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Representations Used by Primary School Students in Solving Math Problems

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Abstract

This study examined the types of representations that primary school children create in solving different types of problems, the relationship between these types of representations, and the accuracy rates in solving the problems. The study was conducted with 80 students attending primary school's 2nd and 3rd grades. An achievement test prepared by the researchers, containing eight different problem types, was used as a data collection tool. A total of 279 visual representations students used in problem-solving were examined. Descriptive analysis and content analysis were used in data analysis. As a result, it was seen that the students used 244 operational representations, 217 pictorial representations, and 62 schematic representations and made 64 mental solutions. It was determined that the accuracy rates of problem solutions were 22% for students using operational representation, 33% for students using pictorial representation, 35% for students using schematic representation, and 31% for students using mental solutions. The use of representation skills in the correct solution of problems has been discussed in the light of the literature, and some suggestions have been developed.

Introduction

In our era, the success of societies stems from the importance they place on fundamental sciences, especially mathematics. The history of science and its current state demonstrate that progress in other fields of science is impossible without the aid of mathematics. Thus, the education provided to foster advancement in mathematics from the very beginning of schooling holds great significance. There are various skills that mathematics education aims to improve. These include problem-solving, effective communication and connection through mathematical language, educational materials, mathematical modelling, reasoning, and effectively using information and communication technologies (Van de Walle & Lovin, 2015). These skills form the core group of main competencies, interrelate, and developing one skill aids in advancing others. The mathematical process criteria identified by the National Council of Teachers of Mathematics (NCTM) such as problem-solving, establishing connections, communication, representation, reasoning, and proof are founded on these designated skills. According to NCTM, these skills are critical for understanding mathematics and should be taught at every grade level (NCTM, 2000).

Learning mathematics is akin to learning a language where students discover new symbols, terms, and ways to communicate mathematically (Gowers et al., 2008). Mathematics is a language composed of numerous formulas, symbols, patterns, and expressions. Consequently, if students struggle to understand the meaning of words in problems, communicate, or grasp questions, they will likely encounter challenges in problem-solving or mathematics in general. Students of all ages tend to face difficulties with problem-solving. The main issue with problems often lies in failing to comprehend the question. Therefore, mathematics education fundamentally begins with teaching the language (Gray, 2004). While math may appear as merely numbers and operations, it requires a solid grasp of the mathematical language first (Durkin & Shire, 1991). The more proficient students are in mathematical language, the more successful they will be in problem-solving. Besides knowledge of mathematical language, "representation" is another essential skill for mastering mathematics and solving problems.

Representation and visualization are at the heart of understanding mathematics. "Representation" often refers to mental constructs that describe an object through its content. It is something that stands in for something else (Duval, 1999). Representations are observable products, symbols, and objects created in individuals' minds. Mathematical representations can be depicted through images, concrete materials, numbers, models, written words, and other tangible items representing mathematical ideas or relationships. The mathematical representations that students create are often unique to them. However, forms and meanings of representations can be negotiated during problem-solving (Goldin, 2020). In mathematics, representations are internal abstractions of mathematical ideas or cognitive schemas developed through experience (Pape & Tchoshanov, 2001). The Organisation for Economic Co-operation and Development (OECD) describes representation as a tool that enables individuals to use diagrams, tables, graphs, images, equations, formulas, and concrete materials to interpret and understand mathematical objects and situations for any given problem (OECD, 2019). Solving and Mayer suggest that representation is one of the four main components in the conceptual framework used in problem-solving. In the first problem-solving stage, students are expected to read and explain the problem in their own words. In the second stage, relationships among the problem parts are expressed through visual representation. In the third stage, after creating an appropriate representation, the design phase identifies the necessary operations and equations for solving the problem. Lastly, during the implementation phase, previously determined actions are carried out, and the accuracy of the solution is verified (Solving & Mayer, 1985). Research on using visual representation methods to learn mathematics and solve problems has long been conducted. Using visual representations during problem-solving is a factor linked to solving efficiency (Uesaka et al., 2007). In mathematics, while students have various ways to solve problems, those who are better at problem-solving can often find a mathematical representation that supports their understanding of the problem.

Students who prefer using schematic representations during problem-solving are shown to be more successful (Guoliang & Pangpang, 2003; Hegarty & Kozhevnikov, 1999; Van Garderen & Montague, 2003). However, the type of representation used in problem-solving may vary depending on the problem type. Choosing representations relevant to the concept in the problem and distinguishing between effective and ineffective representations are essential for solving the problem (Davis, 1986). Visual, symbolic, verbal, contextual, and physical representations of mathematical content areas provide different ways to access mathematics. At the same time, connecting these representations is crucial for deepening students' mathematical understanding (Lesh et al., 1987). In learning and problem-solving in mathematics, models adapted to real-life situations can make learning meaningful. Using multiple representations rather than a single one makes learning more meaningful and strengthens students' understanding (Amoah & Laridon, 2004). Mathematical representations are more effectively utilized when associated with students' interests and experiences (Desai et al., 2021). Representations, especially visual ones that allow viewing events from different perspectives, can influence mathematical success.

