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Self-Organizing Neural Network Map for the Purpose of Visualizing the Concept Images of Students on Angles

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Abstract

The purpose of the study is to perform a less-dimensional thorough visualization process for the purpose of determining the images of the students on the concept of angle. The Ward clustering analysis combined with Self-Organizing Neural Network Map (SOM) has been used for the dimension process. The Conceptual Understanding Tool, which consisted of the open-ended question "write the first ten things you remember when the term 'angle' is mentioned" to the study group, which consisted of 250 seventh grade students. The analysis results showed that students mostly explained the concept of angle by associating it with mathematics and other sciences with the terms like "reflection and incidence angle, numbers, light, graphics, fraction, area, speed, algebraic expressions, energy, sound, four operations, viewpoint, electricity and natural numbers". The students mostly established relations with the static side of the angle. At the end of the study, the dataset obtained from the Conceptual Understanding Tool was used for training the SOM, and a structure that reveals the images of the students on angle has been recommended.

Introduction

According to the National Mathematics Teachers Council, geometry, which is an important field of mathematics, provides students with a mathematical viewpoint that is different from the world of numbers but associated with it (NCTM, 2000). In this context, geometric models or examples have an important place in teaching mathematics (Sherard, 1981). Geometry has important roles especially in reaching the general purposes of mathematics as an efficient tool as well as understanding/making sense of and explaining the environment around us in which we live. When geometry is considered as a learning environment consisting of two and three-dimensional shapes and figures, the "concept of angle" constitutes the center of the geometrical knowledge development (Clements & Burns, 2000).

Angle is the basic element of geometry that appears before us in many instances in our daily lives and has to be dealt with multiple variations. Because of this property, we can see it sometimes in scissors and sometimes in the inclination of a slope (Mitchelmore & White, 2000). With the broadest meaning, an angle is described as the gap or the space that never stretch through a straight line and is formed by two lines on a plain; and in addition, it is also known as the gap that meet each other when its two ends revolve (Euclid, 1956). Aside from this, during the past decades, many researchers investigated the concept of angle, which is the basic study area of geometry, with different viewpoints, and made some definitions. Henderson and Taimina (2005) defined the angle concept in three different viewpoints as *(i) a geometric shape, (ii) a changing and dynamic structure, (iii) a measurable attribute*. The modern definition of the angle concept on the other hand, is *(1) the measurement of the revolving of a beam from one point to another in a position change (dynamic dimension) (2) a geometrical shape formed by two beams whose starting points are common (static dimension)* and *(3) the gap between two beams (static dimension)* (Keiser, 2004). In addition to the above mentioned viewpoints, it has been claimed that the angle was a compound structure; and mathematically, an angle was formed by joining of two beams and was first of all, a geometrical shape (Freudenthal, 1973). In the light of these definitions, an angle has been defined as a concept in various forms throughout years, and depending on its mathematical position, it is still being defined in various dimensions with different meanings.

