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Abstract

This paper investigates the connection between the spiral teaching method and mathematical rigor in K-12 education. Defining rigor as a balanced integration of conceptual understanding, application, and procedural fluency (contrasting it with rote learning), the paper posits that the spiral approach cultivates this rigor. By revisiting mathematical concepts with increasing complexity, this method enhances retention, deepens comprehension through the building of knowledge, narrows learning disparities, and refines problem-solving abilities. The significance of mathematical rigor is underscored by the growing sophistication of contemporary mathematics and its broad applicability. The central inquiry of the paper is whether spiraled mathematics promotes rigorous mathematical thinking. To address this, the study will undertake a literature review. The theoretical foundation rests primarily on Bruner's Spiral Curriculum theory, which champions the repeated engagement with topics at progressively challenging levels. The paper also integrates insights from cognitive development and the role of scaffolding in facilitating mathematical learning within a spiral framework. Recognizing the iterative nature of the spiral, including potential temporary regressions for struggling learners, the paper ultimately seeks to establish the alignment and supportive relationship between the spiral method and the development of mathematical rigor in students. While the spiral curriculum is widely used in the US, its effectiveness is debated despite its adoption in other countries. The spiral approach, utilizing spaced repetition, is argued to deepen understanding, advance academic levels, and ultimately promote mathematical rigor, as seen in countries like Finland and Singapore, although its direct application to all students versus advanced learners differs.

Introduction

Spiral teaching method elevates mathematical rigor by encouraging a deep, connected, and long-lasting understanding of mathematical concepts for solving math problems and applying them through repeated exposure and increasing complexity. This strategy helps students not just memorize but advances them to an adaptable proficiency. The spiral teaching method can be especially effective when striving to teach complex, abstract concepts, mathematical formulas, or scientific theories. By revisiting these topics repeatedly, students gain a greater understanding of the underlying principles and can apply this knowledge more effectively in practical or real-world situations (Main, 2022). The spiral approach promotes better student retention, problem-solving skills, a deep understanding of concepts, and flexible thinking. Spiral teaching contributes significantly to mathematical rigor by fostering procedural fluency, conceptual understanding, and application. Mathematical rigor is defined in different forms depending on the perspective from which we observe it. This study is concerned more with the definition of mathematical rigor in high school education. Mathematical rigor pursues conceptual understanding, procedural skills and fluency, and application with equal intensity (Ride, 2021). Mathematical rigor is important for students because it helps them develop a deep understanding of concepts, strengthens critical thinking, improves clear communication, prepares them for advanced math and STEM fields, and encourages intellectual honesty.

Spiral Teaching Method (Revisiting Topics Progressively with Increasing Complexity)

In general, the spiral teaching method is employed over the long term, typically spanning several years. In a spiral approach, learning is spread out over a longer period rather than concentrated in a short amount of time (Burns, 2021). Nonetheless, we are interested in analyzing how the spiral approach fosters mathematical rigor during an academic year in PreK to 12. This method involves introducing mathematical concepts and then revisiting them regularly over time to reinforce understanding, retention, and application in solving math problems and in real-world situations. Teaching and learning within the spiral approach begin with the step-by-step introduction of mathematical ideas, progressively revisiting topics in various contexts with gradually increasing complexity.

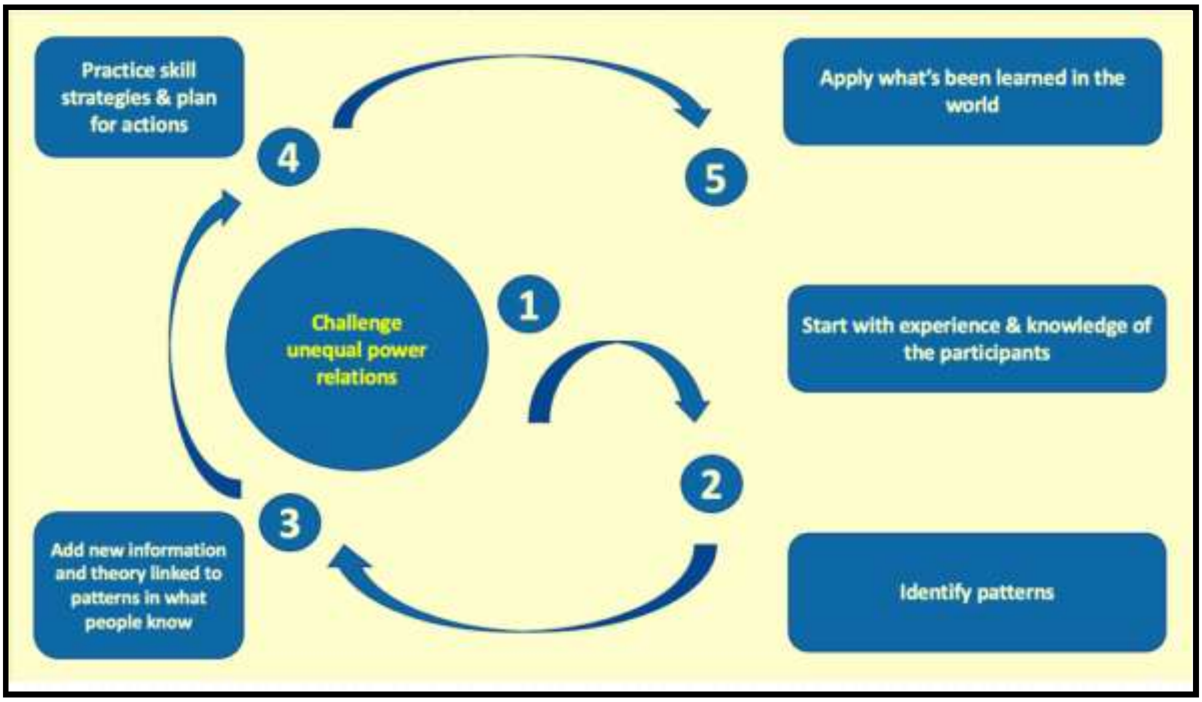

Figure 1. Five Steps of the Spiral Teaching and Learning Method (the Cycle repeated in Topics progressively with Increasing Complexity). The Figure modified from Smith (2022).

The rationale for applying the spiral method follows the steps described in Figure 1, which balances the teaching and learning of mathematical concepts, problem-solving, and real-world applications. Balancing these elements begins with the first step: a diagnostic exam to recognize students' existing experience and knowledge. The second step involves identifying patterns in what students know well, or where their existing knowledge of the subject converges. The third step involves adding new information (mathematical concepts or theories) linked to the students' existing knowledge (patterns). Spiral review ensures students retain information while connecting it with new material (Thomas et al., 2025). The fourth step is practicing skill strategies (solving mathematical problems) and planning for actions (easy, medium, or hard examples). The fifth step is applying the learned mathematical concepts and procedural fluency in real-world situations.

These five steps repeat progressively with increasing complexity of the topics. Revisiting topics combats the forgetting curve in mathematics, reinforcing understanding and retention. The human brain is not inherently structured to retain strict, factual information—what teachers often categorize as “level-two knowledge”—such as facts, definitions, and trivial knowledge (Kollman, 2023). The spiral method plays a crucial role in balancing mathematical concepts, the process of solving mathematical problems, application in real-world situations, and preparing students for any exam, including summative exams. The spiral approach significantly fosters mathematical rigor. To support a deep understanding of mathematics, college-and-career-ready standards are designed to appropriately balance the development of all three aspects of rigor in math within and across grades (Resanovich, 2023). Mathematical rigor enables students to become better critical thinkers by understanding key math concepts and applying them in solving or applying math in the real world.