Most of the research on problem-solving using multiple representations has focused on primary school students' ability to use multiple representations (Castro et al., 1999; Fennell & Rowan, 2001; Gürbüz & Şahin, 2015; Kara & İncikabı, 2018; Özbas, 2023). These studies are generally conducted with 4th-grade students, where groups are classified based on the types of representations they use. It is emphasized that students using visual representations had higher problem-solving performance (Uesaka et al., 2007). Cultural factors can also influence mathematical skills. For example, numerical word structures and linguistic features can limit mathematical competencies. Strategies adopted for operations like addition and subtraction may develop in connection with a culture's counting system. Brain-imaging studies have shown relationships between language, math learning, and culture. Comparative studies indicate that children from some cultures perform better in mathematics (Menteş & Akman, 2022). There is a limited amount of research in Turkey on this topic. One study by Çilingir Altıner and Önal found that students preferred symbolic representation when solving given problems, with higher accuracy rates for problems solved using schematic representation (Altıner Çilingir & Önal, 2022). Ergan and Özsoy identified that students, especially, struggled to create meaningful structures that would represent the given elements in the problem (Ergan & Özsoy, 2021). In a study by Kartallıoğlu on modelling verbal math problems, some students reported that they either lacked the skills to draw shapes or viewed it as time-consuming; when they were uncertain of the correctness of their chosen operation, they directly chose to draw (Kartallıoğlu, 2005). Using representations facilitates reaching the correct answer in problem-solving, and children can use various representations from an early age (NCTM, 2000). Early use of representations can enhance children's problem-solving success, enabling them to enjoy learning mathematics and develop positive attitudes toward it. Therefore, it can be said that studies on the use of representations in problem-solving should be conducted starting from a young age. Identifying the types of representations used by primary school children in Turkey when solving math problems may offer a cultural contribution to the field.

Children are naturally curious and try to make sense of their mathematical surroundings. They should be given opportunities to see the connections between mathematics and their daily lives to appreciate its beauty. Teachers should provide students with experiences that allow them to understand mathematical relationships by using multiple mathematical representations, working to develop students' skills in using various representations fluidly (Desai et al., 2021). From the 1st grade, it is essential to practice, under teacher guidance, how to use representations in problem-solving. Initially, using representations may seem like a waste of time. However, as students become more proficient in using representation skills, they can minimize time loss by developing different strategies. By reaching the correct answer to problems, students can experience the satisfaction of success and develop a positive attitude toward mathematics. Therefore, it is essential to understand which representations students use and conduct studies to encourage using different types. This research focuses on determining the representations used by 2nd and 3rd grade Turkish students in primary school when solving mathematical problems, differentiating it from studies conducted with older age groups. The findings are anticipated to guide teachers' efforts to encourage the use of representations in children's mathematics education. The following questions are explored in this study:

- What types of representations do 2nd and 3rd-grade students use in solving mathematical problems?
- What are the accuracy rates of problems solved using operational, pictorial, schematic representation, and mental solutions?

Method

Model of the Study

In this study, the primary research method was preferred because of the types of representations that primary school children created in solving different types of problems, and the relationship between these types of representations and the accuracy rates in solving the problems were examined. The primary research methods in the qualitative research model are "What?" and "How?" they search for answers to questions and try to understand the nature of the current phenomenon while examining the subject. The primary research method aims to understand, explain, and try to find the main structure of the research subject (Patton, 2014).

Ethics

After review by the Turkish Ministry of National Education (MEB), approval was obtained from schoolteachers and parents of the children. The study followed the guidelines of the Scientific Research and Ethics Committee Regulation by The Council of Higher Education in Turkey (YÖK). Participant confidentiality was ensured through anonymization. Parents were allowed to withdraw their consent at any time, and children could refuse to participate verbally. This study was carried out with the permission of Karabuk Provincial Directorate of National Education (Guidance and Research Center Directorate), dated 12/01/2024 and numbered 94221994.

Working Group

The study was conducted in the first semester of the 2023-2024 academic year. After permission was obtained, the research was scheduled with 2nd and 3rd-grade teachers at the selected school. Based on a pilot study, a 40-minute lesson was found sufficient for data collection. Students were informed about the study, and their consent was obtained with the support of their class teacher. Under teacher supervision, problem sheets were distributed to the children, who were given 40 minutes to complete them. The study group consisted of 2nd and 3rd-grade students aged 8-9 from a state school under the MEB in Turkey. The application was carried out with two 2nd-grade and two 3rd-grade classes.

To save time and resources, convenience sampling was used, selecting the school based on ease of access and the willingness of administrators and teachers. Initially, three researchers provided the necessary information to four class teachers. The study group included 80 students, 40 in 2nd and 40 in 3rd grades. Students were assigned codes based on their grade level (starting with "2" for 2nd grade, "3" for 3rd grade) and class section (with "A" for section A, "B" for section B), e.g., "2AÖ1," "3BÖ20".

Data Collection Tools

A questionnaire with ten visual-verbal math problems was developed as a data collection tool. The problems were chosen based on their potential to be solved using multiple representation formats. The problems were reviewed by two primary school teachers, two elementary mathematics teachers, a mathematics education specialist, and a child development expert, who provided feedback for refinement. After revisions, a pilot study with 15 students was conducted, removing two questions based on expert feedback. Symbols such as ☆, ∆, ⌂, ♡, □, ■, ▲, ● were used instead of sequential numbering for the questions. To prevent fatigue and distraction, the questions were arranged in a different order for each student. For instance, one student might start with the ☆ problem while another begins with the ∆ problem. Following the pilot study, it was confirmed that students had no issues understanding the questions, and an eight-problem questionnaire was finalized for the main study.