The concept of angle, as well as many other concepts used in teaching mathematics, includes mathematical definitions, and therefore, these definitions have important characteristics for the mathematical development of students. On the one hand, mathematical concepts help the grouping of knowledge in a systematic manner, and on the other hand, they ensure that an efficient mathematical language is formed. By so-doing, an individual who learns a concept may know the examples related with this concept in a better manner, and expand his/her knowledge system (Erden & Akman, 2003). Otherwise, it a common understanding for concepts is not ensured in learners, this may cause contradiction in terms. In order to analyze the understanding of the learners on a concept and their thoughts on learning Tall and Vinner (1981) suggested concept image and concept definition models. The concept image is defined as the total cognitive structure of a certain concept (Tall & Vinner, 1981). The concept image consists of a series of processes, properties and as well as some structures of the concept. However, the definition concept and the concept image must not be confused. While the concept definition is the whole of words used to discriminate a concept from another one; the concept image contains the stimuli in the mind about that concept (Erşen & Karakuş, 2013). According to Vinner (1983), there are two different cells, which are the concept definition and concept image of a certain concept. These cells are in the form of mental structures that are activated in tasks related with that concept. As a matter of fact, when an individual hears the name of a concept or when a concept is seen, the image of that concept is always associated, not the definition of it (Herskowitz, 1989; Shwarz & Herskowitz, 1999). In this context, when individuals are learning various subjects in mathematics, they develop an image about the concepts in their minds. When this image is defined as the mental codes encoded in the memory of an individual about the mathematical thought, these may be pictures, figures, graphics, shapes, mathematical shapes, operations, verbal depictions, and even some examples from daily life. For example while the definition of a fraction is considered in a manner that is different from other concepts, when fraction is mentioned, some structures such as dividend, denominator, fraction line, percentages, decimal fractions, integral, simple and mixed fraction and some structures from daily life [one and a half liter of milk, one third of a cake, half pizza]. Tall and Vinner (1981) interpreted this situation as the information on a concept stimulating our brain when the name of the concept is heard or seen. The basic justification of this understanding is the holistic images formed by the concept in the brain, i.e. the properties and structures of the concept rather than the definition of the concept. The image of the concept is mostly formed of the cognitive structures of that concept, which may include visual presentations, experiences, and mental pictures about the name of the concept (Tall & Vinner, 1981). This cognitive frame matures with the environmental factors (i.e. school, family, peers, etc.) or gains a new dimension. By doing so, the concept image develops as a result of students' experiences and acquisitions. The concept image is an informal definition and may include misconceptions as well. For this reason, the concept image does not necessarily have to be in the same structure with the concept or does not be suitable to it; it may also include different and conflicting thoughts about the concept, which may not be deliberate on the side of students (Rösken & Rolka, 2007). In this context, the formal definition of the relevant concept, the definition that is structured by students, the mental images of the concept and all introvert processes make the quality of the concept image important. After the study conducted by Tall and Vinner (1981), in which the definition of the concept and the concept image model were used, many studies were conducted in the international field on similar models. When these studies are examined it is observed that the majority of them were on quadrangles and misconceptions of students (Cansız-Aktaş & Aktaş, 2012; De Villiers, 1994; Erez & Yerushalmy, 2006; Erşen & Karakuş, 2013; Fujita & Jones, 2007; Fujita, 2012; Heinze & Ossietzky, 2002; Monaghan, 2000; Nordlander & Nordlander, 2012; Okazaki & Fujita, 2007; Özsoy & Kemankaşlı, 2004; Türnüklü, Alaylı & Akkaş, 2013; Usiskin, Griffin, Witonsky & Willmore, 2008; Vinner, 1991); however, there are limited studies conducted on how students perceive angles (Casas-García & Luengo-González, 2013; Clements & Burns, 2000; Davey & Pegg, 1991; Keiser, 1997; Mitchelmore & White, 2000; Schwarz & Hershkowitz, 1999; Vinner, 1991).