Mathematical Rigor (Depth, Precision, Logical Reasoning, and Conceptual Understanding)

Mathematical rigor is used by both pre-K to 12 and in higher education, however, this study deals with mathematical rigor in high school as is mentioned above. Rigor is associated with “advanced courses,” comprehensive presentation and lectures on all possible topics and techniques, high-stakes summative tests, low pass rates and/or low grades (Dana Center, n. d.). In higher education rigor is used sometimes as college algebra. A Common Vision (Saxe & Braddy, 2015) in a particular place describe: Current college algebra courses serve two distinct student populations: 1) the overwhelming majority for whom it is a terminal course in mathematics, and 2) the relatively small minority for whom it is a gateway to further mathematics (Saxe, K., & Braddy, L. (2015). The high school algebra (rigor) is a gateway to mathematics and a key to answer the Aristotelian question: how do we know?

There is not a consensus on how to define rigor mathematics; the best definition for the rigor in K to 12 education is presented in a presentation in EdWEb. Based on the presentation “What is Mathematical Rigor?” It defines, “Rigor Mathematics means Teaching grade levels - standards with a balance, or equal intensity, of conceptual understanding, application, and procedural fluency (Toncheff, 2025). Balancing the three components of the mathematical rigor can be reached in several ways such as systematic, linear, mastery, spiral teaching and learning, etc. Rigor is not memorizing a myriad of procedures but being able to flexibly analyze and apply mathematics to different situations with a focus on concepts and relationships (Wilkerson, 2021).The focus of the study sheds light on the spiral approach fosters mathematical rigor. The key benefits of the spiral and mathematical rigor strategies are retaining and understanding mathematical concepts, expanding knowledge by gaining new knowledge upon the old one, reducing learning gap, mastering solving and applying mathematical techniques.

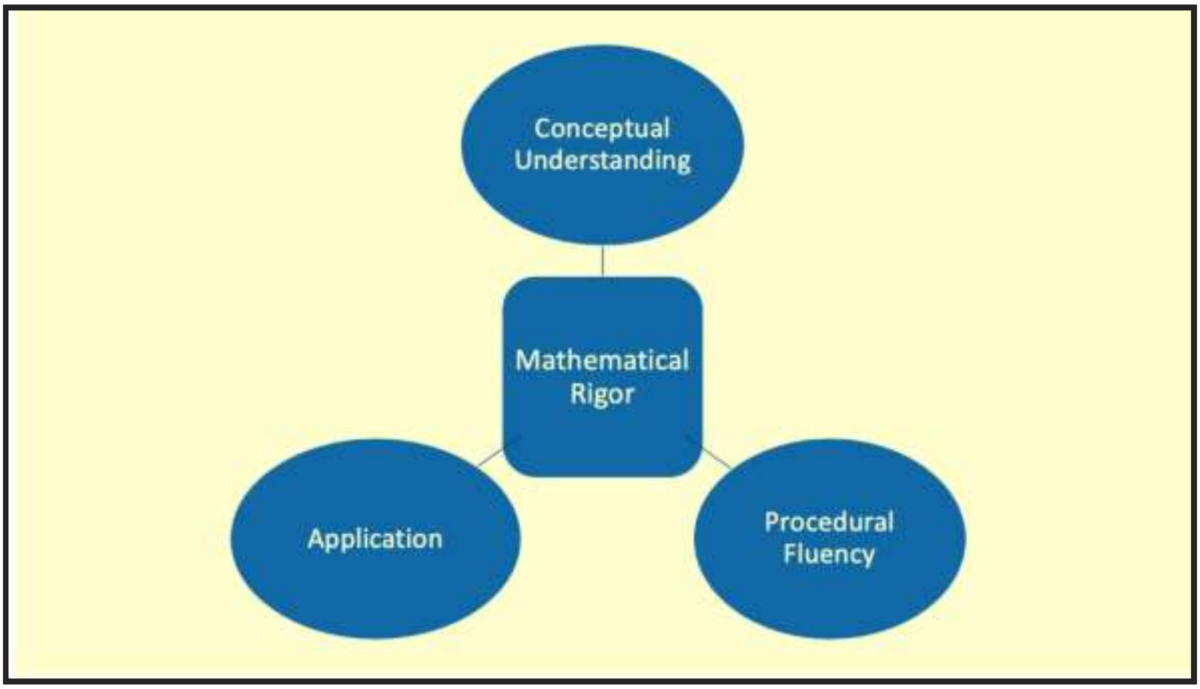

Figure 2. Description of Mathematical Rigor with its Three Main Components that is defined in the Pre K to 12 in the System of Education

Importance of the Topic in Contemporary Math Education

Contemporary mathematics progresses in giant steps and requires rigorous mathematics as a crucial tool. The relevance of verifying truth through mathematics, both within and outside the field, demands accuracy and efficiency. The problem of rigor is particularly relevant today because contemporary mathematics is increasingly sophisticated and is pushing the boundaries of what proofs constitute (Burgess and Toffoli, 2022). Contemporary mathematics at the high school level or in higher education requires advanced strategies within mathematical rigor for solving mathematical problems, applying mathematical concepts in various situations, and proving statements. A significant factor in fostering mathematical rigor is a spiral teaching and learning method. Harden and Stamper identify four key features of Bruner's spiral curriculum: topics are revisited over time, levels of difficulty increase, new learning is related to previous learning, and as a result, the competence of students increases (Severs, 2025). Mathematical rigor and increased student competency lead to deeper understanding and greater success for students in advanced, respectively contemporary, mathematics.

The Research Question(s) and Purpose of the Review

There are numerous methods in teaching and learning that result in different learning outcomes regarding mathematical rigor. Each method has both advantages and disadvantages related to rigorous math; however, the focus of this paper is to explore the relationship between the spiral teaching/learning method and rigor, specifically how a spiral approach promotes rigorous math. The Spiral Progression Approach, if implemented or used, helps in the reinforcement of learning because repetitive exposure to concepts can solidify foundational knowledge and facilitate deeper understanding over time (Bacud and Futalan, 2024). A solid foundation is crucial for developing deep understanding. Without a strong grasp of the basic building blocks, it becomes difficult to construct more complex knowledge and make meaningful connections. Conversely, striving for deep understanding reinforces and solidifies foundational knowledge. When students explore the "why" behind concepts, their initial understanding becomes more robust and memorable. Revisiting topics in mathematics from different perspectives enables students to increase their angle of analyzing relationships of complex mathematical concepts and their applications. Since there are some similarities in the objective of the spiral teaching and learning method with mathematical rigor, the question arises: Does spiraled mathematics relate to and foster rigorous mathematics? To answer this question, the paper will explore a literature review and identify facts that support or refute their relationship.

Theoretical Framework

The paper utilizes several theories that support Spiral Education in mathematics. The main theory relies on Bruner's Spiral Curriculum. The spiral curriculum was created by Jerome Bruner in 1960 for major topics in education by revisiting them repeatedly with increasing complexity. This approach allows for the earlier introduction of concepts traditionally reserved for later, more specialized courses. This occurs after students have mastered some fundamental principles that are often very theoretical and likely to discourage those eager to apply their learning to real-world applications (University of Detroit Mercy, n. d.). Another factor related to Bruner's theory is cognitive development and scaffolding in math learning. Cognitive development imparts a solid foundation for math learning; on the other hand, scaffolding offers the steps to build new knowledge upon this foundation. By blending and utilizing both, teachers can create a more effective learning environment for students.

