ISSN: 2148-9955

International Journal of Research in Education and Science (IJRES)

www.ijres.net

Reasoning and Proving Opportunities in Textbooks: A Comparative Analysis

Dae S. Hong, Kyong Mi Choi
University of Iowa

To cite this article:
Hong, D.S. & Choi, K.M. (2018). Reasoning and proving opportunities in textbooks: A comparative analysis. *International Journal of Research in Education and Science (IJRES), 4*(1), 82-97. DOI:10.21890/ijres.382937

IJRES International Journal of Research in Education and Science
Volume 4, Issue 1, Winter 2018 DOI:10.21890/ijres.382937

Reasoning and Proving Opportunities in Textbooks: A Comparative Analysis

Dae S. Hong, Kyong Mi Choi

Article Info

Article History

Received:
01 March 2017

Accepted:
27 June 2017

Keywords

Proof
Textbooks

Abstract

In this study, we analyzed and compared reasoning and proving opportunities in geometry lessons from American standard-based textbooks and Korean textbooks to understand how these textbooks provide student opportunities to engage in reasoning and proving activities. Overall, around 40% of exercise problems in Core Plus Mathematics Project (CPMP) ask for reasoning and proving activities while 20% of exercise problems in Korean textbooks are about reasoning and proving activities. One of the major findings is that Korean and CPMP students have different opportunities in learning geometry. It may be interesting to investigate geometric reasoning and proving performance of Korean and CPMP students to see how textbooks influence their learning.

Introduction

In school mathematics, it is recommended that the process of reasoning and proving be present throughout the mathematics curriculum (National Council of Teachers of Mathematics [NCTM], 2000; National Governors Association & Council of Chief State School Officers, 2010). Despite such an emphasis, many studies show that students have difficulties with understanding or constructing reasoning and proving related activities (Chazan, 1993; Haely and Holyes, 2000; Senk, 1985; Soweder & Harel, 2003; Webber, 2001). In attempts to address issues involving reasoning and proving, recent research has examined reasoning and proving in the middle grades (Stylianides, 2009), an integrated standards–based textbook series (Davis, 2010), algebra and precalculus (Thompson et al., 2012), and geometry (Otten et al., 2014).

Although there are different views about the link between what textbooks offer and what students learn (Freeman & Porter, 1989; Fuson, Stigler, & Bartsch, 1988; Li, 2000), researchers generally agree that the content of mathematics textbooks informs us about students' opportunities to learn mathematics (Garner, 1992; Otten et al., 2014; Reys, Reys, & Chavez, 2004; Thopmson et al., 2012). If opportunities to reason and prove are not presented in textbooks, it is unlikely that these opportunities will be created in enacted curriculum of classroom practices (Thompson et al., 2012). Thus, although opportunities provided by textbooks are not the only influencing factors on teaching and learning of mathematics, analyzing reasoning and proving opportunities in textbooks is a first important step to understand students' opportunities to learn reasoning and proving.

The textbook and curriculum studies cited above are limited to American mathematics textbooks examining reasoning and proving opportunities. Expanding understanding of reasoning and proving opportunities in textbooks from other countries will be beneficial (Ottten et al., 2014), particularly from countries whose students have performed well on international studies such as Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA). The purpose of this study is to analyze and compare reasoning and proving opportunities in one American standards–based secondary textbook series, Core Plus Mathematics Project (CPMP), and secondary textbooks from Korea, one of the top performing countries, in geometry, where reasoning and proving is most prevalent (Ottten et al., 2014). Here are the research questions that we attempt to answer.

1. What is the nature of reasoning and proving opportunities in Korean and standards–based American textbooks?
   i) What types of justifications are used in the textbooks?
   ii) What types of reasoning and proving opportunities are provided in the textbooks?

2. What similarities and differences are observed in reasoning and proving opportunities in Korean and standards–based American secondary textbooks?

In an attempt to answer these research questions, we selected Core Plus Mathematics Project (CPMP) textbooks. The algebra sections of the CPMP textbooks were examined by Thompson et al. (2012). Geometry sections in Connected Mathematics, another standards–based textbook series, were examined and analyzed previously (Stylianides, 2009). Thus, it would be meaningful to examine geometry sections of CPMP, an exemplary mathematics program. Once CPMP was selected, we found Korean textbooks that include compatible topics of geometry: Middle School Mathematics I, II and III*.

Related Literature

Teaching and Learning of Reasoning and Proving

Many studies show students' difficulties in writing proofs (Harel & Sowder, 2007; Senk, 1985; Webber 2001). One well–known finding is that students use inductive argument and that empirical evidence is sufficient enough to prove instead of using a deductive proof (Chazan, 1993; Harel & Sowder, 2007). Other studies report similar results, where the majority of students proved a statement by providing a specific example, and only a few students tried to prove general arguments; however, they did not utilize deductive proof schemes (Knuth, Slaughter, Choppin & Sutherland, 2002; Thompson, 1991). According to Fischbein (1982) and Haely and Holyes (2000), students are convinced by empirical evidence but not convinced by deductive form of arguments. More specifically, Haely and Holyes (2000) found that even some able students feel that proof has to have certain features that are provided by the teacher (authority), and students have little feel for purposes for proofs. Moreover, only a few students thought that one counterexample is sufficient to disprove a conjecture (Galbraith, 1981) and that a proof verified a particular case (Vinner, 1983).

Fujita and Jones (2003) reported that students do not realize why proofs are needed even when they are able to write proofs. Since constructing proof is not easy, students may leave some responses with empirical or inductive arguments (Otten et al., 2014). This situation does not improve at the college level. Many college students also consider inductive arguments to be mathematical proofs or they do not differentiate deductive and inductive proofs (Martin & Harel, 1989; Morris, 2002). These findings across secondary and post-secondary school levels and across countries can be serious stumbling blocks for students to accurately understand proofs. With this evidence, it is important to pay systematic attention to reasoning and proving opportunities provided by textbooks, which partially explain students' reasoning and proving experiences in mathematics classes.

Textbook Studies on Reasoning and Proving

Several studies have examined reasoning and proving opportunities in various textbooks (Davis, 2010; Fujita & Jones, 2013; Otten et al., 2014; Stylianides, 2009; Thompson et al., 2012). Stylianides (2009) found in his analysis of the Connected Mathematics series that about 40% of tasks are related to reasoning and proving and rationale. Providing Non–Proof Argument was the most frequent type of reasoning and proving task. Otten and his colleagues (2014) discovered that geometry sections include a substantially high portion of reasoning and proving tasks compared to other mathematical areas. In addition, while there are more general statements in textbook narratives, there are more reasoning and proving exercises about particular statements than general statements, which, they claimed, could cause students' difficulties in constructing proofs.

There are varying results in the analysis of Algebra and Precalculus textbooks. There are more general statements in Algebra I textbooks, but there are more general argument problems than specific cases in Algebra II and precalculus textbooks (Thompson et al., 2012). Thompson et al. (2012) found that less than 6% of the exercises are about proof–related reasoning. Also, standards–based textbooks offer more reasoning and proving opportunities, which was also confirmed by Davis (2012). Fujita and Jones (2013) examined Japanese geometry textbooks on reasoning and proving opportunities and showed that a larger portion of exercises, compared to American textbooks, are devoted to reasoning and proving. However, this finding may be skewed because their study analyzed chapters that explicitly address proofs, triangle congruence, and parallelogram as argued by Cai

* Korean high school geometry topics are three dimensional coordinates and figures and vectors. Some of these topics are included in Core Plus Mathematics Course 4. These topics were not considered in previous studies (Davis, 2010; Otten et al., 2014) so we did not consider them in this study either.