When the literature is examined it is observed that the results of many studies conducted on angle concept showed that students had some difficulties in understanding the angle concept, and the results also indicate that the angle concept is extremely difficult concept (Battista, 2007; Clements & Battista, 1992; Keiser, 2004; Mitchelmore & White, 2000). For example, many students believe that the size of an angle depends on the length of it (Clements & Burns, 2000; Keiser, 2004; Mitchelmore, 1998; Munier & Merle, 2009); some of them have difficulties in recognizing straight angle with different directions (Mitchelmore, 1998) and in understanding the angles such as 0^0, 90^0, 180^0 and 360^0 (Keiser, 2004). No doubt, one of the most important reasons underlying in these difficulties is the lack of full understanding of an angle by students. As a matter of fact, common misconceptions like "a small angle has a short side while a big angle has a long side" indicate this situation (Clements 2003; Fyhn, 2008; Gjone & Nortvedt, 2001). On the other hand, the results of many studies conducted on how students shape the concept image in their minds show that students have misconceptions and misunderstandings on angle concept (Clements & Burns, 2000; Keiser, 2004; Mitchelmore & White, 1998, 2000; Munier & Merle, 2009). There may be many reasons for this situation. However, general understanding claims that the angle concept is multifaceted and has different movement and forms (Close, 1982; Kieran, 1986). For this reason, there have been many studies and discussions conducted on the nature of the concept image and still, there are studies being conducted on the same topic (Matos, 1990). In this context, it is observed that the studies conducted on angles are explained over two important viewpoints. The first one is Piaget et al. conducted a study with a viewpoint of students (Piaget, Inhelder & Szeminska, 1960), and the other one is the Van Hiele geometric thinking level (van Hiele, 1986). Piaget et al. (1960) conducted a study and tried to explain the geometrical thinking skills of students with the model that suggests a transformation from concrete operational period to formal operational period. The van Hiele theory, on the other and, claimed the hypothesis that geometric thinking levels of students occurred in the form of stages that supplemented each other. In van Hiele model, five levels are defined which are *(0) recognition and imagining/visualization, (1) analysis, (2) organizing or informal deduction, (3) conclusion or formal deduction (4) difficulty/precision* (van Hiele, 1986). The consensus of the researchers is that this multi-faced concept (Mitchelmore & White, 2000) settles down in the minds of students in a slow manner and in stages (Lehrer, Jenkins & Osana, 1998; Mitchelmore, 1998; van Hiele, 1986). In addition to this, it has also been claimed that the concept of angle is shaped as a result of the performances of students based on their verbal expressions (Siegler, 1981). In addition, it has also been suggested that the angle concept in the minds of students develop with the perception of the static (configurationally) and dynamic (moving) direction; and by so-doing, students start to discover and question the angle concept (Kieran, 1986; Scally, 1990).

When the literature is examined, it has been observed that Davey and Pegg (1991) conducted a study and examined the answers of students from 1st to 10th grades given in response to the definition of an angle. At the end of the study, they collected the meanings expressed by the students on the definition of an angle under four categories as; *(1) sharp and strict corner, (2) the point where two lines meet, (3) the area of the distance between two lines, (4) the difference between the inclinations of lines*. Similarly, Clements and Battista (1989) conducted another study on 3rd graders and asked the definition of an angle to students, who defined the angle as the point where two inclined lines meet. According to Mitchelmore and White (2007), students see the concept of angle both as abstract-separate and as abstract/general. While abstract/general angles represent the general properties of the world, abstract/separate angles consist of representative diagrams and similar properties. Casas-García and Luengo-González (2013) investigated how students represented the angle concept in their cognitive structures through years. In this context, 11 concepts that were related to the general angle concept were applied to 458 Olympics, primary, secondary, and undergraduate level mathematics students with networks, and the representative knowledge maps of each group were formed. According to the findings, it has been determined that the similarities increased in knowledge maps on angle concept depending on the increases in their ages and experiences. Keiser (1997) conducted another study and tried to determine what students understood from the angle concept and tried to define their misconceptions. The findings obtained in the study revealed that students had difficulties in comprehending the corners of the angles, the beams that constituted the angle and the internal areas of angles. In addition, it was also determined that the students who defined an angle as the area between two beams could not observe the corners and therefore could not define the 180^0 and 360^0 angles; and similarly, when students were given an angle that was smaller than 180^0, they could not find the angle that complemented this angle to 360^0. The fact that students associate the measurement of the angle with the length of the beams and think that as the beams extend so will the measurement of the angle, which is among the findings of the study. Another study on angles which attracts attention was conducted on Mitchelmore and White (2000) with 144 students from second, fourth and sixth grades. The notion of how students recognized angles with nine examples, which consisted of the names of objects used in daily lives such as “wheel, door, scissors, hand-fan, sign post, hill, crossroad, roof and wall”, constituted the purpose of the study. The theoretical results obtained in the study showed that the conceptual development of students on angle were collected in three sets according to the physical status of angles, which are visible angle arms (crossroads, wall, scissors, roof, hand-fan, panel); slopes (hills, sign posts), openable and revolvable objects (wheel, hand-fan, door, scissors). In addition it was reported that, in situations where angle is formed, students cannot see holistic movements and cannot think about the angle. Özsoy and Kemankaşlı (2004) conducted a study on learning levels, mistakes and misconceptions on angles in circles with 70 students from 11th grade. It was reported in the study that the students had difficulties in establishing associations between the internal, external, central and circumferential angle concepts in the circle; had difficulties in applying some properties in angle concepts in triangular and tetragonal areas in the circle; and could not analyze the data in the questions in due manner. In addition, Schwarz and Hershkowitz (1999) and Vinner (1991) investigated the conceptual images of students on basic concepts in geometry and reported that each concept had one or more prototypes (samples).