Educational Theories Supporting Spiral Learning (e.g., Bruner’s Theory of Instruction)

The main theory that supports the Spiral approach in education is Jerome Bruner’s Theory. As children grow, Bruner believed curriculum should revisit earlier learned ideas, expanding upon them until a child reaches a more complete understanding of individual ideas and how they relate to one another (Clark, n. d.). The purpose of different levels of education is to ensure students’ progress at each level. An education program that incorporates revisiting topics with increasing complexity makes a positive difference in learning outcomes. Programs should be arranged in a spiral so that the student is continually building on what they have learned (Baron and Dela, 2023). In cases where students do not progress academically at the desired pace, educators should go back to revisit missing concepts that students did not understand well. The spiral curriculum may also mean we take steps backwards before progressing forward — something to consider when a pupil is not moving forward as we might anticipate (Currell, 2024). In this way, it fills the gap of the unknown with study, fostering familiarity and reducing those gaps.

The Role of Cognitive Development and Scaffolding in Math Learning

Recognizing an imminent need in mathematical subjects, the rote method of teaching and learning shifted towards a model focusing on understanding and reasoning. American cognitive and developmental psychologist Jerome

Bruner revolutionized math education, shifting the psychological mindset from behaviorism to place cognition at the center of modern educational theory and modern educational practices everywhere (Currell, 2024). Learning mathematics with the question 'why' in mind influences the intellectual development of students. The term cognitive development refers to the process of growth and change in intellectual/mental abilities such as thinking, reasoning, and understanding (California Department of Education, n. d.). By placing cognitive development at the center of the mathematical curriculum, a significant space is created for the spiral teaching and learning method, which tends towards promoting mathematical rigor.

Literature Review

The paper centers on exploring the relationship between the spiral teaching and learning method and mathematical rigor in literature-based research papers. In the United States, the spiral curriculum is the most widely used structure for school mathematics (Seely, 2009; Snider, 2004). Despite poor test results in the United States, many countries have applied a spiral approach in their mathematical curricula. Nevertheless, the practice of retaining concepts from previous courses is closely aligned with the philosophy of the spiral curriculum and delivers significant benefits to students, to the extent that it serves as a vindication of the spiral curriculum's success (Masters and Gibs, 2007). Regardless of how well students master material, after a period of time they will forget a significant portion of it. Even if educators use a mastery teaching method, the forgetting curve will affect students' memory, leading to the forgetting of at least 70% of the learned material after a six-month period. Hermann Ebbinghaus theorized that the forgetting curve occurs due to several psychological factors, including the difficulty of the learning material and the limitations of long-term memory. Luckily, one of the methods Ebbinghaus hypothesized could help retain learning has since been proven very effective: spaced repetition, or spiral practice, as we at SMASH Maths call it (Smash Maths, n. d.)! The benefits of a spiral approach are multidimensional; this method deepens understanding, advances students to a higher academic level, and tends to promote rigor in math. Spiral teaching and learning have significant applications in many countries, such as Finland and Singapore. Singapore Math could be described as a program with elements of spiral learning and mastery, but which focuses on helping students learn through a clear, step-by-step methodology that builds upon each step so that students can develop the skills to move from easy to rigorous problems (Singapore Math, n. d.). While the spiral teaching method could be applied to all students, mathematical rigor is more applicable for advanced students; however, the spiral method promotes mathematical rigor.

Spiral Approach in Mathematics

Usually, the teaching method corresponds with effective learning. Spiral Learning is a tried-and-tested method for enhancing comprehension and memory of new knowledge (Doron, 2024). Spiral approach differs from linear or cyclic teaching and learning. Spiral approach never ends like a circle, it always progresses outward. According to Bruner, curriculums should be designed to revisit basic ideas, building upon them over time until students have full understanding of them (Bruner, 1977). Revisiting topics with progressing toward its complexity approaches not just to a better understanding but mastering these topics to an advanced level. Spiral teaching supplies students with significant tools that students can apply their knowledge in different situations.

Jerome Bruner's Original Concept

Burner's original theory of spiral model is defined with clear statements, but its application might interconnect and prompt other learning methods. Both Hardin (1999) and Bruner (1960) concluded that the concept of a spiral curriculum merits careful consideration Neumann, et. al., 2017). Researchers analyzed deeply the main components of Spiral methods with the purpose to apply them easily in the educational curriculum. According to EPI (n. d.) key features of Spiral curriculum based on Bruner's work are:

The student revisits topic, theme, or subject several times throughout their school career.

The complexity of the topic or theme increases with each revisit.

Students are encouraged to apply their early knowledge to later course objectives.

The Burner's original concept is described with an original paragraph of Jerome Burner. According to (Harden and Stamper, 1999) this concept was described first by Jerome Bruner in 1960:

> I was struck by the fact that successful efforts to teach highly structured bodies of knowledge like mathematics, physical sciences, and even the field of history often took the form of a metamorphic spiral in which at some simple level a set of ideas or operations were introduced in a rather intuitive way and, once mastered in that spirit, were then revisited and reconstrued in a more formal or operational way, then being connected with other knowledge, the mastery at this stage then being carried one step higher to a new level of formal or operational rigor and to a broader level of abstraction and comprehensiveness. The end state of this process was eventual mastery of the connexity and structure of a large body of knowledge . . . .

Based on the paragraph above, Burner claims the pathway of teaching and learning requires a structured body of knowledge. The structure (Spiral approach) enables students to learn fundamental concepts, and then by revisiting topics, learners get deeper in the subject by adding new knowledge in the old one. The spiral of learning can continue further to advance learning to a higher level. The new level incorporates more abstract concepts or more rigor processes.

Implementations in Various Curricula (e.g., Singapore Math, Common Core)

Implementation of spiral curriculum is adopted in many countries, mostly is implemented in developed countries and international schools. There does not exist a statistic the number of the countries that apply a spiral curriculum; dozens of countries implement a form of curriculum with spiral learning in a part of their education system. In Chinese schools' students revisit each of the basic sciences each year of their secondary school experience (EPI, n. D.). This is a reason why their academic performance is great. China mixes other teaching methods such as mastery teaching method and spiral approach.

The Shanghai city in China whose students perform outstanding results in the PISA test use a mastery method combined with a spiral method. This is essentially a 'teaching for mastery' approach: covering less and making smaller incremental movements forward, ensuring the class moves together as one and that you go over stuff again and again until it's truly understood (Low, 2017). China's education system does blend elements of both the mastery teaching method and the spiral curriculum, particularly in math and science education.