& Cirillo (2013). Overall, researchers agree that students should be exposed to various types of reasoning and proving tasks and activities (Davis, 2010; Stylianides, 2009). These results are the foundation to the current study to determine the different reasoning and proving opportunities CPMP and Korean textbooks offer to students.

Textbook Comparison Studies

Textbook comparison studies have revealed contrasting results. Some demonstrate that in both elementary and secondary mathematics textbooks, textbooks from Asian countries contain more challenging problems, while American standards–based (e.g., *Everyday Mathematics*) and traditional textbooks contain more single–step problems (Fan & Zhu, 2007; Li, 2000; Son & Senk 2010). However, other studies detected that standards–based textbooks include more problems with higher level cognitive demands than traditional American textbooks and Korean secondary textbooks (Cai et al., 2010; Hong & Choi, 2014). Results also differ by response types of problems. Son and Senk (2010) found that Korean textbooks contain more problems requiring explanation while Li (2000) and Hong and Choi (2014) revealed that American textbooks contain more problems requiring explanation. Charalambous and his colleagues (2010) found that Taiwanese textbooks include more challenging problems and problems requiring explanations compared to textbooks from Cyprus and Ireland. Thompson et al. (2012) discovered that most problems in CPMP are related to proofs. Hong and Choi (2014) also revealed that CPMP contains more problems that require explanation and reasoning. One common finding in these studies is that CPMP contains more reasoning and proving problems than other American textbooks and Korean textbooks (Davis, 2012; Hong & Choi, 2014; Thompson et al., 2012). In all, textbook studies present different results.

Methodology

Data

Geometry lessons in five textbooks, two standards–based American textbooks and three Korean secondary textbooks, were analyzed for reasoning and proving opportunities. Table 1 displays the total number of lessons and number of exercise problems that were analyzed. CPMP is a popular standards–based textbook in America. It was named an exemplary program by the U.S. Department of Education and is currently used by more than 500 schools. Geometry lessons were found in CPMP Course 1 and 3. The chapter titles are Pattern in Shapes (Course 1), Reasoning and Proof, and Similarity and Congruence (Course 3). Korea has a centralized educational system. The Korean government oversees the textbook publishing process so the content of textbooks are almost identical. In Korean textbooks, compatible geometry lessons to the selected CPMP topics were found in Middle School Mathematics I, II, and III from Dusan Publishing. The chapter titles are Basics in Polygon and Two and Three Dimensional Figures (Mathematics I), Properties of Polygon and Similarity of Polygon (Mathematics II) and Pythagorean Theorem (Mathematics III).

Table 1. Textbooks and number of selected lessons and exercises

Course and Textbooks	**Number of Lessons**	**Number of Exercises**
Core Plus 1	7	356
Core Plus 3	10	584
Middle School Mathematics I	8	283
Middle School Mathematics II	10	311
Middle School Mathematics III	4	164
Total	**39**	**1698**

Analytical Framework

When textbooks are analyzed both exposition (the paragraphs, text boxes that contain definition formulas and theorems and worked examples in each lesson) and exercise problems (mathematical items that students are expected to solve) should be examined because teachers can use exposition to introduce mathematics content and students can have opportunities to engage in mathematical tasks (Li, 2000; Otten et al., 2014). Few studies have shown the framework to analyze reasoning and proving opportunities in textbooks (Davis, 2012; Otten et al., 2014; Stylianides, 2009; Thompson et al., 2012). Otten and his colleagues adopted Thompson et al.'s framework but added a few more codes for geometry content. Since our study attempts to analyze reasoning and proving opportunities of geometry content, we adopted the framework used by Otten et al. (2014). Table 2 features the analytic framework used in this study.

Table 2. Analytic framework

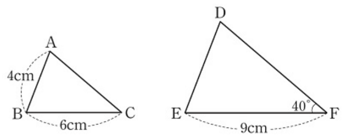
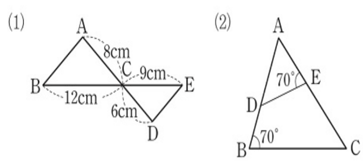
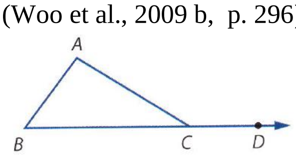
	Exposition	Exercise
Statement Type	Particular General General with particular instantiation provided	Particular General General with particular instantiation provided
Expected Activity		Make a conjecture Investigate Conjecture Evaluate Argument Construct a proof Fill in the blanks Find a counterexample
Justification Types	Deductive Empirical No Justification	

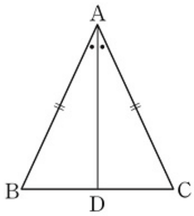
Analysis of Mathematical Statements

Mathematical Statements by their Characteristics

Based on findings on students' reasoning and proving ability, it is important to know what opportunities are provided for students to reason and prove both inductive and deductive arguments. Both Thompson et al. (2012) and Otten et al. (2014) categorized mathematical statements to general and specific or particular cases. Otten et al. added one more category, *particular instantiation provided*. Since in geometry, a particular diagram is often used to satisfy the hypothesis stated in the statement (Otten et al., 2014), we felt that this code was appropriate in our analysis. Two examples in Table 2 prove two general statements about isosceles triangle and exterior angle of a triangle with particular cases. We also believe that providing these different statements are important for students, so these three categories were used to code mathematical statements in the textbooks analyzed. Other samples for each code are provided in Table 3.

Table 3. Mathematical characteristics codes with sample statements

Code	Description	Exposition Examples	Exercise Examples
General	A statement concerns entire class of mathematical situations or objects	Recall that by definition a parallelogram, opposite sides are congruent (Hirsch, Fey, Hart, Schoen & Watkins, 2007, p. 375)	Prove an argument to justify the statement. A diagonal of a parallelogram divides the parallelogram into two congruent triangles. (Hirsch et al., 2007, p. 375)
Particular	A statement that concerns a specific mathematical object or situation	When triangles ABC and DEF are similar, $m\angle C = 40^o$.  (Woo et al., 2009 b, p. 291)	Find similar triangles and state the reason why.  (Woo et al., 2009 b, p. 296)
General with particular instantiation	A statement that concerns entire class of mathematical objects but a specific case of the objects has been indicated.	Properties of an isosceles triangle: Base angles of an isosceles triangles are congruent.	 Recall that an exterior angle of a triangle is formed when one side of the triangle is extended as shown above. ∠A and ∠B are called remote interior

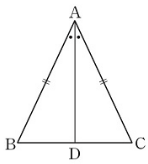
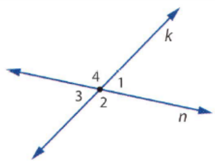
	 In the picture above, explain why $\overline{AB} \cong \overline{AC}$ implies $\angle B \cong \angle C$. Draw $\overline{AD}$, the angle bisector of $\angle A$. In ΔABD and ΔADC, $\overline{AB} \cong \overline{AC}$ (Given), $\overline{AD} \cong \overline{AD}$ by reflective property. . $\angle BAD \cong \angle CDA$ because $\overline{AD}$ is the angle bisector. $\Delta ABD \cong \Delta ADC$ by SAS $\cong$ SAS. Therefore, $\angle B \cong \angle C$ because they are corresponding angles of congruent triangles (Woo et al., 2009b, p. 237)	angles with respect to the exterior angle $\angle ACD$. a. How is $m\angle ACD$ related to $m\angle C$ and $m\angle B$. b. Write the argument to support your claim. c. Using the term "remote interior angles," write a statement of the theorem you have proved. This theorem is often called Exterior Angle Theorem for a Triangle (Fey et al., 2007, p. 46)

Mathematical Statements by Justifications

Mathematical statements can be justified inductively and deductively (Otten et al., 2014). Thus, two codes that Otten et al. (2014) used in their study were also employed in this study. These codes are defined as follows:
• *Deductive justification*—the textbook provides a logical argument building on definitions, postulates, or previously established results to support or prove a mathematical claim.
• *Empirical justification*—the textbook provides a confirming example to a mathematical claim or infers the truth of the claim from a subset of the relevant cases (Otten et al., 2014, p. 62).