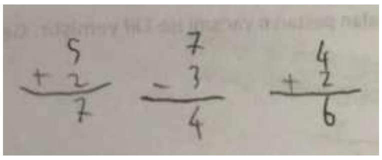
Data Analysis

Descriptive analysis and content analysis methods were used to analyze the data. Descriptive analysis involves conveying the data with direct quotations in its original form (Kümbetoğlu, 2008). Content analysis aims to interpret the data more analytically and causally, deciding on expressions and themes within the data and making inferences about the relationships between these themes (Kümbetoğlu, 2008; Yıldırım & Şimşek, 2008). To ensure coding reliability, disagreements were resolved, resulting in a 98% coder reliability as calculated by the formula suggested by Miles and Huberman (Miles & Huberman, 1994). Two coding schemes were used to address the research questions. First, each question was analyzed using document analysis, a qualitative research technique, and coded as follows: (1) type of visual representation created (pictorial-schematic) and whether the answer was correct, (2) type of nonvisual representation used (symbolic-verbal) and whether the answer was correct, and (3) whether the representation was external (i.e., drawn on paper) or internal (i.e., mental) and whether the answer was correct. For example, if students drew an image or schema or reported a mental image in solving a verbal problem, the solution was classified as visual or pictorial and checked for accuracy. If a student used a schematic representation and answered correctly, the item was coded as "schematic correct."

Additionally, to analyze the quantitative aspect of the research, various scores were recorded: the total number of correct answers as the problem-solving performance score, the number of pictorial representations used as the pictorial representation score, the number of schematic representations as the schematic representation score, the number of symbolic representations as the symbolic representation score, the number of verbal representations as the verbal representation score, and the number of mental representations as the mental representation score. Visual representations created by students were examined and coded, then classified according to their structural characteristics. The types of representations used in problem-solving, examples of representations created by students, and code explanations are presented in Table 1.

In the data analysis, the types of representations, the number of representations, and example representations used by students about the problems were included. Thematic analysis was used to examine which visual and nonvisual representation type students preferred while solving verbal problems and whether these choices led to correct answers. This analysis identified the common representation types selected by students for each question, the frequency of each type's selection, and the accuracy rates associated with different types of representations.

Table 1. Definition, Example, and Explanation of Representation Types

Code name and description	Example of code	Description of the code
Operational representation Solving the problem with numerical values.		It is a type of representation in which the problem is solved with numerical values. It does not contain mental solutions or pictorial or schematic representation. The operation is found.
Pictorial representation It is a solution made by drawing the image of what is given in the problem text.		It is a type of representation formed by depicting the elements in the problem. It helps to understand the problem clearly.
Schematic representation These drawings express what is given in the problem text in a representative context without depicting it precisely.		It does not contain a detailed view of what is given in the problem. It is a representation consisting of shapes roughly representing the given things. They are elements related to problems.
Solution from mind Only numerical results are available.		It does not contain any operations. It contains only numerical values.

Results

In this section, the findings obtained in line with the sub-problems of the study are interpreted in tables and supported with sample representation images. Table 2 shows the types of visual representations students created while solving problems, their mental solutions, and their distribution according to whether the problem was solved correctly or incorrectly.

Table 2. Types of Representations and Correct/Incorrect Distribution of Problems with Mental Solutions

Problem	Operational representation				Pictorial representation				Schematic representation				Solution from mind			
	Correct		Incorrect		Correct		Incorrect		Correct		Incorrect		Correct		Incorrect	
	f	%	f	%	f	%	f	%	f	%	f	%	f	%	f	%
☆	18	53	16	47	2	18	9	82	6	38	10	62	13	68	6	32
∆	11	24	34	76	4	31	9	69	4	100	0	0	1	9	10	91
⌂	0	0	18	100	9	20	36	80	2	100	0	0	0	0	5	100
♡	4	36	7	64	18	42	25	58	1	100	0	0	0	0	5	100
□	12	35	22	65	18	64	10	36	4	40	6	60	2	67	1	33
■	8	23	27	77	5	25	15	75	4	22	14	78	3	43	4	57
▲	1	3	31	97	6	19	26	81	1	25	3	75	0	0	7	100
●	0	0	35	100	10	40	15	60	0	0	7	100	1	14	6	86
Total	54	22	190	78	72	33	145	67	22	35	40	65	20	31	44	69

When Table 2 is examined, it is seen that the students use the most operational representations in solving the ☆, ∆, □, ■ and ● problems. They use the most pictorial representations in the solution of the ⌂ and ♡ problems. They use equal numbers of operational and pictorial representations to solve the ▲ problem. It has been determined that students generally use operational representation (f=244) the most in solving problems, followed by pictorial representation (f=217) and mental solution (f=64), and they use schematic representation (f=62) the least.