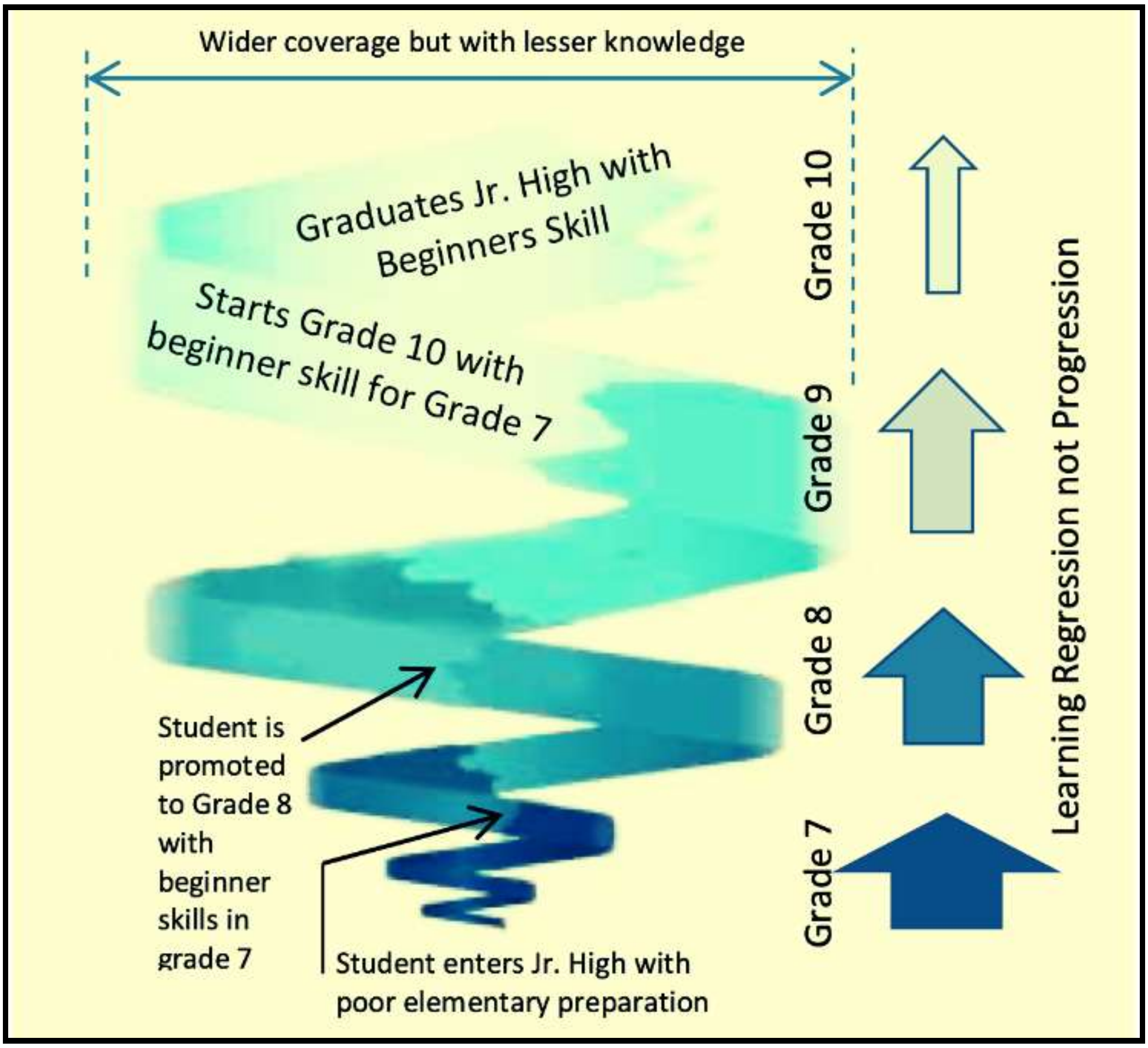

Figure 3. Application of Spiral Curriculum from Grade 8 to Grade 10 in Philippine Modified from Orale (2018)

The Filipino curriculum in the K to 12 education system is based on the Spiral Approach. It is highly important to apply the spiral curriculum in a rational and structured manner. One feature of the new curriculum is the spiral approach to learning, which is believed to provide a smooth transition between grade levels (DepEd, 2012). Figure 4 represents the mathematics curriculum and its inclination toward the spiral method. Beginning in Grade 8, it enables students to revisit topics from Grade 7, especially those closely connected to the Grade 8 curriculum.

However, students in Grade 10 tend to forget a considerable amount of math material from Grades 7, 8, and 9. Each skill is reviewed and revisited throughout different levels of math, continually building upon prior learning (Dragon Times, 2024). Revisiting material from previous grades substantially contributes to combating the forgetting curve while also introducing new material in Grade 10.

In the UK, a spiral mathematics curriculum is implemented in two-week blocks before shifting to other topics. Christopher Boyd, in his article, argues that it would be more effective to extend topics over longer periods, shifting from weeks to months. He suggests that combining a mastery method with the spiral method would be beneficial. This approach would still involve revisiting old topics to prevent forgetting, but the primary focus of learning would remain on the same theme for months rather than weeks (Boyd, 2021). In their first year, students at Surrey University in the