Samples for each category are seen in Table 4. Other statements are *Justification is left for students* or *No justification*. Among these two, we did not find *Justification is left for students* after the initial pilot analysis, so only *No Justification* was included in our analysis.

Table 4. Mathematical justification codes with sample statements

	Deductive Justifications	Empirical Justifications
General Statement In an isosceles triangle, an angle bisector drawn to the base is the perpendicular bisector of the base (Woo et al., 2009 b, p. 239)	 ΔABC is an isosceles triangle with $\overline{AB} \cong \overline{AC}$. $\overline{AD}$ is the angle bisector of $\angle A$. $\overline{AD}$ is the perpendicular bisector of $\overline{BC}$. Explain In triangles ΔABD and ΔADC, $\overline{AB} \cong \overline{AC}$ and $\overline{AD}$ is shared by both triangles. $\angle BAD \cong \angle CDA$ because $\overline{AD}$ is the angle bisector. $\Delta ABD \cong \Delta ACD$ by SAS $\cong$ SAS. Because of congruent triangles, $\overline{BD} \cong \overline{CD}$ and $\angle ADB \cong \angle ADC$. Since $\angle ADB + \angle ADC = 180^o$, $m\angle ADB = 90^o$. Thus, $\overline{AD}$ is perpendicular bisector (Woo et al., 2009 b, 239)	Draw isosceles triangles and their angle bisectors and measure base and angles to confirm.
Particular Statement *If lines n and k intersect at the point shown, then* $m\angle 1 = m\angle 3$.(Fey, Hirsch, Hart, Schoen & Watkins, 2009, p. 31).	 Since lines n and k intersect, $\angle 1$ and $\angle 2$ are a linear pair. So $m\angle 1 + m\angle 2 = 180^0$.	Draw intersecting lines and measure angles 1 and 3 with a protractor to see they have the same measure.

	Since lines n and k intersect, $\angle 2$ and $\angle 3$ are a linear pair. So $m\angle 2 + m\angle 3 = 180^0$ If $m\angle 1 + m\angle 2 = 180^0$ and $m\angle 2 + m\angle 3 = 180^0$, then $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$. If $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$, then $m\angle 1 = m\angle 3$ (Fey et al., 2007, p. 31).	

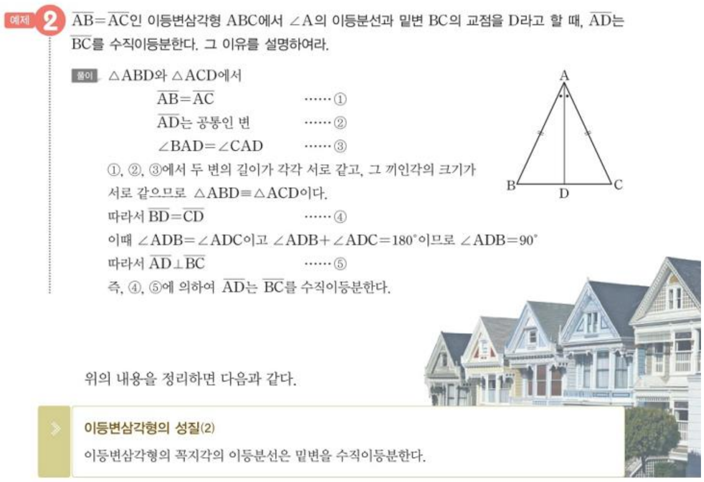
Analysis of Exercises

The following codes for exercise problems were adopted from Thompson et al. (2012).

- *Make a conjecture*—students are asked to formulate a mathematical claim or modify a false conjecture into one that the student believes is true. If students are also asked to support the resulting claim, an additional code or codes captures this supporting activity.
- *Investigate a conjecture*—students are asked to determine the truth-value of a given conjecture or to determine the truth-value of something they just conjectured themselves.
- *Evaluate an argument* —an argument or proof is presented and students are asked to determine whether it is valid or to find the error(s) and correct them.
- *Find a counterexample to a mathematical claim*—students are asked to supply a counterexample that disproves a given mathematical claim (Otten et al., 2014, p. 62)

These codes and examples are illustrated in Table 5. Otten et al. (2014) added three more codes, *Fill in the blanks of a conjecture, Outline,* and *Fill in the blanks of an argument or proof*. After our pilot analysis, we determined not to include *Outline* and *Past and Future* that were not found in the selected lessons. Instead, we found that problems related to conjectures and arguments can be more specified to the categories of *general* and *particular conjectures* and *arguments*. Since we are only able to examine potential opportunities for students when they solve these problems, "potential" justification types that each problem potentially provides were not considered in our analysis.

Construct a proof was also used by Otten et al. However, in our pilot analysis, we noticed that Korean textbooks do not contain any problems that specifically ask to "construct a proof." Instead, they use the word "explain" even if a deductive proof is provided (see Figure 1). Problems like these are also found in the exercise section. For these problems, although we can assume that students can provide deductive proofs, we decided not to code these items as *construct a proof* because we were not considering potential justification. Instead, we coded these items as *evaluate an argument* because we felt that the definition of the code (an argument or proof is presented and students are asked to determine whether it is valid) provided by Otten et al. was appropriate.


예제 2 $\overline{AB}=\overline{AC}$인 이등변삼각형 ABC에서 ∠A의 이등분선과 밑변 BC의 교점을 D라고 할 때, $\overline{AD}$는 $\overline{BC}$를 수직이등분한다. 그 이유를 설명하여라.

풀이 △ABD와 △ACD에서

$\overline{AB}=\overline{AC}$ ……①

$\overline{AD}$는 공통인 변 ……②

∠BAD=∠CAD ……③

①, ②, ③에서 두 변의 길이가 각각 서로 같고, 그 끼인각의 크기가 서로 같으므로 △ABD≡△ACD이다.

따라서 $\overline{BD}=\overline{CD}$ ……④

이때 ∠ADB=∠ADC이고 ∠ADB+∠ADC=180°이므로 ∠ADB=90°

따라서 $\overline{AD}\perp\overline{BC}$ ……⑤

즉, ④, ⑤에 의하여 $\overline{AD}$는 $\overline{BC}$를 수직이등분한다.

위의 내용을 정리하면 다음과 같다.