Table 3. Distribution of Problems Using Operational Representation

Problem	Correct		Incorrect		Total
	f	%	f	%	f
☆	18	53	16	47	34
∆	11	24	34	76	45
⌂	-	-	18	100	18
♡	4	36	7	64	11
□	12	35	22	65	34
■	8	23	27	77	35
▲	1	3	31	97	32
●	-	-	35	100	35
Total	**54**	**22,13**	**190**	**77,87**	**244**

Table 3 includes information about the rates of students using operational representations in solving problems and the accuracy of the problems.

Table 4. Examples of Operational Representation

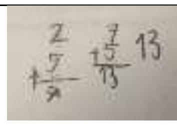
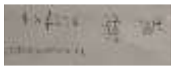
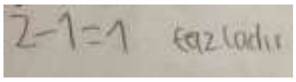
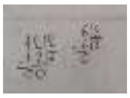
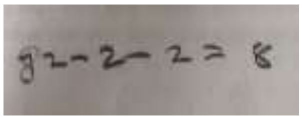
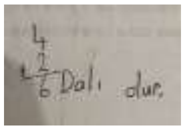
Problem	Student	Correct Solution	Student	Incorrect Solution
☆	3AÖ3		2AÖ2	
Δ	3AÖ13		3AÖ8	
⌂	-	-	3BÖ7	
♡	2BÖ14		2BÖ2	
□	3AÖ3		2AÖ19	
■	3AÖ8		3AÖ10	
▲	2AÖ18		3AÖ8	
●	-	-	3BÖ4	

Table 3 includes sample representations that reach correct and incorrect solutions. When Table 3 is examined, it is seen that the students used 244 operational representations for the eight problems they solved; 54 (%22,13) reached the correct solution of the problem, and 190 (%77,87) of them resulted in the wrong solution. In other words, approximately 78% of the solutions to the problems using operational representations resulted incorrectly. As seen in Table 4, some students used only four operations and supported their judgments regarding the operation in writing.

Table 5 contains information about students' use of pictorial representation in solving problems and the accuracy of the problems. In Table 5, sample representations with correct and incorrect solutions are given. When Table 5 is examined, it is seen that the students used 217 pictorial representations for the eight problems they solved; 72 (%33,18) of them reached the correct solution of the problem, and 145 (%66,82) resulted in the wrong solution. In other words, approximately 67% of the solutions to the problems using pictorial representation resulted incorrectly. As seen in Table 6, some students used pictorial representation in addition to the four operations with numbers, and in some cases, they supported the pictures in writing.

Table 5. Distribution of Problems Using Pictorial Representation

	Correct		Incorrect		Total
Problem	f	%	f	%	f
☆	2	18	9	82	11
∆	4	31	9	69	13
⌂	9	20	36	80	45
♡	18	42	25	58	43
□	18	64	10	36	28
■	5	25	15	75	20
▲	6	19	26	81	32
●	10	40	15	60	25
Total	**72**	**33,18**	**145**	**66,82**	**217**

Table 6. Examples of Pictorial Representation

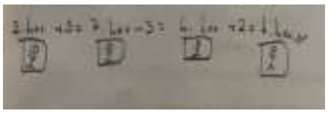
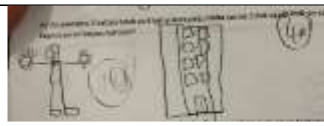
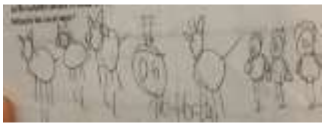
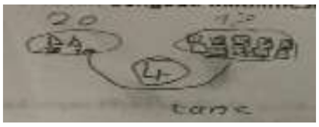
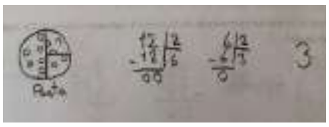
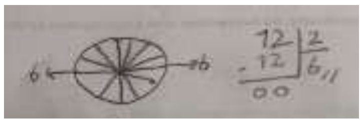
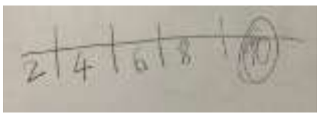
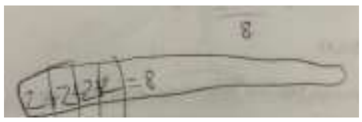
Problem	Student	Correct Solution	Student	Incorrect Solution
☆	3A Ö17		2AÖ4	
∆	2BÖ17		3AÖ15	
⌂	2BÖ5		2AÖ4	
♡	3AÖ3		2BÖ7	
□	3AÖ8		3AÖ5	
■	3AÖ16		3AÖ12	
▲	3AÖ16		3AÖ13	
●	2BÖ21		2AÖ19	

Table 7. Distribution of Schematic Representation

Problem	Correct		Incorrect		Total
	f	%	f	%	f
☆	6	38	10	62	16
∆	4	100	0	0	4
⌂	2	100	0	0	2
♡	1	100	0	0	1
□	4	40	6	60	10
■	4	22	14	78	18
▲	1	25	3	75	4
●	0	0	7	100	7
Total	**22**	**35,48**	**40**	**64,52**	**62**