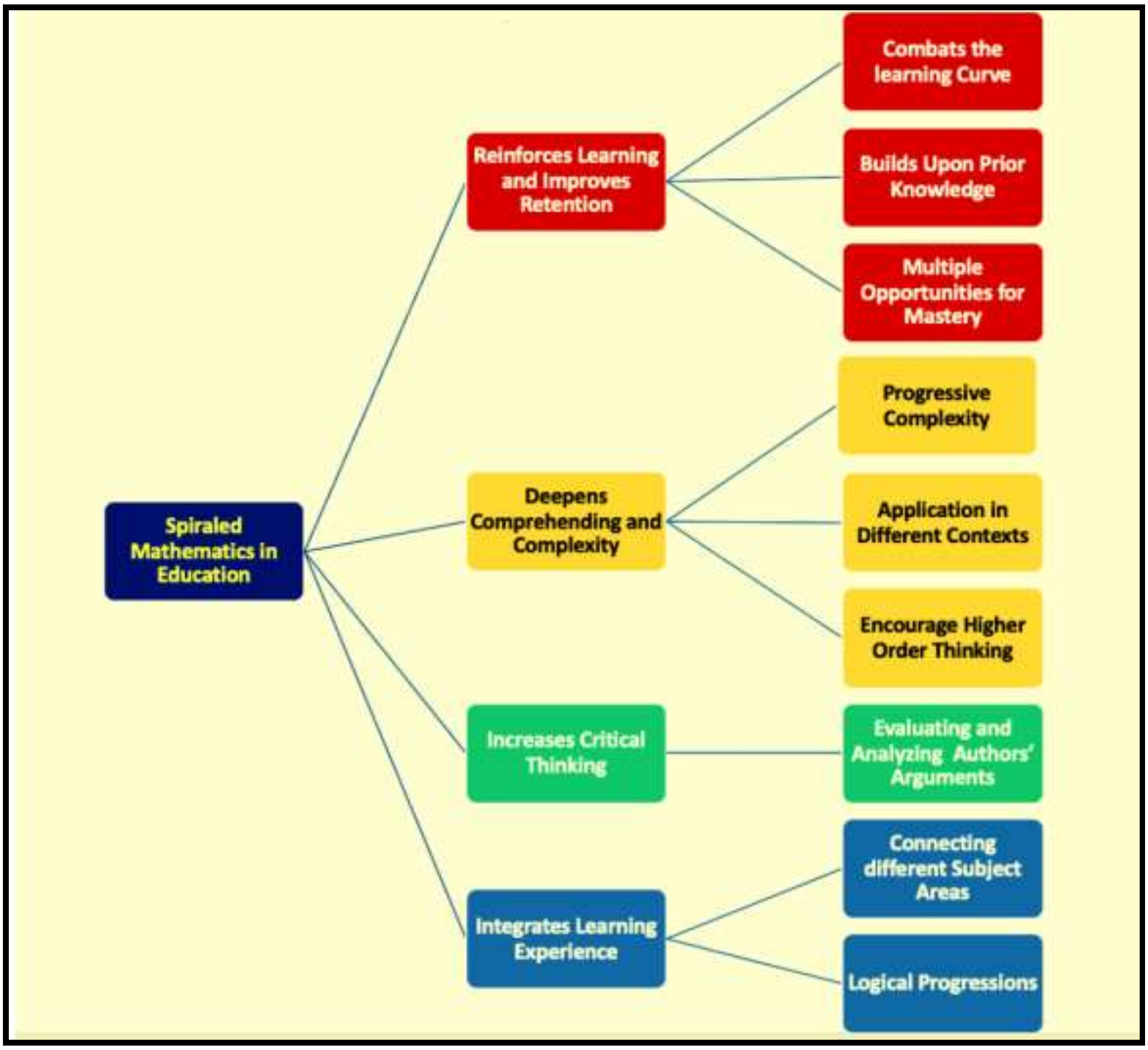

Figure 4. Spiraled Education Analyzes Through Four (the Components Described in the Figure) Components that Fosters Mathematical Rigor

UK participates in a single module. While they then move on to a different topic, the spiral approach ensures that students return to previous topics later in their studies (Jia Doulton, 2022), allowing them to revisit these topics and gain a deeper understanding of the material.

Singapore ranks alongside top-performing countries like Finland in the standardized PISA tests. It applies a spiral learning method similar to that of the United States. However, while the U.S. covers a large number of math concepts in one academic year—traditionally about 30 concepts introduced and revisited with increasing complexity (Schools that Can, n.d.)—Singapore covers only 10 to 14 math concepts per academic year. Singapore spends about 2 to 3 weeks on each concept before moving on. Notably, Singapore modified its curriculum by blending the spiral approach with the mastery method. Furthermore, they did not import textbooks from other countries but instead developed their own materials tailored to the specific needs of students to help them excel in mathematical understanding.

Finland updates its National Core Curriculum (NCC) approximately every 10 years. In the 2000 PISA test, Finland ranked first in reading, and its results in science and mathematics were also impressive, placing third and fourth, respectively. In 2003, Finland improved even further, ranking first in all three areas (Franko, 2011). Recently Finish math scores of student's performances fell for 56 points in PISA. In math, Finland's average scale scores went down by 79 points between 2003 and 2022 - lapping the other laggards by miles (Daly, 2024). While the Finnish curriculum does not explicitly label itself as using a spiral curriculum, it incorporates several elements of the approach. The Finnish model naturally embodies spiral curriculum principles such as revisiting key ideas, increasing complexity over time, cross-subject integration, and a strong emphasis on deep understanding over broad coverage.

In the U.S., the spiral curriculum is mostly applied in mathematics and science from elementary through middle grades. It also makes a significant positive impact in high school by gradually increasing the complexity of mathematical problems and concepts. To achieve this, the level of difficulty and complexity must continually increase until the concept is fully mastered (Gifford, 2020).

Furthermore, standardized testing in the U.S., such as state assessments aligned with Common Core or other standards, also influences how the spiral curriculum is implemented. Teachers often need to balance revisiting concepts with preparing students for broad assessments, which can create pacing challenges. The spiral curriculum is applied in numerous countries around the world. In some countries, it is explicitly named as part of their educational system, while in others, it is embedded within the curriculum and blended with other teaching and learning methods. In either case, the spiral curriculum plays a significant role in education, particularly in improving student performance in examinations.

Mathematical Rigor

Mathematical Rigor is defined in different forms depending on which angle is observed. The paper focus is to define from the K to 12 perspectives. Mathematical rigor in K–12 means balancing conceptual understanding, procedural fluency, and application—so that students not only know how to do math, but also understand why it works and how to use it. The Department of education and workforce in Ohio define Rigor in this form: Students use mathematical language to communicate effectively and describe their work with clarity and precision, Students demonstrate how, when and why their procedures work and why they are appropriate. Students can answer the question, "How do we know?" Answering the question "How do we know?" corresponds with the metacognition process; it encompasses the awareness, understanding, and control of one's cognitive processes

Definitions and Importance of Rigor in National and International Standards

American Institute for Research defines mathematical rigor in a low mathematical rigor and high mathematical rigor for the K to 12 educations:

> Low-rigor items require students to use simple mathematical skills such as recalling mathematics definitions or applying mathematics formulas. High-rigor items require students to use cognitively demanding skills, including reasoning through the information to formulate a problem mathematically. Rigor can also be defined in terms of the difficulty of mathematics items (Ginsburg, et. al., 2003 p. V).

Regardless of low or high mathematical rigor it is necessary to adjust the expectation of students' learning outcome with the content of the lessons. In general, it is necessary to balance the three components of the mathematical rigor as the definition of rigor for K to 12 states. Rigor: Pursuing conceptual understanding, procedural skill & fluency, and application all with equal intensity (Blackburn, 2018). Oftentimes math educators prepare students for the PISA test and they try to apply mathematical rigor into mathematical concepts, problem solving skills, and application. But it too often is of low quality and fails to ask students to apply math to complex, real-world problems, as PISA does (Hughes, 2023). Most likely, students will struggle with a successful mathematical test performance in PISA.

Mathematics rigor defined with respect to instructions and content explains (Miles, 2013):

> Content: Mathematical rigor is the depth of interconnecting concepts and the breadth of supporting skills students are expected to know and understand.
>
> Instructions: Mathematical rigor is the effective, ongoing interaction between teacher instruction and student reasoning and thinking about concepts, skills, and challenging tasks that results in a conscious, connected and transferable body of valuable knowledge for every student.

Rigor in mathematics, as mentioned earlier in this paper, is defined differently across K–12 and higher education. The Charles A. Dana Center outlines five different views of rigor; however, this research focuses on one of them. In this context, rigor is associated with *advanced courses*, comprehensive presentations and lectures covering a wide range of topics and techniques, high-stakes summative tests, low pass rates, and/or low grades (Dana Center, n.d.).

Advanced math courses (such as Pre-Calculus, Calculus, or Advanced Placement courses) delve deeper into algebra, functions, and problem-solving skills—areas often assessed on standardized exams like the SAT, ACT, national high school assessments, and college entrance exams. Students who study these topics in greater depth are better prepared for the complex problems found on these assessments, including international ones like the PISA math test.

Carlson, in her book, defines rigor as a means to guide students toward applying the highest levels of thinking. It moves them beyond simple recall of facts or procedures and encourages them to investigate, draw from existing conceptual and procedural knowledge, and create their own solutions (Carlson, n.d., p. 13). The purpose of mathematical rigor is to help students become more adept at applying creativity in various situations.