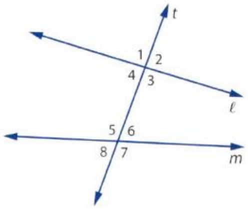
이등변삼각형의 성질(2)

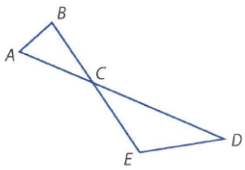
이등변삼각형의 꼭지각의 이등분선은 밑변을 수직이등분한다.

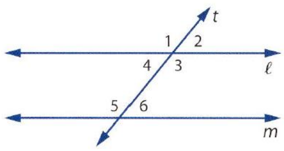
Translation: In an isosceles triangle with $\overline{AB} = \overline{AC}$, AD bisects $\angle A$. Explain why $\overline{AD}$ is perpendicular bisector of $\overline{BC}$.
A property of an isosceles triangle: The angle bisector from the vertex angle to the base of an isosceles triangle is perpendicular bisector of the base.
(Woo et al., 2009 b, p. 239)

Figure 1. Deductive proof of an isosceles triangle theorem

Table 5. Codes for expected activities and examples

<table>
<tr><th></th><th>Example</th></tr>
<tr><td>Make General Conjecture</td><td>Write an if –then statement about linear pairs of angles that you think always correct. You may want begin as follow. If two angles are a linear pair, then… . (Fey et al., 2009, p. 31)</td></tr>
<tr><td>Make Particular Conjecture</td><td>

What condition on a pair of alternate interior angles would guarantee that line *l* is parallel to line *m*? Write you conjecture in if - then form (Fey et al., 2007, p. 38)</td></tr>
<tr><td>Investigate General Conjecture</td><td>Claim: Two perpendicular lines form four right angles. Is this claim true or false? (Fey et al., 2009, p. 32)</td></tr>
<tr><td>Investigate Particular Conjecture</td><td>Find out whether right triangles with side lengths 3 and 4 are always congruent or not (Woo et al., 2009 b, p. 243)</td></tr>
<tr><td>Find a counterexample to a mathematical claim</td><td>

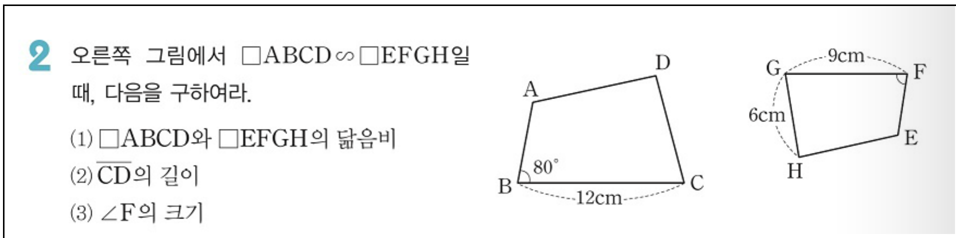
In the diagram at the above, $\overline{AD} and\ \overline{BE}$ intersect at point C and $m\angle ECD = m\angle D$
Is $m\angle A = m\angle D$? If so prove it. If not, give a counterexample. (Fey et al., 2007, p. 34)</td></tr>
<tr><td>Evaluate General Argument</td><td>Explain why not all rectangles are similar (Fey et al., 2007, p. 167)</td></tr>
<tr><td>Evaluate Particular Argument</td><td>In two triangles ABC and DEF, $\angle B = \angle E = 60^0,\ \angle C = \angle F = 70^0, \overline{BC} = 4cm, \overline{EF} = 8cm$. Explain why ΔABC is similar to ΔDEF (Woo et al., 2009 b, p. 295)</td></tr>
<tr><td>Fill in the blanks</td><td>Given: ℓ ∥ m; t is a transversal cutting ℓ and m
Prove: ∠4 and ∠5 are supplementary. ∠3 and ∠6 are supplementary.


<table>
<tr><th>Statements</th><th>Reasons</th></tr>
<tr><td>1. ℓ ∥ m; t is a transversal cutting ℓ and m</td><td>1.</td></tr>
<tr><td>2. m∠4 + m∠1 = 180°</td><td>2.</td></tr>
<tr><td>3. m∠1 = m∠5</td><td>3.</td></tr>
<tr><td>4. m∠4 + m∠5 = 180°</td><td>4.</td></tr>
<tr><td>5. ∠4 and ∠5 are supplementary.</td><td>5.</td></tr>
<tr><td>⋮</td><td>⋮</td></tr>
</table>
ii. Continue the two-column statement and reason proof to show that ∠3 and ∠6 are supplementary (Fey et al., 2007, p. 36)</td></tr>
</table>

Analytic Procedures and Reliability of Coding

In all, we analyzed and coded 161 pages, 74 statements, and 940 exercise problems from the CPMP textbooks and 211 pages, 180 statements, and 758 exercise problems from te Korean textbooks. For textbook exposition, we included theorems, postulates, properties, and other claims of geometric statements. Similar to Otten et al. (2014), worked examples are included as exposition. Each statement was coded for their statement type and justification type. If several sentences represent one geometric statement, we considered those as one statement.

Exercise problems in these textbooks were also coded similarly based on statement type and expected activities. Similar to Otten et al., we excluded items related to finding missing angles and lengths even if we thought that reasoning might be required to solve the problem correctly. In Figure 2, for example, students may need to rotate the second quadrilateral to find the scale factor. We thought this process could be "Evaluating Particular Argument;" however, it is also possible that some students may be able to solve this without rotating the second figure, so we only looked at what is required in the problem, finding missing length and angle.



Translation: When quadrilaterals ABCD and EFGH are similar, answer following questions.

1) Find the scale factor of ABCD and EFGH
2) The length of $\overline{CD}$
3) The measure of ∠F (Woo et al., 2009 b, p. 292).

Figure 2. An example of Item that we did not include

We first conducted a pilot coding to determine whether existing coding schemes are appropriate (e.g. Thompson et al., 2012; Otten et al., 2014). After pilot coding, we refined the codes for exercise problems as described earlier: adding *general* and *particular conjectures* and *arguments* and deleting *Justification is left for students, Outline,* and *Past and Future*. For coding reliability, the two authors, fluent in both English and Korean, independently coded each problem in the textbooks. Next, a third rater, a doctoral student in mathematics education, randomly chose one textbook from each country and independently coded each problem. When the two authors disagreed, items were coded based on majority rule using the third rater's codes. There were no items in which all three raters disagreed. The percent agreement of the two raters was between 86% and 92%.

Results

In this section, we present the results of our analysis and compare them to those of previous studies. The findings from textbook exposition and textbook exercise follow.

Reasoning and Proving Opportunities in the Exposition

Statements in textbook exposition provide students with opportunities to read and reflect on mathematical arguments rather than opportunities to prove or evaluate any statements directly (Thompson et al., 2012). These statements can be the basis that students use when they are working on proving and reasoning exercise problems. Table 6 shows the percent distribution of reasoning and proving statements from each textbook.