When all the above-mentioned studies are considered, it is observed that images of typical shapes, properties of the shape and the image properties direct the formation of the geometric concepts and in establishment of relations between these concepts. In this aspect, the issue of what the cognitive structures on angle concept is in the literature and a structure that would reveal the concept images and cognitive structures of students on angles constituted the starting point of the study.

Artificial Neural Networks (ANNs)

Although the working principle, functionality and information-processing stages of the brain have not been understood fully today, it is known that billions of neurons that constitute the brain play important role in these processes. In recent years, in order to understand this structure better, artificial models that take the function of the brain to its center have been produced in many sciences (anatomy, biopsychology, neurology, neurosurgery and psychophysiology) especially in cognitive sciences. One of these models is the Artificial Neural Networks, which was simulated to the working principle of the brain. Artificial Neural Networks (ANNs) is the computer programming language that imitates the structure of the biological neural networks. ANNs are also known as the Parallel Distributed Coupling Processing Models (Kulkarni, 1994). These models that have the traces of a mathematical understanding consist of previously learned networks. These networks are the mechanisms that consist of parallel connections (synapse) between themselves and simple processor elements (neurons) in order to store the information obtained with the experience and to use it (White, Southgate, Thomson & Brenner, 1986). In this context, ANN is a systematic organization that established interaction with real-world examples and consists of many neurons connected to each other in a parallel manner (Haykin, 1994; Kohonen, 2001). For this reason, "artificial neural systems or neural networks have been defined as the system with physical cells that receive, store and use the empirical knowledge" (Zurada, 1992). In 1943, McCulloch and Pitts defined ANNs as the algorithmic models that include logical processes of the weighted sums of the inputs; and these networks gained a new dimension with the addition of a learning rule to the model by Hebb (1949). In further years, Minsky (1954) developed a learning machine that could apply the weights of the connection in an automatic manner; and right after this, Rosenblatt (1958) developed a perceptive model. Widrow and Hoff (1960) developed the ADALINE (ADAptive LInear NEuron) model as the calculation model for the elements and Hopfield started neural network analysis with feedback in 1982 and 1984. These processes continued with the development of neural network model with feed-forward with multiple layers as an alternative to the algorithm developed by Rumelhart, Hinton & Williams (1986). The human brain is known as an information processing system distributed in an intense-parallel manner, nonlinear, and extremely complex due to its abilities like learning, joining, adapting and generalization (Fine, 1999; Kulkarni, 1994). ANNs show similarities to the working principle of the human brain in that it also receives the information from the environment and passes through learning process and stores this information with the help of the strength of the connection between the neurons (nerve cell) (Baldi & Hornik, 1989; Haykin, 1994; Kohonen, 2001). In ANNs, learning occurs with the changing of the weights between the neurons (Elmas, 2016). As a matter of fact, ANNs are fictionalized on the formal and functional side of the neurons and therefore they model the learning ability that is specific to developed creatures, which occurs by trial (by doing/experiencing) and provides many advantages in finding solutions to real world problems. The other advantages of ANNs are that they can define the relations between the data, change the connection weights, process missing data, may be used in solving non-linear complex problems, they are tolerant to errors, they can be re-trained, adapted and generalized, and they facilitate analyses and designs (Elmas, 2016; Haykin, 1994; Nabiyev, 2016; Noyes, 1992; Öztemel, 2003). On the other hand, the success of ANN may be possible when the weights in the network structure take the best value. For this reason, ANNs are trained, in other words, it is aimed that the optimum connection weights are achieved among the neurons which will ensure that the output values calculated as a result of the vector set and the output values targeted are approximated to each other (Whitley, Starkweather & Bogart, 1990). In this context, the architecture property of the network structure, the algorithm structure and usage type may be considered as important factors in the success of the artificial neural networks. In Figure 1, there are five components, which are inputs (x_i), weights (w_i), adding function (Σ), activation function ($f(x)$) and output ($y=f$ (net)), in the threshold logic, which was developed by McCulloch and Pitts (1943). The synapses in the biological neural network refer to the weights, the dentrits refer to the adding function, cell body refers to the transfer function, and axons refer to the cell output.