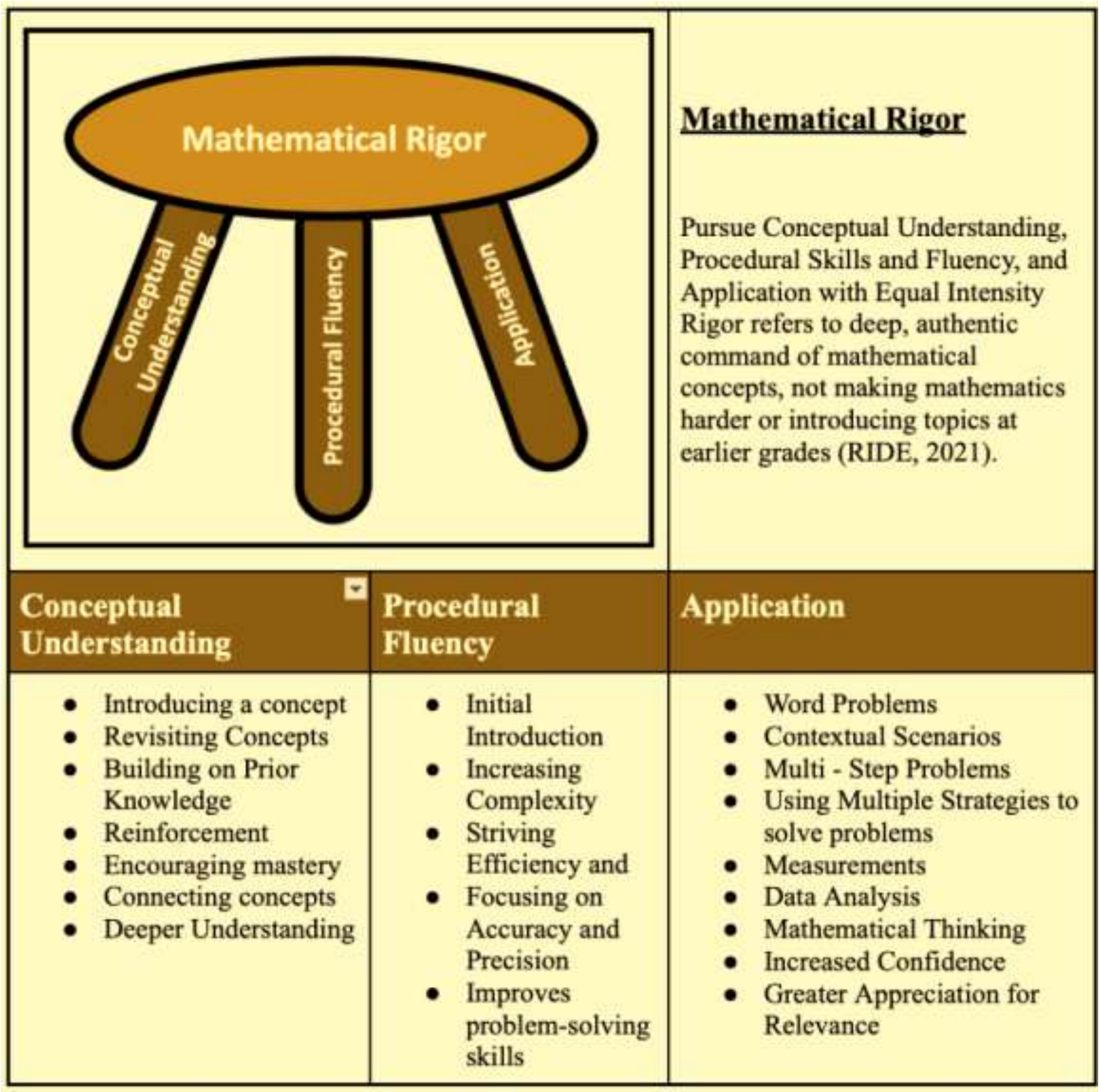

Figure 5. Three Components of Mathematical Rigor followed by their Elements described in the Column of Each Component. The Figure is Modified by Toncheff (2025)

Challenges in Achieving Rigor in Mathematics

Achieving rigor in mathematics education presents several challenges across different educational levels. One major difficulty lies in balancing procedural fluency with deep conceptual understanding. While many curricula emphasize memorization and rote learning, true rigor requires students to engage in higher-order thinking, make connections between concepts, and apply their knowledge to unfamiliar problems. Ensuring equitable access to rigorous coursework is also critical, as students from underserved communities often lack the resources or support systems needed to succeed in advanced math classes.

The unequal access to quality education is highlighted even in PISA. PISA demonstrates quite convincingly that some countries do a much better job than others at ensuring all students have roughly equal access to rigorous mathematics content, which includes formal mathematics (Schmidt & Burroughs, 2015). The challenge of implementing mathematical rigor often lies in the methods used to teach and learn mathematics. Jessica Carlson, in her blog, identifies five key challenges teachers face when implementing mathematical rigor:

1. Direct instruction is not enough
2. Mistakes indicate a lack of understanding
3. There is only one way to find the answer
4. Memorization is better than reasoning
5. Perfect scores equate to mastery (Carlson, n.d.b)

Exploring multiple strategies to solve mathematical problems—by revisiting topics and analyzing them from different points of view—enables students to deepen their mathematical knowledge. In math class, students should have opportunities to discuss various approaches. Comparing these strategies helps learners think not only about what works in mathematics but also about how and why it works (Tamer, 2015).

The mathematics curriculum is designed to encourage solving problems in two or more different ways. For example, solving quadratic equations can involve several methods: vertex form, standard form, factored form, and graphing. Other mathematical concepts also offer multiple approaches, such as using long division or synthetic division when working with rational expressions.

Mistakes are a natural part of learning new ideas, concepts, and problem-solving strategies. They help students identify what is going wrong and what adjustments are needed to improve. In her article *The Power of Good Mistakes*, Jaster writes:

> As you strive to implement standards designed to ensure that students master mathematical practices and content—whether they are the Common Core or your own state or provincial standards—remember that how you respond to “good” mistakes has the potential to discourage students or to help them become more confident in their ability to do mathematics (Jaster, 2013).

Memorization is useful for remembering formulas, procedures, and mathematical vocabulary. However, the development of reasoning spans several domains: additive thinking, multiplicative reasoning, proportional reasoning, functional reasoning, etc. Each of these builds on the previous and becomes increasingly complex and sophisticated, with more variables and interactions taking place (Harris & McElhanney, 2020). Revisiting mathematical topics through these domains of reasoning, in order of increasing complexity, enhances student learning outcomes.

Perfect scores are often equated with mastery across a wide spectrum of education. Researchers at the Institute of Education, University of London, associate high test scores with mastery in their study:

> "For example, a key feature of many apparently effective programmer studied to date is that once pupils have completed each block of content they must demonstrate a high level of success on a test, typically at about the 80% level. Pupils not meeting this hurdle receive additional instruction, whilst those who

> succeed engage in enrichment activity that seeks to deepen their understanding of the same topic" (Vignoles, 2015).

In this program, teachers provide students with opportunities to engage in enrichment activities and revisit topics from different perspectives. By reviewing previously learned material using varied examples and scenarios, students can develop additional skills and deepen their understanding. Those who fall behind spend more time reviewing the content to reinforce their learning.

Bridging the Two Teaching Methods:

This is a learning or curriculum design strategy where topics are revisited multiple times over a period, each time at a deeper or more complex level. The key idea is iterative learning: first pass: basic exposure, second pass: add more detail, third pass: apply concepts …and so on. Each stage builds mastery, and revisiting helps deepen understanding, not just repeat it. Students cover material about a subject with greater complexity until they have mastered it (Guest Writer, 2022). They revisit topics and balances mastering the content of topics in three domains: procedural fluency, conceptual understanding, and its applications. The application of mathematical concepts in practical scenarios necessitates insightful vision and strategic thinking (Baez, et. al.,2024).The spiral method is a pathway on mastering the teaching and learning material.