Table 6. Percent distribution of reasoning and proving statements in textbook exposition

	No. of Lessons	No. of Reasoning and Proving Statements and percent	Total No. of Statements
Core Plus 1	7	8 (24.2%)	33
Core Plus 3	10	20(47.6%)	42
Total	17	28(37.3%)	75
Middle School Math I	8	27 (34.1%)	79
Middle school Math II	10	47(61.3%)	77
Middle School Math III	4	12(50%)	24
Total	22	86 (47.8%)	180

We can note several things from this table. Korean textbooks include more reasoning and proving related statements per lesson in exposition: 3.59 in Korean textbooks versus 1.64 in CPMP. This is because the way CPMP is designed, by letting students work on and engage with various mathematics tasks rather than providing worked examples and explanations. In CPMP, instead of providing proofs for different statements and theorems, students conjecture and prove those statements and theorems, which was also found in previous studies that compared other mathematical topics in CPMP (Davis, 2012; Thompson et al., 2012). On the other hand, Korean textbooks prove statements and theorems when they are introduced in textbooks. The number of reasoning and proving statements increased in CPMP 3 and Middle School Mathematics II. These two textbooks include topics of congruent and similar triangles and properties of quadrilaterals. These topics often require proofs to verify theorems and statements, which is why there are more reasoning and proving statements in these textbooks.

Types of Statements in Exposition

Table 7 shows types of reasoning and proving statements in exposition. General statements are more prevalent in both CPMP and Korean textbooks. This finding coincides with previous studies (Otten et al., 2014; Thompson et al., 2012), where 75% and 71% of Reasoning and Proving statements in CPMP and Korean textbooks, respectively, are about general case, very similar to what Otten et al. found. A notable difference between CPMP and Korean textbooks is the number of general statements with particular instantiation in Korean textbooks compared to CPMP. Korean textbooks provide proofs in exposition sections to demonstrate how general statements can be proved (Figure 1) while CPMP devotes proof related problems in exercise sections. This finding indicates that Korean students have opportunities to read and become familiar with proofs provided by textbooks while CPMP students have opportunities to construct proofs themselves.

Table 7. Percent distribution of types of reasoning and proving statements in textbook exposition

	General Statements	Particular Statements	General Statements with Particular Instantiation
Core Plus 1	5 (62.5%)	3 (37.5%)	0 (0%)
Core Plus 3	16 (80%)	4 (20%)	0 (0%)
Total	21 (75%)	7 (25%)	0 (0%)
Middle School Math I	10 (37%)	8 (29.6%)	9 (33.3%)
Middle School Math II	10 (21.2%)	15 (31.9%)	22 (46.8%)
Middle School Math III	1 (8.3%)	2 (16.7%)	9 (75%)
Total	21 (24.4%)	25 (29.1%)	40 (46.5%)

Types of Justifications in Textbook Exposition

Table 8 shows types of justification used in textbook exposition. The major difference between CPMP and Korean textbooks is the number of statements proved deductively in Korean textbooks. About 62% of reasoning and proving exercise problems are justified deductively. This is higher than any other American geometry textbook (Otten et al., 2014). Korean textbooks usually give statements and theorems and prove them deductively. Figure 1 shows the deductive proof of “The vertex angle bisector to the base of an isosceles triangle bisects the base.” This gives students opportunities to become familiar with deductive proofs. We can see that more deductive proofs are included in Middle School Mathematics II because several topics (properties of triangles and quadrilaterals and similar triangles) are treated in Middle School Mathematics II and these topics often require proofs, which is why more deductive proofs are included in this textbook.

Table 8. Percent distribution of types of justifications in textbook exposition

	Deductive Justification	Empirical Justification	Not Justified
Core Plus 1	2 (25%)	1 (12.5%)	5 (62.5%)
Core Plus 3	2 (10%)	0 (0%)	18 (90%)
Total	4 (14.3%)	1 (3.6%)	23(82.1%)
Middle School Math I	10 (37%)	7 (25.9%)	10 (37%)
Middle School Math II	35 (74.5%)	2 (4.3%)	10 (21.2%)
Middle School Math III	9 (75%)	2 (16.7%)	1 (8.3%)
Total	54 (62.7%)	11 (12.7%)	22 (25.5%)

On the other hand, only a limited amount of reasoning and proving statements are justified in CPMP. Again, one of the foci of CPMP is letting students work on various problems so instead of providing proofs in exposition, CPMP lets students conjecture and prove those conjectures. CPMP introduces deductive proofs; however, to allow student become familiar with how deductive proofs are completed, more deductive proofs in exposition may need to be included. Although reasoning and proving statements in exposition sections of CPMP are minimal, deductive proofs are more prevalent in both countries’ textbooks, which does not coincide with Otten et al., who found that deductive and empirical justifications were equally distributed. Around 82% of CPMP statements were not justified while 74% of reasoning and proving statements in Korean textbooks were justified. This finding shows different reasoning and proving opportunities provided by these textbooks. Korean students have opportunities to become familiar with reasoning and proving by reading textbook exposition while CPMP offers these opportunities in exercise problems.

Reasoning and Proving Exercise in Textbooks

The exercise problems in textbooks offer different potential opportunities, practicing and testing their understating of a lesson, to students compared to textbook exposition. Thus, students have opportunities to engage in various reasoning and proving related problems. Table 9 shows the percent distribution of reasoning and proving problems in textbooks. As previously mentioned, CPMP devotes a substantial portion of its textbooks to exercise problems so that students can explore various mathematical topics themselves. In contrast to exposition in these textbooks, CPMP includes many more problems related to reasoning and proving compared to Korean textbooks. CPMP includes 22.3 reasoning and proving problems per lesson while Korean textbooks include 7.54 reasoning and proving problems per lesson. Approximately 40% of exercise problems are related to reasoning and proving. This is higher than any other geometry and algebra textbooks seen in previous studies (Otten et al., 2014; Thompson et al., 2012). Again, CPMP Course 3 and Middle School Mathematics II include more reasoning and proving problems because these two textbooks include properties of triangles and quadrilaterals and congruent and similar triangles. This finding confirms what Cai and Cirillo (2013) found in Japanese geometry textbooks.

Table 9. Numbers and percent of reasoning and proving problems

	No. of Lessons	No. of Reasoning and Proving Problems and percent	Total No. of problems
Core Plus 1	7	123 (34.6%)	356
Core Plus 3	10	256 (43.8%)	584
Total	17	379 (40.4%)	940
Middle School Math I	8	36 (12.7%)	283
Middle school Math II	10	108 (34.7%)	311
Middle School Math III	4	22 (13.4%)	164
Total	22	166 (21.9%)	758

Type of Reasoning and Proving Problems in Textbooks

Table 10 shows types of reasoning and proving exercise problems. Unlike exposition in textbooks, particular statements are more prevalent in textbook exercises, which was also found in previous studies (Otten et al., 2014; Otten, Males & Gilbertson, 2014). The percentages of particular statements are 71.8% for CPMP and 92.7% for Korean textbooks, which are higher than what was previously found by Otten et al. (2014). In Korean textbooks, the majority of general statements are already proved in exposition sections so in the exercise sections, only a limited number of problems about general statements are presented.