Table 8. Examples of Schematic Representation

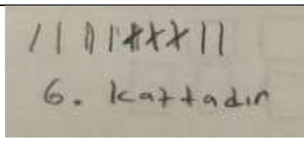
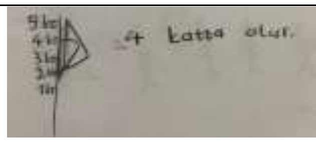
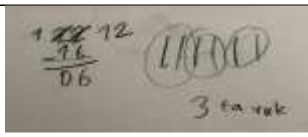
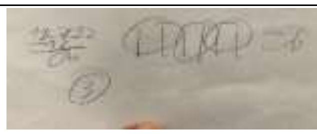
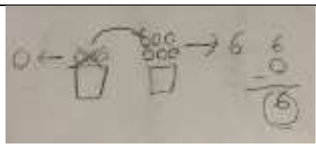
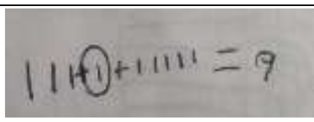
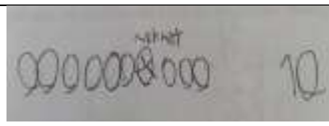
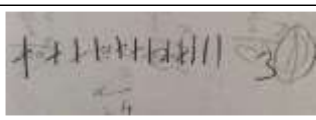
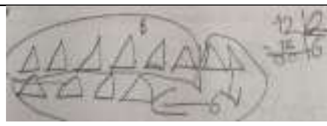
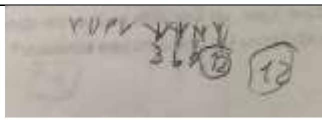
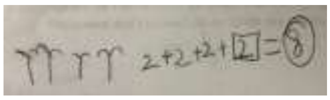
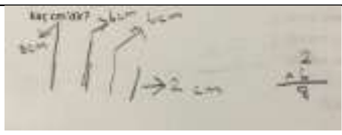
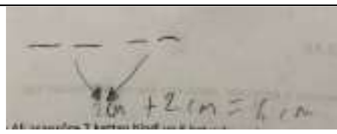
Problem	Student	Correct Solution	Student	Incorrect Solution
☆	3BÖ9		3AÖ5	
∆	2BÖ4		2BÖ1	
⌂	2BÖ4		-	-
♡	3BÖ5		-	-
□	2BÖ16		2AÖ14	
■	3BÖ7		3AÖ14	
▲	2BÖ5		2BÖ6	
●	3AÖ17		3AÖ15	

Table 7 contains information about students' use of schematic representation in solving problems and the accuracy of the problems. In Table 8, sample representations with correct and incorrect solutions are given. When Table 7 is examined, it is seen that the students used 62 schematic representations for the eight problems they solved; 22 (%35,48) of them reached the correct solution of the problem, and 40 (%64,52) of them resulted in the wrong solution. In other words, approximately 65% of the solutions to the problems using schematic representations resulted incorrectly.

As seen in Table 8, some students used pictorial representation in addition to the four operations with numbers, and in some cases, they supported the pictures in writing. Table 9 contains information about the students' mental solutions and the accuracy of the problems in solving the problems. When Table 9 is examined, it is seen that the students used 64 mental solutions for the eight problems they solved; 20 (%31,25) reached the correct solution of the problem, and 44 (%68,75) of them resulted in the wrong solution. In other words, approximately 77% of the solutions for the problems in which mental solutions were used resulted in errors.

Table 9. Distribution of Mental Solutions

	Correct		**Incorrect**		**Total**
Problem	f	%	f	%	f
☆	13	68	6	32	19
Δ	1	9	10	91	11
⌂	0	0	5	100	5
♡	0	0	5	100	5
□	2	67	1	33	3
■	3	43	4	57	7
▲	0	0	7	100	7
●	1	14	6	68	7
Total	**20**	**31,25**	**44**	**68,75**	**64**

Discussion

Developing problem-solving skills is a crucial aspect of mathematics education and a primary concern for teachers (Garzon & Casinillo, 2021). The teaching strategies employed by mathematics teachers are vital to enhancing students' problem-solving abilities (Leonard, 2018). A proficient mathematician often uses representations to understand a problem, visualize approaches, and guide the planning of a solution (Stylianou & Silver, 2004). This study aimed to determine the types of representations preferred by students, identify which representations assist in problem-solving, and demonstrate the type of representation that most positively impacts problem-solving performance.

According to the findings, in solving verbal problems, students predominantly used operational representation, followed by pictorial and schematic representations. Mental solutions were also used. Analysis of the problems solved by students showed that the highest accuracy rate was achieved with schematic representations, followed by pictorial representation, mental solutions, and operational representation. Although schematic representation was used less frequently, its accuracy rate was higher than other types. Similarly, Van Garderen and co-workers found that students who could solve problems effectively were twice as likely to create schematic representations compared to others (Van Garderen et al., 2014). Ergan and Özsoy also observed that schematic representations yielded the highest accuracy rate among primary school students in problem-solving (Ergan & Özsoy, 2021). Likewise, Antıner Çilingir and Önal reported high accuracy rates with schematic representation despite its infrequent use (Altıner Çilingir & Önal, 2022).

