, 5-25.https://doi.org/10.2307/749817.

van Putten, C. M., Van den Brom-Snijders, P. A. & Beishuizen, M. (2005). Progressive mathematization of long division strategies in Dutch primary schools. *Journal for Research in Mathematics Education, 36*(1), 44-73. https://doi.org/10.2307/30034920.

Verschaffel, L., Greer, B. & De Corte, E. (2007). Whole number of concepts and operations. In: Lester, F.K. & M. National Council of Teachers (Eds.), *Second handbook of research on mathematics teaching and learning: A project of the national council of teachers of mathematics* (pp. 577–628). IAP.

Yackel, E., Underwood, D. & Elias, N. (2007). Mathematical tasks designed to foster a reconceptualized view of early arithmetic. *Journal of Mathematics Teacher Education, 10*(4–6), 351–367. https://doi.org/10.1007/s10857-007-9044-x.

Zazkis, R. (2011). *Relearning mathematics: A challenge for prospective elementary school teachers*. IAP.

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