Table 10. Numbers and percent of each type of statement

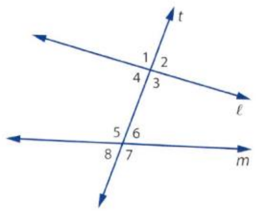
	No. of General Statements	No. of Particular Statements	No. of General Statements with Particular Instantiation
Core Plus 1	27 (21.9%)	96 (78%)	0 (0%)
Core Plus 3	45 (17.5%)	176 (69%)	35 (13.6%)
Total	72 (18.9%)	272 (71.8%)	35 (9.23%)
Middle School Math I	7 (19.4%)	29 (80.5%)	0 (0%)
Middle School Math 11	5 (4.6%)	103 (95.3%)	0 (0%)
Middle School Math III	0 (0%)	22 (100%)	0 (0%)
Total	12 (7.2%)	154 (92.7%)	0 (0%)

Types of Reasoning and Proving Activities in Exercises

Tables 11 and 12 show types of reasoning and proving activities in exercise problems. “Evaluate particular arguments” and “Construct a proof” are two of the most prevalent activities in CPMP while “Evaluate particular arguments" is the predominant activity in Korean textbooks. One apparent difference between CPMP and Korean textbooks is the number of different activities that CPMP presents. Compared to Korean textbooks, CPMP includes various reasoning and proving activities. In addition to different types of activities, CPMP lets students make and investigate conjectures while Korean textbooks rarely allow this. The exercise problems in the Korean textbooks provide opportunities to investigate conjectures but opportunities to make conjectures are very rare. Figures 3 and 4 show different learning opportunities that CPMP and Korean textbooks offer on the same topic. Figure 3 shows one example from CPMP Course 3, showing that students are asked to make a conjecture about parallel lines and related angles and to prove their conjecture. In contrast to CPMP's approach, instead of giving students opportunities to conjecture and prove statements, the Korean textbook provides an empirical justification, measuring angles with protractor, in textbook exposition (Figure 4). In another textbook comparison study, it was also found that CPMP provides students with more reasoning and explanation problems than Korean textbooks (Hong & Choi, 2014) as well as other American textbooks (Thompson et al., 2012). It appears that this finding is one of the characteristics of CPMP textbooks.

Table 11. Numbers and percent of each type of justification in CPMP series

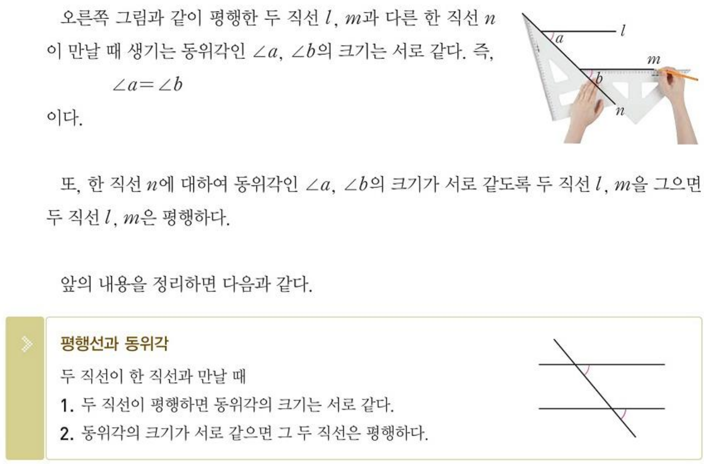
	Core Plus 1	Core Plus 3	Total
Evaluate particular Argument	81	61	142 (37.4%)
Evaluate General Argument	16	14	30 (7.9%)
Construct a Proof	1	110	111 (29.2%)
Make Particular Conjecture	3	20	23 (6%)
Make General Conjecture	2	4	6 (1.6%)
Find a Counterexample	1	7	8 (2.1%)
Investigate Particular Conjecture	10	20	30 (7.9%)
Investigate General Conjecture	9	17	26 (6.7%)
Fill in the blanks	0	3	3 (0.7%)



a. What condition on a pair of alternate interior angles would guarantee that line *l* is parallel to line *m*? Write you conjecture in if - then form.

d. Working with a classmate, write a proof for one of the statements in parts a – c (Fey et al., 2007, p. 38)

Figure 3. An example of CPMP that lets students make conjectures and prove statements


오른쪽 그림과 같이 평행한 두 직선 l, m과 다른 한 직선 n이 만날 때 생기는 동위각인 $\angle a$, $\angle b$의 크기는 서로 같다. 즉,

$$\angle a = \angle b$$

이다.

또, 한 직선 n에 대하여 동위각인 $\angle a$, $\angle b$의 크기가 서로 같도록 두 직선 l, m을 그으면 두 직선 l, m은 평행하다.

앞의 내용을 정리하면 다음과 같다.

평행선과 동위각

두 직선이 한 직선과 만날 때

1. 두 직선이 평행하면 동위각의 크기는 서로 같다.
2. 동위각의 크기가 서로 같으면 그 두 직선은 평행하다.

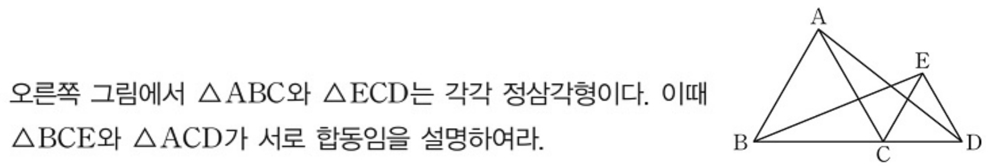
Translation: In the diagram on the right, when line *l* and *m* are parallel, corresponding angles, $\angle a$ and $\angle b$ are congruent. Also, if lines are drawn so that corresponding angles $\angle a$ and $\angle b$ are congruent, then two lines l and m are parallel (Woo et al., 2009, p. 212).

Figure 4. An example of empirical justification in Korean textbook

Another finding that we notice from Table 11 is the number of "Constructing Proof" problems in Korean textbooks. It is very odd to find no exercise problems are related to proofs, especially in geometry. This is not because Korean textbooks do not have any proof problems but because Korean textbooks do not use word "prove." In Figure 1, although a property of an isosceles triangle is proved deductively, the wording in the textbook is "Explain the reason." In Figure 5, the problem asks to explain why two triangles, ΔBCE and ΔACD, are congruent rather than "Prove that two triangles, ΔBCE and ΔACD, are congruent." Problems like these are coded either as "Evaluate Particular Argument" or "Evaluate General Argument." Although it is plausible to assume that students are asked to present a "Deductive Proof," we can only code these as "Evaluate Argument" rather than "Construct a Proof."

Table 12. Numbers and percent of each type of justification in MSM series

	Middle School Math I	Middle School Math II	Middle School Math III	Total
Evaluate particular Argument	13	62	13	88 (53%)
Evaluate General Argument	7	5	0	12 (7.2%)
Construct a Proof	0	0	0	0 (0%)
Make Particular Conjecture	0	0	0	0 (0%)
Make General Conjecture	2	0	0	2 (1.2%)
Find a Counterexample	2	0	0	2 (1.2%)
Investigate Particular Conjecture	12	25	6	37 (25.9%)
Investigate General Conjecture	0	0	0	0 (0%)
Fill in the Blanks	0	16	3	19 (11.4%)



Translation: ΔABC and ΔECD are equilateral triangles. Explain why ΔBCE and ΔACD are congruent (Woo et al., 2009 b, p. 243).

Figure 5. An example of item that uses the word "explain" in Korean textbook

What Exposition and Exercise Problems Offer in These Textbooks

Analysis of textbook exposition and exercise problems shows that textbooks from two countries provide quite different reasoning and proving opportunities for students. CPMP lets students engage with various reasoning and proving activities by providing different exercise problems. On the other hand, Korean textbooks provide opportunities to read and become familiar with deductive proofs in their exposition. However, Korean textbooks provide only a few opportunities for students to make conjectures and prove statements themselves. In CPMP textbook exposition, general statements are more prevalent while particular statements are prevalent in both countries' textbook exercise problems. Senk (1985) found that students were more successful in proving particular statements. Although it has been almost 30 years since Senk published her work, particular statements are more prevalent in these textbooks, which may explain students' difficulty in proving general statements. Deductive proofs are prevalent in textbook exposition while possible justifications for exercise problems were not examined because with words such as "prove" or "explain" without referring to specific justification, it would be difficult to see what is expected from students.