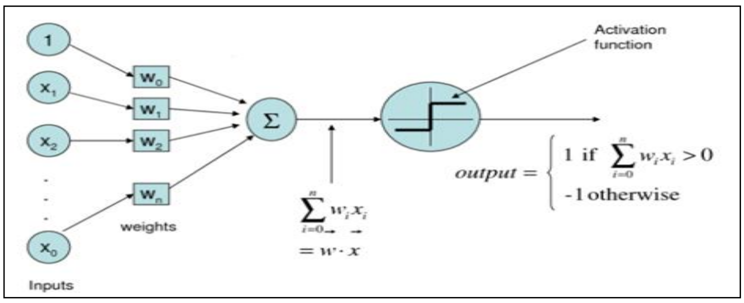

Figure 1. Fundamental unit of artificial neural network

The models that are formed with ANNs are generally used in analyzing statistical data or optimization of various systems. They are especially preferred intensely in regression, prediction and classification problems (Warner & Misra, 1996). ANNs is divided into two classes as feed-forward and feed-back according to the architecture of the network. This structure may be divided into four according to the leaning rule as Hebb, Hopfield, Delta and Kohonen; into three according to the learning algorithm as counseling, non-counseling and reinforced; and into two according to application style as off-line and on-line. The feed-forward networks are generally used in parent recognition problems, and feed-back networks are used in optimization problems in a common manner (Elmas, 2016).

Self-Organizing Map (SOM)

Self-Organizing Map (SOM) was developed in 1982 by Kohonen and is non-counseling neural networks that are non-linear, and is used most frequently in clustering studies (Kohonen, 2001). The purpose of the self-organized map which is known as the Kohenen Map is ensuring that the size of the datasets that are complex or that have multiple variables. With the help of the competitor learning algorithm, it is used in an efficient manner in modeling, classifying and visualization of the classified data of the elements in the dataset (Kohonen, 2001). SOM has a single-layer network structure and includes entry and output neurons. The number of the entry neurons depends on the number of the variables in the dataset. The connections between the entry neurons and output neurons are defined by reference vectors [code-book vectors]. The reason for this is that as the entry vectors are entered to the network, the network organizes itself (Fausett, 1994). The output neuron that wins the competition is called as *the winning neuron* or as the best-matching unit (BMU) (Haykin, 1994; Kohonen, 2001; Koikkalainen & Oja, 1990). The neurons that are in competitive learning process are separated into various input designs or to the classes in input designs. In this way, the areas where winning neurons are placed [different input places] are ranked in a way that will form a meaningful coordinate on the grid (Kohonen, 2001). For this reason, SOM networks are characterized with the formation of the topographical map of input designs. In this topographical map, the spatial placement of neurons in the grid reveals the settled statistical properties in the input designs (Haykin, 1994; Kohenen, 2001; Nabiyev, 2016).