Boonen and colleagues further emphasized that correct visual-schematic representations were linked to higher accuracy in solving verbal problems. In contrast, those relying on pictorial and nonvisual representations had lower accuracy rates. This situation highlights the importance of the content and presentation of visual representations (Boonen et al., 2014). Hegarty and Kozhevnikov also found that using schematic representations improved problem-solving success, whereas pictorial representations had a negative effect (Hegarty & Kozhevnikov, 1999). It was emphasized that students who constructed accurate schematic representations were six times more likely to solve the problem correctly. Conversely, inaccurate schematic representations or reliance on pictorial representation reduced the likelihood of correct solutions by approximately three times, suggesting that students' attention may be diverted when using pictorial representation. Some students may lack drawing skills, leading them to focus on the drawing rather than the problem itself. Pictorial representations sometimes focus on irrelevant details, distracting students from the main problem. For instance, focusing on the branches and leaves of a tree or a person's clothing and hair can detract from solving the problem itself, potentially impeding the formation of a coherent model and hindering appropriate relational thinking (Boonen et al., 2014).

Montenegro et al. noted that solving a problem using visual representation serves as an initial stage in problem-solving, helping to develop other forms of representation (Montenegro et al., 2018). Lowrie and Kay found that students’ preferences for representations varied with problem difficulty; they tended to use visual representations for more challenging problems. Students might prefer nonvisual representations when the necessary information is easily accessible, especially with complex or novel problems. This study also found that students tended to favour operational representations, although their accuracy rate was lower than that of schematic representations. This may stem from the nature of the problems easy but non-routine. Also, they emphasized that students with sufficient prior knowledge and capacity could use nonvisual representations, highlighting that a progression from visual to nonvisual representation is necessary (Lowrie & Kay, 2001).

Lowrie and Clements proposed that primary school students should transition from visual to nonvisual representations. Students who effectively use visual representation could also use other types (Lowrie & Clements, 2001). Ahmad et al. suggested that students should be allowed to use each type of mathematical representation to solve verbally presented problems. Contrasting with these findings, some students in this study first solved problems mentally and accurately before representing them pictorially, potentially reflecting a generational tendency toward quick access to information (Ahmad et al., 2010). Altıner Çilingir and Önal observed a similar pattern in which algebraic or numerical answers were more common than others. Likewise, this study found that operational representations were the most frequently used, yet had the lowest accuracy rate. Possible reasons include a lack of ability to create other types of representation, reluctance to use visual representations due to drawing skill limitations, and the perception of drawing as time-consuming (Altıner Çilingir & Önal, 2022). Some teachers also perceive pictorial representation as time consuming and feel pressured to complete lesson content within time constraints, leading to an emphasis on operational representation in problem-solving (Bilgili & Çiltaş, 2018; Uesaka et al., 2007).

In mathematics, visual representation involves designing and creating models that reflect mathematical data (Van Garderen & Montague, 2003). Montenegro et al. described visual representation as an initial stage that aids in expanding the scope of other visual processes, facilitating transformations, and supporting flexibility in converting these into symbolic and verbal representations (Montenegro et al., 2018). Lowrie and Kay similarly argued that students access nonvisual applications once they reach the required knowledge, prior experience, or comprehension capacity. Thus, nonvisual representations can be seen as a more complex process. Given these findings, supporting problem-solving with visual representations before moving to nonvisual representations may be beneficial (Lowrie & Kay, 2001). Lowrie and Clements similarly suggested that primary school students should first acquire skills in solving problems with visual representations before being introduced to symbolic strategies, including algebraic representations, indicating a transition from visual to nonvisual representations (Lowrie & Clements, 2001). However, Anwar and Rahmawati highlighted the importance of verbal representations for understanding problems. Verbal representations help students clearly define problems, making symbols created by students highly useful in problem-solving. Verbal representation is used primarily in the comprehension phase, often by noting all known information in a familiar language. Students can plan and execute calculations based on known information, enabling accurate solutions (Anwar & Rahmawati, 2017).

The contribution of representation types (e.g., visual or concrete) to students' mathematical skills and reasoning has long been recognized (Presmeg, 2020). While some students prefer visual representations for complex problems (Ho & Lowrie, 2014), others avoid them in complex scenarios (Uesaka et al., 2007). Some students who are capable problem solvers avoided using visual representations altogether, as observed through teacher feedback. A study by Van Garderen and co-workers found that high-achieving students could create suitable examples for representing problems, while low-achieving students struggled (Van Garderen et al., 2014).

These findings offer valuable insights for primary mathematics instruction. According to the study results, students may find creating visual representations unnecessary or challenging in solving verbal problems, or they may not have been taught to use them effectively. The limited use of visual representation to solve verbal problems is notable. However, it is concerning that students who preferred nonvisual representations demonstrated lower problem-solving performance than those using schematic representations. According to Johar and Lubis, this may be due to students' high error rates in using representations, lack of familiarity with problems requiring representation, or inexperience with problem types found in assessments like PISA and TIMSS. Multiple representations are essential for understanding various aspects of complex mathematical ideas (Johar & Lubis, 2018). Ahmad et al. emphasized the importance of providing students with opportunities to use various representations to solve mathematical problems (Ahmad et al., 2010).

Conclusion

Relying on a single representation type restricts students' ability to explore problems from multiple perspectives, potentially hindering their mathematical development. Therefore, instructional strategies in mathematics should aim to balance students' knowledge and skills in both visual and nonvisual representations. Teachers should incorporate activities that develop students' abilities to use diverse representation types. Students' choice of representation often depends on what their teachers emphasize in class.