Discussion and Conclusion

This study analyzes and compares reasoning and proving opportunities in geometry lessons from American standard–based textbooks and Korean textbooks to understand reasoning and proving opportunities. Overall, around 40% of exercise problems in CPMP ask for reasoning and proving activities while 20% of exercise problems in Korean textbooks are about reasoning and proving activities. For CPMP, it is higher than what was previously found in geometry textbooks (Otten et al., 2014) as well as in other mathematics textbooks (Thompson et al., 2012). It is also higher than other lessons in CPMP (Thompson et al., 2012). Around 5% of problems in other geometry textbooks were about constructing a proof (Otten et al., 2014). CPMP also has higher a percentage of constructing proof problems. This reveals a well–known fact that geometry lessons typically focus on reasoning and proving activities than other mathematical topics (Hanna & Bruyun, 1999; Herbst, 2002). However, only a limited number of geometric statements in CPMP were justified deductively. Reasoning and proving opportunities in Korean textbooks are quite different because they provide fewer reasoning proving problems but more statements are proved deductively in exposition sections. Compared to CPMP, many general statements are proved deductively to give students opportunities to read and become familiar with deductive proofs. On the other hand, CPMP provides opportunities for students to make, investigate, and prove conjectures by solving exercise problems. Reasoning and proving opportunities provided by these textbooks are clearly different. However, as Otten and his colleagues (2014) stated, it is beyond the scope of this study to discuss the ideal proportion of reasoning and proving statements and exercises in textbooks or to think about which textbooks would give more opportunities to students. Nonetheless, it is clear that Korean and CPMP students have different opportunities in learning geometry. It may be interesting to test Korean and CPMP students with different types of geometry problems to see how textbooks influence their learning.

Another interesting finding is the lack of “prove” problems in Korean textbooks. Rather than using the word “prove,” Korean textbooks used “explain” in both exposition and exercise sections. We also examined the curriculum guidelines from the Korean Ministry of Education and found that the word “prove” is not used in geometry lessons. It is plausible to think that deductive proofs are expected for those problems because numerous general statements are proved deductively in exposition sections of Korean textbooks. However, predicting what students will do to solve each problem is beyond the scope of this study. Based on this result, it may be interesting to see how Korean students perform on “prove” problems and “explain” problems. While “prove” problems are lacking in Korean textbooks, deductive justification is lacking in CPMP.

Taking into consideration all of these findings, we can see how textbooks from these two countries approach geometry teaching differently. Whether students have opportunities to read or construct deductive proofs and make and investigate conjectures and arguments, textbooks need to provide equal opportunities for various reasoning and proving activities so that students can be exposed to various types of reasoning and proving problems (Davis, 2010; Stylianides, 2009). Stylianides (2009) developed a model that places reasoning and proving opportunities, from empirical arguments to proofs, hierarchically so that students can experience them at different grade levels. Such a model may be needed for students to experience various reasoning and proving opportunities. Since what textbooks offer is an influencing factor, opportunities might not be presented to students if they are not included in textbooks (Thompson et al., 2012). Textbook authors and publishers need to consider findings from the current study as well as previous studies on reasoning and proving opportunities in textbooks.

As previously mentioned, one well-known fact about teaching and learning of proofs is that students are not comfortable with providing proofs; rather, they use specific cases (Knuth, Slaughter, Choppin & Sutherland, 2002; Thompson, 1991). Our results support this fact because the most prevalent problems in both CPMP and Korean textbooks are evaluating particular statements. This result partially explains why students have difficulties in constructing proofs because of the opportunities provided by these textbooks. Again, equally providing general and particular cases to students may need to be considered when textbooks are developed and written.

The link between what textbooks potentially bring and what students actually learn is what teachers do in their classes (Bieda, 2010; Hong & Choi, 2014; Johnson, Thompson & Senk, 2010; Son & Senk, 2010). Since these textbooks provide different learning opportunities, the teachers’ role in teaching geometry with these textbooks also needs to be considered because teachers need to be sensitive about these different possibilities and ensure students have opportunities to experience them (Johnson et al., 2010). However, Bieda (2010) discovered that teachers were not able to implement proof related opportunities even if those opportunities were provided by textbooks. Thus, whether students are using CPMP or Korean textbooks, what teachers need to do in their classes is critical in teaching geometry. Since there are only a limited number of deductive proofs in exposition sections of CPMP, teachers should guide students through conjectures and arguments until students are able to construct deductive proofs when using CPMP. For Korean students, teachers need to give them opportunities to make and investigate conjectures as well as opportunities to read and become familiar with deductive proofs in exposition. It would be interesting to examine how teachers use these textbooks in their classes in terms of teacher preparation programs.

This study also gives some insights into “textbook signature.” Charalambous, Delaney, Hui-Yu and Mesa (2010) defined “textbook signature” as “the uniform distinctive features in the textbooks within a particular country." They proposed analyzing other mathematics topics from different textbooks for further understanding about characteristics of textbooks from different countries. One of the findings from this study that coincides well with previous studies is about CPMP. This study found that CPMP includes more reasoning and proving problems than Korean textbooks, which was also found when CPMP was compared to other algebra, geometry precalculus textbooks (Davis, 2012; Thompson et al., 2012). Hong and Choi (2014) also discovered that CPMP provides more reasoning and explaining problems when quadratic equations sections were compared. These findings in total can be interpreted as the "textbook signature" of CPMP, where students have numerous opportunities to reason, prove, and explain mathematical concepts.

Finally, we can think of textbooks' potential influence on American students' mediocre performances on TIMSS and PISA. Hong and Choi (2014) stated in their textbook comparison study that textbooks may not be the reason for American students' performances on TIMSS and PISA. We cannot confirm whether American students who participated in TIMSS and PISA used CPMP or any other standards–based textbooks; however, previous findings and this current study show that, compared to other textbooks, CPMP offers ample reasoning and proving opportunities to students. At the same time, our findings reveal that Korean textbooks also provide different reasoning and proving opportunities to students in reading various deductive proofs. Making conjectures and proving particular statements in CPMP and deductive proofs in Korean textbooks are different but they can both provide good learning opportunities to students. A TIMSS video study illustrated that in American mathematics classes, proof opportunities are very rare (Hiebert et al., 2003) and another showed that teachers are not able to provide proof related opportunities to students (Bieda, 2010). While we are not able to say which textbooks provide more meaningful learning opportunities to students, CPMP may provide comparable amount of reasoning and proving opportunities to students, and it is possible that enacted curriculum of class practices rather than potentially implemented curriculum of textbooks may play a role in American students' performances on TIMSS and PISA. For further study, it would be interesting to compare CPMP and other top performing East Asian countries' textbooks to determine how comparable those learning opportunities are. Such studies will provide a better picture about what learning opportunities CPMP and other textbooks bring to students.