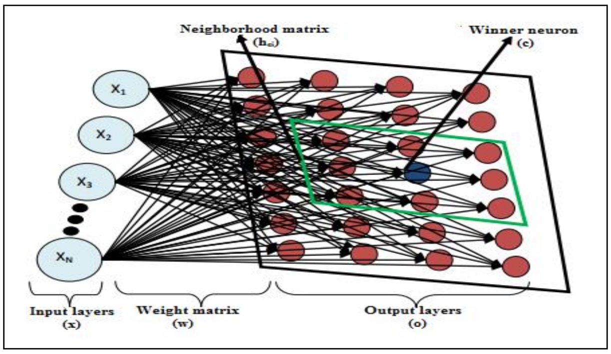

Figure 2. Structure of Kohonen self-organizing map

In the 2-dimensional algorithmic structure given in Figure 2 there is entry layer (*x*), weight vector *(w)* and output layer *(o)*. In each *i* entry design, and in each *t* iteration there is a weight vector in the form of $w_i=(w_{i1}, w_{i2}, w_{i3},.......w_{im})$. The whole of the weight vectors obtained constitute the memory of the SOM (Yang, Cheu, Guo & Romo, 2014). On the other hand, there is a winning neuron *(c)* for each *x* entry sample and the equation given in equation (1) is used to calculate the winning neuron. In this equation, ‖.‖ refers to the Euclidian distance measurement.

$$\|x - w_c\| = min_i \|x(t) - c_i(t)\| \tag{1}$$

After the winning neuron is determined, both *c* and the weight values of the determined neighboring neurons are found with the equation given in (2).

$$w_i(t+1) = w_i(t) + \alpha(t) h_{ci}(t)[x(t) - w_i(t)] \tag{2}$$

In the equation above; (*t+1*) refers to the next iteration, α*(t)* refers to the change speed (learning rate) of the weight vectors h_{ci} refers to the neighboring function of the decreasing neuron. The number of these units that are included in the interaction decreases in time, and towards the end of the training process, only the winning unit enters interaction (Kohenen et al., 2000). In this context, the neighboring function that may be formed in the form of Gauss function is given in equation (3).

$$h_{ci}(t) = exp(-\frac{\|r_c - r_i\|^2}{2\sigma^2(t)}) \quad (3)$$

In the above equation, the r_i shows the place of the unit *i* the winning r_c unit in the map. The $\|r_c\text{-}r_i\|$ expression refers to the distance between the winning unit *c* in active training station and the *i* in the output space. On the other hand, it is important to care for some assumptions for clustering work in SOM model. Firstly, the model that had the lowest average error [the difference between the entry vector and reference vector] was formed (Kohenen, 2001). For this purpose, it is adequate that there are output neurons as many as 10% of the number of the elements in the dataset (Deboeck & Kohonen, 1998). Aside from this, the following issues must be cared for; normalization of the varying values in the dataset (Graupe, 1997), assigning the first value (between [0,1] in random order) to the reference vectors (Kohenen, 2001), calculation of the distance between the entry vectors and reference vectors, and learning coefficient and neighboring variables (Kohenen, 2001). The study was conducted in this context, and the SOM method was preferred because it organizes the datasets in complex structure in a neat manner and it forms data maps that may be interpreted easily.

Ward Clustering Analysis

Ward clustering analysis is a grouping and hierarchical clustering method. Since it is not possible to know how many clusters will be formed in advance during the application of hierarchical clustering methods, Error Sum of Squares (ESS) in the groups (in the clusters) is cared for rather than group connections (Chatfield & Collins, 1980). For this reason, the objects that resemble each other at the highest level in the dataset are placed in the same cluster. In Ward method, the clusters with the lowest variance [with deviation from the center] are taken in order to catch the homogeneity in the cluster and the information loss is kept at the lowest level; the basic purpose here is to keep the sum of the squares in the cluster (Aldenderfer & Blashfield, 1984; Gan, Ma & Wu, 2007; Johnson & Wichern, 2002; Sharma, 1996; Ward, 1963). For this reason, ESS is used in the cluster as the measurement of the homogeneity. When the two clusters are combined, if the error in the cluster will be less when compared with the combining with the other cluster, the application is terminated; otherwise, another third cluster is tried for combination (Sharma, 1996). In the first step of the Ward technique, ESS is zero because each observation result refers to a cluster (Everitt, 1980). Two sub-clusters are merged in each step to form the next level. The merger (i.e. the combination) is started with the clusters that have the lowest variability. In this case, it is assumed that *k(k-1)* is the sub-group. The sum of the Euclidian distances to the averages vector of the n_i point of the *k* cluster is ESS, and is shown with W_k. The formula of the sum of the error squares is given in equation (4).