Studies that Explore how Spiral Methods Affect Mastery and Depth

Bruner’s theory allows for the combination of different types of learning. *The Process of Education* contrasts with Jean Piaget’s more rigid theory of child development, as Bruner’s approach enables various types and stages of learning to occur simultaneously and blend into one another (APS, 2016). The spiral approach outlines a pathway for teaching and learning across different content areas and topic structures. As purposive participants in the knowledge acquisition process, learners choose structures, remember information, and transform it (Tirol, 2022). The application of the mastery teaching method can be implemented throughout an academic year by administering a diagnostic test and assessing students' prior knowledge. Teaching and learning begin with what students already know, and new knowledge is added each time they revisit material. The spiral approach offers educators a framework to enhance the quality of student learning as an investment in the future (Veladat & Mohammadi, 2015).

> This method allows for increasing topic complexity while supporting mastery of the material within a set timeframe. Regardless of how well students initially understand a subject, forgetting is a natural human tendency. Regularly revisiting material refreshes learning, deepens understanding, and enriches knowledge over time.

Research on Retention and Long-term Understanding

Mastering mathematical content in a subject involves recalling mathematical concepts and solving problems—both of which require consistent practice and systematic review of topics. Reviewing material at increasing intervals helps reinforce it in long-term memory.

In education, learning retention plays a vital role in enabling learners to successfully grasp concepts taught both inside and outside the classroom. According to Valderama and Oligo (2021), the amount of retained learning decreases over time: by an average of 4% per week for concepts at the definition/knowledge level, up to 17% per week for concepts at the comprehension level, and up to 21% per week for concepts at the application/analysis level. Mathematics builds new knowledge upon previous learning. Since every new concept is connected to earlier mathematical ideas, retaining information from prior lessons is essential.

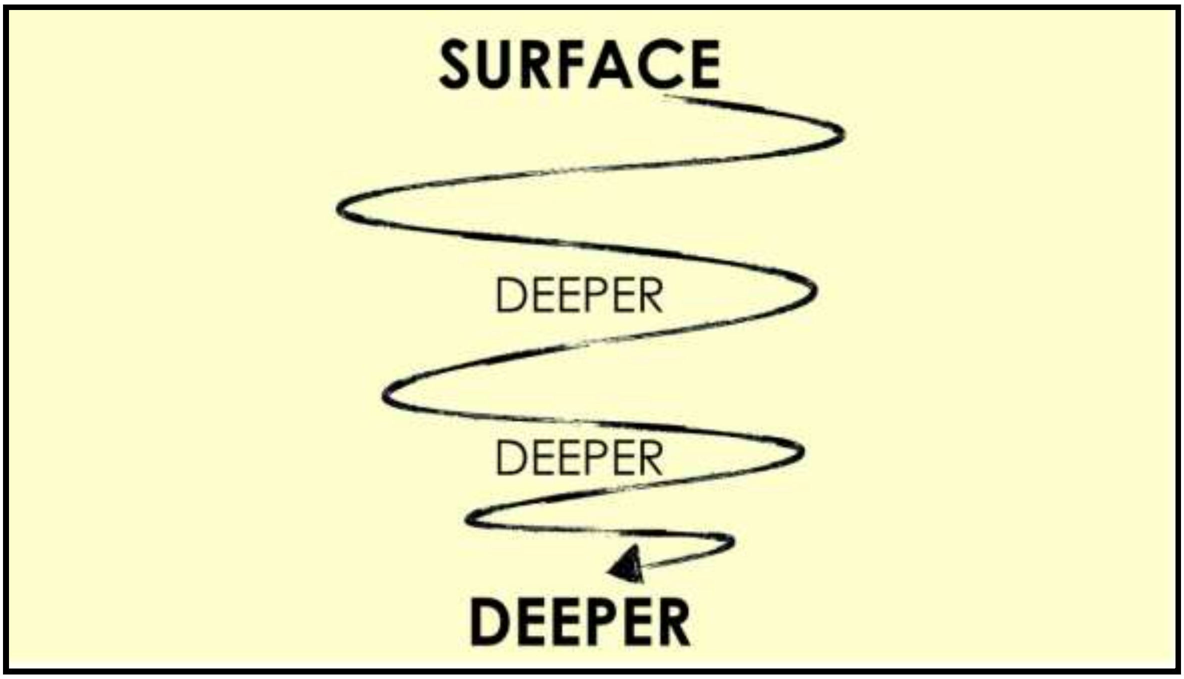

Figure 6. Spiral Teaching and Learning Start from Easy Topics and Get Deeper, Deeper, and Deeper with a Tendency to Master the Content of Topics (Orr, 2018)

Possible Criticisms of Spiral Methods in Promoting Superficial Repetition rather than Depth

Spiral methods differ significantly from their superficial repetition. In a spiral method, the radius of knowledge expands continuously with each cycle. In contrast, with simple repetition, the radius of knowledge remains constant across cycles. Harden and Stamper, in their article *"What is a Spiral Curriculum,"* state:

> "A spiral curriculum is one in which there is an iterative revisiting of topics, subjects, or themes throughout the course. A spiral curriculum is not simply the repetition of a topic taught. It also requires the deepening of understanding, with each successive encounter building on the previous one." — Harden & Stamper, 1999 (originally described by Jerome Bruner, 1960)

Repeating learning material in exactly the same form follows the natural principle that doing the same thing every day yields the same results. Repetition using the same topics and examples does not lead to meaningful progress. However, revisiting topics with increasingly challenging mathematical problems enhances learners' skills and equips them to handle more advanced examples—in other words, it produces different, improved learning outcomes.

Discussion

Spiral learning is a curriculum design strategy where topics are revisited multiple times, each time with greater depth and complexity. Each time a learner is asked to revisit a topic, they strengthen their memory (Knight, 2021). This iterative process helps students gradually build mastery by deepening understanding rather than simply repeating content. It emphasizes three key learning domains: procedural fluency, conceptual understanding, and real-world application.