References

Bieda, K. (2010). Enacting proofs in middle school mathematics: Challenges and opportunities. *Journal for Research in Mathematics Education*, *41*, 351–382.

Cai, J. & Cirillo, M. (2013). What do we know about reasoning and proving? Opportunities and missing opportunities from curriculum analyses (in press). *International Journal of Educational Research.*

Cai, J., Nie, B., & Moyer, J. C. (2010). The teaching of equation solving: Approaches in Standards-based and traditional curricula in the United States. *Pedagogies: An International Journal, 5*, 170-186.

Charalambous, C., Delaney, S., Hui-Yu Hsu, H., & Mesa, V. (2010). A comparative analysis of the addition and subtraction of fractions in textbooks from three countries. *Mathematical Thinking and Learning, 12*, 117-151.

Chazan, D. (1993). High school geometry students' justification for their views of empirical evidence and mathematical proof. *Educational Studies in Mathematics*, *24*, 359–387.

Davis, J. D. (2010). A textual analysis of reasoning and proof in one reform-oriented high school mathematics textbook. In P. Brosnan, D. B. Erchick, & L. Flevares (Eds.), *Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 844–851). Columbus, OH: Ohio State University.

Davis, J. D. (2012). An examination of reasoning and proof opportunities in three differently organized secondary mathematics textbook units. *Mathematics Education Research Journal, 24,* 467–491.

Fan, L., & Zhu, Y. (2007). Representation of problem-solving procedures: A comparative look at China, Singapore, and US mathematics textbooks. *Educational Studies in Mathematics*, *66*, 61-75.

Fey, J. T., Hirsch, C. R., Hart, E. W., Schoen, H. L., & Watkins, A. E. (2007). *Core-Plus Mathematics: Contemporary mathematics in context, Course 3*. Glencoe/McGraw-Hill: Columbus, OH.

Fischbein, E. (1982). Intuition and proof. *For the Learning of Mathematics*, *3*(2), 9–24.

Freeman, D., & Porter, A. (1989). Do textbooks dictate the content of mathematics instruction in elementary schools? *American Educational Research Journal, 26,* 403–421.

Fujita, T. and Jones, K. (2003). *Interpretations of national curricula: The case of geometry in Japan and the UK.* Paper presented at the *British Educational Research Association Annual Conference*, Heriot-Watt University, 10-13.

Fujita, T. & Jones, K. (2013). Reasoning-and-proving in geometry in school mathematics textbooks in Japan (in press). *International Journal of Educational Research*.

Fuson, K. C., Stigler, J. W., & Bartsch, K. B. (1988). Grade placement of addition and subtraction topics in Japan, Mainland China, the Soviet Union, Taiwan, and the United States. *Journal for Research in Mathematics Education, 19*, 449-456.

Galbraith, P. (1981). Aspects of proving: A clinical investigation of process. *Educational Studies in Mathematics, 12*, 1–28.

Garner, R. (1992). Learning from school texts. *Educational Psychologist, 27*, 53-43.

Hanna, G. & de Bruyn, Y. (1999). Opportunity to learn proof in Ontario grade twelve mathematics texts. *Ontario Mathematics Gazette, 27*(4), 23–29.

Harel, G., & Sowder, L. (2007). Toward comprehensive perspectives on the learning and teaching of proof. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 805–842). Charlotte, NC: Information Age Publishing.

Healy, L., & Hoyles, C. (2000). A study of proof conceptions in algebra. *Journal for Research in Mathematics Education, 31*, 396–428.

Herbst, P. (2002). Establishing a custom of proving in American school geometry textbooks: evolution of the two - column proof in the early twentieth century. *Educational Studies in Mathematics, 49*, 283–312.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J. K., et al. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 videostudy, (NCES 2003-013 Revised)*. Washington, DC: U.S. Department of Education, National Center for Education Statistics.

Hirsch, C. R., Fey, J. T., Hart, E. W., Schoen, H. L., & Watkins, A. E. (2007 a). *Core-Plus Mathematics: Contemporary mathematics in context, Course 1*. Glencoe/McGraw-Hill: Columbus, OH.

Hong, D. & Choi, K. (2014). A comparison of Korean and American secondary school textbooks: The case of quadratic equations. *Educational Studies in Mathematics, 85*(2), 241–263.

Johnson, G., Thompson, D., & Senk, S. (2010). Proof–related reasoning in high school mathematics textbooks. *Mathematics Teacher, 103*(6), 410–418.

Knuth, E., Choppin, J., Slaughter, M., & Sutherland, J. (October, 2002). Mapping the conceptual terrain of middle school students' competencies in justifying and proving. *Proceedings of the Twenty-fourth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* 1693-1700.

Li, Y. (2000). A comparison of problems that follow selected content presentations in American and Chinese mathematics textbooks. *Journal for Research in Mathematics Education, 31*, 234–241.

Martin, W. G., & Harel, G. (1989). Proof frames of preservice elementary teachers. *Journal for Research in Mathematics Education, 20*, 41-51.

Morris, A. (2002). Mathematical reasoning: Adults' ability to make the inductive-deductive distinction. *Cognition and Instruction, 20,* 79-118.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Author.

Otten, S., Gilbertson, N., Males, L., & Clark, D. (2014). The mathematical nature of reasoning – and – proving opportunities in geometry textbooks. *Mathematical Thinking and Learning, 16*(1), 51–79.

Otten, S., Males, L., & Gilbertson, N. (2014). The introduction of proof in secondary geometry textbooks. *International Journal of Educational Research, 64*, 107–118.

Reys, B. J., Reys, R. E., & Chavez, O. (2004). Why mathematics textbooks matter. *Educational Leadership, 61*(5), 61–66.

Senk, S. L. (1985). How well do students write geometry proofs? *Mathematics Teacher, 78,* 448–456.

Son, J., & Senk. S. (2010). How standards curricula in the USA and Korea present multiplication and division of fractions. *Educational Studies in Mathematics, 74,* 117-142.

Stylianides, G. J. (2009). Reasoning-and-proving in school mathematics textbooks. *Mathematical Thinking and Learning, 11*, 258–288.

Thompson, D. R., Senk, S. L., & Johnson, G. J. (2012). Opportunities to learn reasoning and proof in high school mathematics textbooks. *Journal for Research in Mathematics Education, 43*, 253–295.

Vinner, S. (1983). The notion of proof – some aspects of students' views at the senior high level. In R. Hershkowitz (ed.), *Proceedings of the Seventh International Conference on the Psychology of Mathematics Education* (pp. 289-294). Rehovot, Israel: The Weizmann Institute of Science.

Weber, K. (2001). Student difficulty in constructing proofs: The need for strategic knowledge. *Educational Studies in Mathematics, 48*, 101–119.

Woo, J. et al. (2009 a). *Joonghakyo Soohak 1*[Middle School Mathematics 1]. Seoul, Korea: Dusan Publishing Co.

Woo, J. et al. (2009 b). *Joonghakyo Soohak 2*[Middle School Mathematics 2]. Seoul, Korea: Dusan Publishing Co.

Woo, J. et al. (2009 c). *Joonghakyo Soohak 3*[Middle School Mathematics 3]. Seoul, Korea: Dusan Publishing Co.

Author Information

Dae S. Hong
University of Iowa
Teaching and Learning, College of Education
Contact e-mail: *dae-hong@uiowa.edu*

Kyong Mi Choi
University of Iowa
Teaching and Learning, College of Education