The researchers recommend that teachers carefully consider representations that aid students in understanding and solving verbal mathematical problems and be prepared to integrate both visual and nonvisual representations into mathematics teaching and learning. Program developers are also encouraged to include the use of multiple representation types in the curriculum. Future research could investigate factors influencing representation type preferences across different age groups. This study, conducted with 2nd and 3rd-grade students, presented data descriptively. Future studies could explore the relationship between representation types used by 2^{nd}, 3^{rd}, and 4^{th} graders in problem-solving and variables such as grade level and gender.

References

Ahmad, A., Tarmizi, R. A., & Nawawi, M. (2010). Visual representations in mathematical word problem solving among form four students in Malacca. *Procedia-Social and Behavioral Sciences*, *8*, 356-361. https://doi.org/10.1016/j.sbspro.2010.12.050

Altıner Çilingir, E., & Önal, H. (2022). İlkokul Öğrencilerinin Sözel Problem Çözerken Kullandıkları Görsel ve Görsel Olmayan Temsillerin İncelenmesi. *Yaşadıkça Eğitim*, *36*(1), 16-34. https://doi.org/10.33308/26674874.2022361344

Amoah, V., & Laridon, P. (2004). Using multiple representations to assess students' understanding of the derivative concept. *Proceedings of the British society for research into learning mathematics*, *24*(1), 1-6.

Anwar, R. B., & Rahmawati, D. (2017). Symbolic and Verbal Representation Process of Student in Solving Mathematics Problem Based Polya's Stages. *International Education Studies*, *10*(10), 20-28. https://doi.org/https://doi.org/10.5539/ies.v10n10p20

Bilgili, S., & Çiltaş, A. (2018). İlköğretim Matematik Öğretmeni Adaylarının Matematiksel Modelleme Etkinliği Kurma ve Çözme Becerilerinin Farklı Değişkenler Açısından İncelenmesi. *International Journal of Educational Studies in Mathematics*, *5*(2), 66-77.

Boonen, A. J., van Wesel, F., Jolles, J., & van der Schoot, M. (2014). The role of visual representation type, spatial ability, and reading comprehension in word problem solving: An item-level analysis in elementary school children. *International Journal of Educational Research*, *68*, 15-26. https://doi.org/10.1016/j.ijer.2014.08.001

Castro, E., Morcillo, N., & Castro, E. (1999). Representations produced by secondary education pupils in mathematical problem solving. Proceedings of the Twenty First Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Mexico.

Davis, R. B. (1986). *The Cognitive Science Approach To Mathematics Education*. Ablex Publishing Corporation.

Desai, S., Bush, S., & Safi, F. (2021). Mathematical representations in the teaching and learning of geometry: A review of the literature from the United States. *The Electronic Journal for Research in Science & Mathematics Education*, *25*(4), 6-22.

Durkin, K., & Shire, B. (1991). Primary school children's interpretations of lexical ambiguity in mathematical descriptions. *Journal of Research in Reading*, *14*(1), 46-55. https://doi.org/10.1111/j.1467-9817.1991.tb00005.x

Duval, R. (1999). Representation, Vision and Visualization: Cognitive Functions in Mathematical Thinking. Basic Issues for Learning. 21st North American Chapter of the International Group for the Psychology of Mathematics Education, Morelos.

Ergan, S. N., & Özsoy, G. (2021). İlkokul dördüncü sınıf öğrencilerinin problem çözme sürecinde oluşturduğu görsel temsillerin incelenmesi. *Dokuz Eylül Üniversitesi Buca Eğitim Fakültesi Dergisi*(51), 57-75. https://doi.org/10.53444/deubefd.763452

Fennell, F., & Rowan, T. (2001). Representation: An important process for teaching and learning mathematics. *Teaching children mathematics*, *7*(5), 288-292. https://doi.org/10.5951/TCM.7.5.0288

Garzon, J. R., & Casinillo, L. F. (2021). Visualizing mathematics: the use of block models for strategic problem solving. *Journal of Education Research and Evaluation*, *5*(1), 112-117.

Goldin, G. A. (2020). *Mathematical representations*.

Gowers, T., Barrow-Green, J., & Leader, I. (2008). Introduction. What is mathematics about. *The Princeton companion to mathematics*, 1-76.

Gray, V. D. (2004). *The* language of mathematics: A functional definition and the development of an instrument to measure teacher perceived self-efficacy*. Oregon State University.

Guoliang, Y., & Pangpang, Z. (2003). Visual-spatial representations and mathematical problem solving among mathematical learning disabilities. *Acta Psychologica Sinica*, *35* (5), 643–648.

Gürbüz, R., & Şahin, S. (2015). 8. sınıf öğrencilerinin çoklu temsiller arasındaki geçiş becerileri. *Kastamonu Eğitim Dergisi*, *23*(4), 1869-1888.

Hegarty, M., & Kozhevnikov, M. (1999). Types of visual–spatial representations and mathematical problem solving. *Journal of educational psychology*, *91*(4), 684.