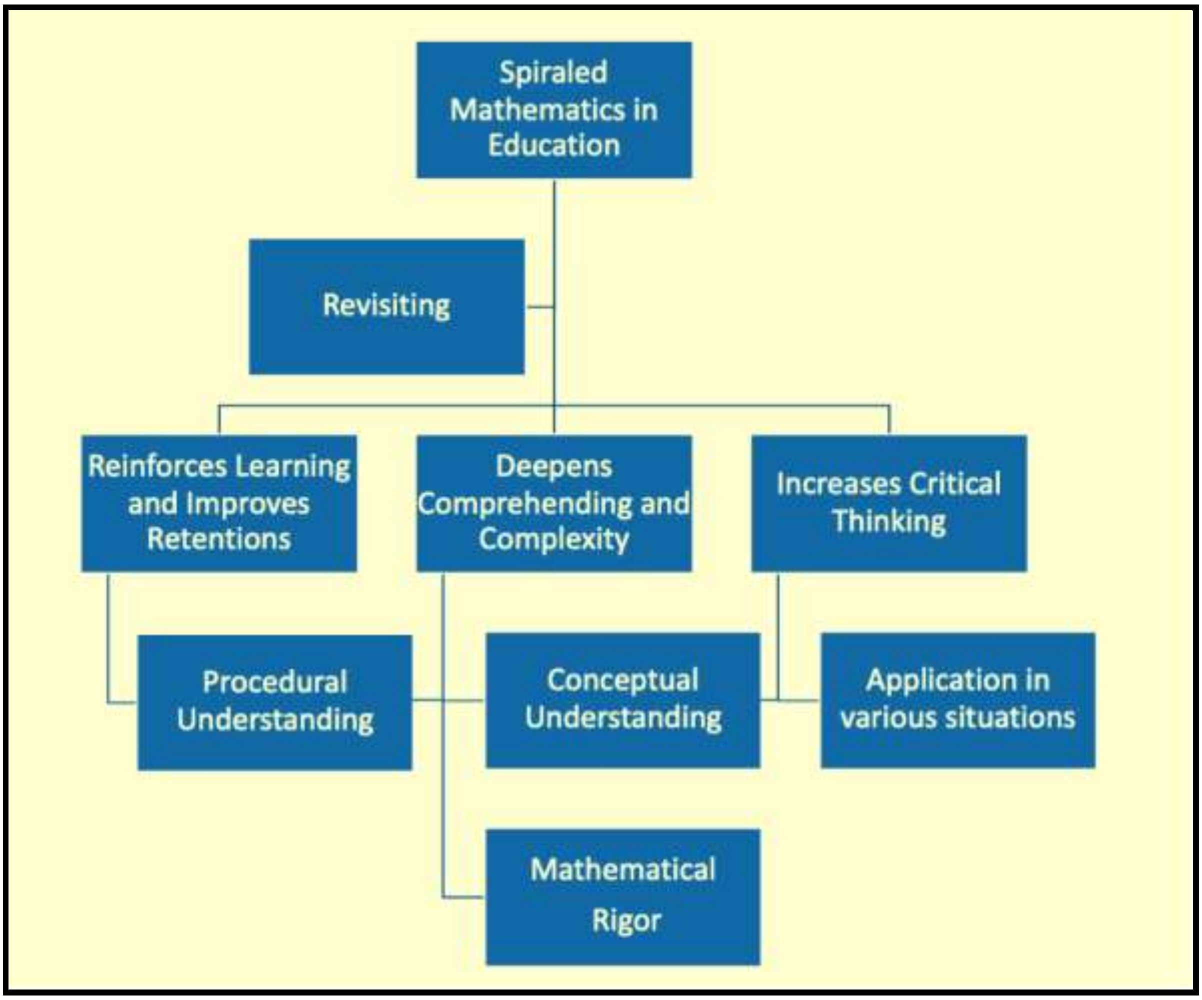

Figure 7. Description of the Relationship of the Spiral Teaching and Learning Method to Mathematical Rigor

Bruner's theory supports this flexible and layered approach, contrasting with Piaget's more rigid developmental stages. The spiral method enables varied learning types to occur simultaneously and progressively, allowing students to build on prior knowledge. This prior knowledge is the foundation on which any future learning will be built (Haward - Jones, 2019). Implementing spiral learning involves assessing students' existing knowledge and continuously adding new layers through diagnostic tests and strategic instruction. This approach enhances retention, combats natural forgetting, and reinforces long-term understanding—especially in subjects like mathematics, where new concepts build upon previously learned material. Research highlights that regular, spaced review with increasing complexity improves knowledge retention, which is essential for achieving true mastery over time.

Synthesizing Findings: Where Do Spiral Methods Align or Conflict with Goals of Rigor?

The spiral method, as presented in the literature, suggests that teaching and learning begin with an introduction (scratching the surface of the content). By revisiting topics in each cycle, learners gradually gain a deeper understanding of the material. The spiral method supports learners in developing deep understanding. Spiral learning methods align with human nature—they are natural, incremental, and allow complex tasks to appear simple and eventually become second nature (Hutcheson, 2023). Spiraling supports students' goals of mastery by building knowledge step by step; each cycle is more complex than the last. Spiral learning is generally flexible and can be applied across various learning contexts.

The mastery method refers to learning a topic thoroughly before progressing to the next. A learner only advances after fully understanding the current concept. The mastery method follows a linear teaching and learning process. Winget and Persky (2022) describe the following steps in mastery learning:

> Mastery learning involves several steps: initial learning, formative assessment, corrective activities, and enrichment activities. The first step is to specify learning goals or objectives. Next is developing formative assessments. The third step is organizing corrective activities, followed by planning enrichment activities. The final step is developing summative assessments.

A key evaluation in mastery learning is determining whether students have mastered the material—typically judged through a summative assessment. While mastery learning is an effective method for ensuring deep understanding of a topic, exams may not always accurately measure long-term learning. Students who mastered the material earlier may forget details over time. In such cases, spiraling can be a solution to combat forgetting and reinforce retention.

The relationship between the spiral method and mathematical rigor is illustrated in Figure 7. Revisiting topics helps reinforce learning and improves retention. Strong foundational knowledge in mathematics supports the understanding of mathematical processes used in solving problems. Spiraling—by reintroducing topics with increasing complexity—deepens both understanding and comprehension. Practicing with complex content leads to a better grasp of underlying concepts.

Deogratias (2018) found that complexity theory, as a model of inquiry, is promising for structuring mathematics classes in ways that support both individual and collective understanding of mathematical concepts. Revisiting content fosters critical thinking. In mathematics, critical thinking involves using reasoning to apply various strategies to solve real-world problems in diverse contexts.

Ridwan (2022) in his research paper, claims:

> Teachers' perceptions of activities that develop students' critical thinking skills in mathematics often involve instructional models or teaching approaches.
>
> Three high school teachers gave consistent responses, though with different interpretations, indicating that critical thinking skills involve the ability to solve mathematical problems by understanding concepts, analyzing, applying, and evaluating information.

Three main components of the spiral method correspond to those of the mastery method, and all three align with the core components of mathematical rigor. Applying the spiral method—while balancing time across understanding mathematical concepts, solving problems, and applying mathematics concepts to real-world scenarios—adequately satisfies the definition of mathematical rigor.

Conclusion

The spiral and mastery methods are two instructional approaches that support deep learning, each with distinct characteristics. The spiral method involves revisiting topics in increasing complexity over time, promoting long-term understanding and retention. It aligns with natural learning processes and supports mastery by gradually deepening knowledge. The mastery method, on the other hand, requires students to thoroughly understand a topic before moving on. It follows a linear sequence involving formative assessments, corrective actions, and enrichment, culminating in a summative assessment. However, mastery learning may not guarantee long-term retention, which the spiral method helps address by reinforcing learning through repetition. In mathematics education, the spiral method enhances mathematical rigor by encouraging repeated practice, critical thinking, and real-world application. Both methods can complement each other, and when combined, they align well with the goals of rigorous mathematical learning.

Employing spiral methods in teaching and learning maintains the balance of mathematical rigor in two directions: vertically and horizontally. Spiraling maintains the horizontal balance with respect to procedural fluency, conceptual understanding, and application in real-world examples. It is very easy to forget the considerable information of the first topic when we move further to the last topic. New information can make it difficult to retain old information. If both sets of information are on the same subject, the newer memories may make it harder, if not impossible, to remember the older ones on the same subject (Najam, 2022). Vertical balance of knowledge throughout all three components of mathematical rigor makes spiraling possible.

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