$$W_k = \sum_{i=1}^{p} \sum_{j=1}^{n_k} (x_{ijk} - \bar{x}_{ik})^2 = \sum_{i=1}^{p} \sum_{j=1}^{n_k} x_{ijk}{}^2 - n_k \sum_{i=1}^{p} \bar{x}_{ik}{}^2 \quad (4)$$

The formula given in equation (4) is applied in the same manner for each set. W_k refers to the Ward value (the sum of the error squares) of the *k* set; *p* refers to the *p* set; n_k refers to the number of the elements in *k* set; *i* refers to the *i. data; j* refers to the *j.* data; *k* refers to any set. On the other hand, W_k value is calculated in *k=1,2,3,.....n* sets; and the intra-cluster ESS sum is calculated with the help of the formula given in equation (5).

$$W = \sum_{i=1}^{n} W_k \quad (5)$$

As a conclusion, when the *r* cluster is obtained by assuming *p* and *q* clusters that have the smallest value in the Ward cluster, this increase in *W* is found with the help of the equation given in (6).

$$DW_{pq} = W_r - W_p - W_q \quad (6)$$

By so-doing, *n* is separated to unit *(n-1)* set. The units are connected is stages by finding *W* increasing values until *k* becomes the least, i.e. “1” (Ward, 1963). Ward method does not calculate the distances between the sets, which is different from the other clustering methods. Instead, it minimizes the sum of the intra-set error squares and forms sets that will make the homogeneity maximum, and joins the ones whose error square sums are the least (Sharma, 1996; Ward, 1963).

Combined SOM-Ward Clusters Analysis

In this study, combined SOM-Ward clustering analysis was used. The basic purpose here is to obtain the lowest variance in the set, and the highest variance among the sets. As it is known, ANN consists of many processing units (processing elements, units, neuron), which are called nodes. Each node makes simple calculations with the activation function it has. Initially, each node is evaluated to define a set in the Ward clustering method in which dispersive criteria are taken as bases. For this reason, the minimum distance between two sets are joined with the variances that have the lowest variance in each step of the algorithm. The formula showing the distance between the two sets is given in equation [7] (Deboeck & Kohonen, 1998; Kohonen, 2001).

$$d_{rs} = \frac{n_r . n_s}{n_r + n_s} . \|\bar{x}_r - \bar{x}_s\|^2 \tag{7}$$

The *r* and *s* in the formula refer to two special sets, n_r and n_s refer to the number of the data points in the sets; $\bar{x}_r$ and $\bar{x}_s$ refer to the center of gravity of the sets. In this context, the formula given in equation (8) is used in order to decide on the center of gravity with the number of the elements of the set that will be formed.

$$\bar{x}_r^{(new)} = \frac{1}{n_r + n_s} . (n_r . \bar{x}_r + n_s . \bar{x}_s), n_r^{(new)} = n_r + n_s \tag{8}$$

The node character of the map and the topological positions of the clusters are used as the measurement tool in SOM-Ward clustering method, which is a modified and special form of the Ward method. In other words, the initial points of the matrix distance, i.e. the recorded data numbers of the nodes that match in the map are considered. When n_r and n_s are taken as the number of the recorded data, $\bar{x}_r$ and $\bar{x}_s$ are taken as the node vectors, the formula given in equation (9) is made use of for the d'_{rs} distance between *r* and s nodes. According to the formula, no two clusters that are adjacent on the map may be joined in any way.

$$d'_{rs} = \begin{cases} d_{rs} \; if \; clusters \; r \; and \; s \; are \; adjacent, \\ \qquad \infty \; differently \end{cases} \tag{9}$$