Ho, S. Y., & Lowrie, T. (2014). The model method: Students' performance and its effectiveness. *The Journal of mathematical behavior*, *35*, 87-100. https://doi.org/10.1016/j.jmathb.2014.06.002

Johar, R., & Lubis, K. R. (2018). The analysis of students' mathematical representation errors in solving word problem related to graph. *Jurnal Riset Pendidikan Matematika*, *5*(1), 96-107. https://doi.org/10.21831/jrpm.v5i1.17277

Kara, F., & İncikabı, L. (2018). Sixth grade students' preferences on multiple representations used in fraction operations and their performance in their preferences. *İlköğretim Online, 17*(4), 2136-2150. https://doi.org/10.17051/ilkonline.2019.506984

Kartallıoğlu, S. (2005). *İlköğretim 3. ve 4. Sınıf öğrencilerinin sözel matematik problemlerini modellemesi: çarpma ve bölme işlemi.* Yayınlanmış Yüksek Lisans Tezi, Abant İzzet Baysal Üniversitesi Sosyal Bilimler Enstitüsü, Bolu.

Kümbetoğlu, B. (2008). *Niteliksel araştırmalarda analiz ve yorumlama, Sosyolojide ve antropolojide niteliksel* *yöntem ve araştırma.* İstanbul, Bağlam Yayıncılık.

Leonard, J. (2018). *Culturally specific pedagogy in the mathematics classroom: Strategies for teachers and students*. Routledge. https://doi.org/https://doi.org/10.4324/9781351255837

Lesh, R., Post, T., & Behr, M. (1987). Representations and Translations among Representations in Mathematics Learning and Problem Solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33-40). Lawrence Erlbaum Associates.

Lowrie, T., & Clements, M. K. (2001). Visual and nonvisual processes in Grade 6 students' mathematical problem solving. *Journal of Research in Childhood Education*, *16*(1), 77-93. https://doi.org/10.1080/02568540109594976

Lowrie, T., & Kay, R. (2001). Relationship between visual and nonvisual solution methods and difficulty in elementary mathematics. *The Journal of Educational Research*, *94*(4), 248-255. https://doi.org/10.1080/00220670109598758

Menteş, H., & Akman, B. (2022). Erken Yıllarda Matematik, Kültür ve Dil İlişkisi. *Okul Öncesi ve Temel Eğitim Dergisi*, *3*(1). https://doi.org/https://doi.org/10.29329/jpee.2022.457.1

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Sage.

Montenegro, P., Costa, C., & Lopes, B. (2018). Transformations in the visual representation of a figural pattern. *Mathematical thinking and learning*, *20*(2), 91-107. https://doi.org/10.1080/10986065.2018.1441599

NCTM. (2000). *Standards for school mathematics*. https://www.nctm.org/uploadedFiles/Standards_and_Positions/PSSM_ExecutiveSummary.pdf

OECD. (2019). *Education at a Glance 2019*. https://doi.org/10.1787/19991487

Özbas, B. (2023). *8. sınıf öğrencilerinin Problem çözme sürecindeki Matematiksel Temsil Becerileri Ile Matematik okuryazarlığı arasındaki ilişkinin Incelenmesi* Dokuz Eylul Universitesi (Turkey)].

Pape, S. J., & Tchoshanov, M. A. (2001). The role of representation (s) in developing mathematical understanding. *Theory into practice*, *40*(2), 118-127. https://doi.org/https://doi.org/10.1207/s15430421tip4002_6

Patton, M. Q. (2014). *Qualitative research & evaluation methods: Integrating theory and practice*. Sage publications.

Presmeg, N. (2020). Visualization and learning in mathematics education. *Encyclopedia of mathematics education*, 900-904.

Solving, M. P., & Mayer, R. E. (1985). *DEFINITION OF PROBLEM SOLVING*.

Stylianou, D. A., & Silver, E. A. (2004). The role of visual representations in advanced mathematical problem solving: An examination of expert-novice similarities and differences. *Mathematical thinking and learning*, *6*(4), 353-387. https://doi.org/https://doi.org/10.1207/s15327833mtl0604_1

Uesaka, Y., Manalo, E., & Ichikawa, S. i. (2007). What kinds of perceptions and daily learning behaviors promote students' use of diagrams in mathematics problem solving? *Learning and Instruction*, *17*(3), 322-335. https://doi.org/https://doi.org/10.1016/j.learninstruc.2007.02.006

Van de Walle, J. A., & Lovin, L. A. H. (2015). Teaching Student-Centered Mathematics: Grades K-3. *Education Review*. https://doi.org/https://doi.org/10.14507/er.v0.832

Van Garderen, D., & Montague, M. (2003). Visual–spatial representation, mathematical problem solving, and students of varying abilities. *Learning Disabilities Research & Practice*, *18*(4), 246-254. https://doi.org/10.1111/1540-5826.00079

Van Garderen, D., Scheuermann, A., & Poch, A. (2014). Challenges students identified with a learning disability and as high-achieving experience when using diagrams as a visualization tool to solve mathematics word problems. *ZDM*, *46*, 135-149. https://doi.org/https://doi.org/10.1007/s11858-013-0519-1

Yıldırım, A., & Şimşek, H. (2008). *Sosyal bilimlerde nitel araştırma yöntemleri* (6. Baskı). Ankara: Seçkin Yayıncılık, 188.

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