Mapping Process of Cognitive Structures with SOMs

Cognitive mapping is accepted as a method that is used to understand, analyze and structure real life problems, and is formed from the relations between the qualitative, quantitative situations and ideas (Hasıloğlu, 2009; Kwahk & Kim, 1999). All the claims whose cause-result variables are the same in this structure are combined by reducing to one single causal relation depending in their sections (Axelrod, 1976). The starting point of many studies conducted on cognitive structures is the focusing of the objects on the positions in space (Golledge & Stimson, 1997; Llyod, 2000). This situation is based on the understanding claiming if a certain cell shows a certain structure in the SOM uncounselled learning algorithm, their neighbors also show similar structures (Önsel-Sahin, 2002). The basic target is ensuring that more than one cell are learned in each learning step. As a matter of fact, SOM is based on two very simple learning principles, which claims that the connection weights of the winning neuron and the observations that have equal and similar entry values with the learnt information (Llyod, 2000). With the help of these two learning principles, the position in which learning is realized in the entry layer is determined, and SOM is formed (Kohonen, 2001). On the other hand, SOM learning algorithm steps are as follows (Lippmann, 1987): *(1) assign the initial weight values of the connections from n number inputs to m number of outputs [0,1] as random and small values; (2) introduce the new input to the network; (3) calculate the distance for all cells; (4) determine the output cell with the smallest distance; (5) update the weight values of the neighboring cells and output cell; (6) if the weight values are not fixed, return to step two.* In the light of the above-mentioned facts, SOM is the ideal selection tool because it requires an uncounselled learning algorithm and determines the spatial area where learning occurs. Meanwhile, SOM, which is a spatial structure in its nature, matches with the neighboring locations of the map that is formed with the self-organization of entry vectors (Llyod, 2000). For this reason, this approach models the learning process in a cognitive structure, and helps that the concept image is formed.

In this study, the answer to the question of how the total cognitive structures (concept image) of secondary school seventh grade students are formed on angle concept has been sought. In this context, the purpose was to visualize the cognitive structures of students on angle concept by using the combined SOM-Ward clustering analysis technique.

Method

The Study Model

In the scope of the SOM-Ward clustering, open-ended questions were asked to the students, and a qualitative approach was adopted. In this context, the perceptions of the students were determined with a phenomenographic approach in terms of revealing the thoughts of the students on angles. Phenomenographic studies are generally preferred to explain how various phenomena are understood via different ways in a qualitative manner, and to classify various understandings according to the categories that emerge (Ashworth & Lucas, 1998; Trigwell, 2006).

The Study Group

The study group consisted of 250 students who were studying at 7^{th} grades in a state secondary school in the city center of Izmir in 2015-2016 academic year. 128 of the students were female (51,2%) and 122 were male (48,8%). The socio-economic status of the students who were selected randomly was generally in middle level.

Data Collection Tool

The Conceptual Understanding Tool, which consisted of the open-ended question *"write the first ten things you remember when the term 'angle' is mentioned",* was used. In order to ensure the content and fitness validity of the measurement tool, which was developed by the authors, the viewpoints of 8 mathematics instructors who were specialists in their fields were received. In addition to this, the viewpoints of the teachers and students were also received in order to obtain detailed information and to clarify unclear concepts. The Matlab and Viscovery SOMine programs were used in analyzing the data.

Results

Conceptual Understanding Measurement Tool Results

In this study, a different way was followed in terms of the analysis of the data and in terms of the data collection and the concept images of the students on angle concept were determined. When the first 10 answers of the students on angle concept are examined respectively, it is observed that the most-frequently repeated statements are scissors, wall, door opening, clock and frame, which are observed frequently in daily life. In addition, edge, internal and external area, beam and similar structures that form an angle were also used frequently by the students. These results show similarities with the results of the study conducted by Mitchelmore and White (2000), which revealed that the students defined the angle with 9 objects "wheel, door, scissors, hand-fan, sign post, hill, crossroads, roof and wall", which were used in daily life. Solid, acute, obtuse, straight angles; square, rectangular, triangle, circle and similar geometric shapes; degree, protractor, compass, cm, km, volume and symbols/signs and units; and abstract concepts like intelligence, fear, excitement, and difficulty were observed. Speed, energy, sound, viewpoint, graphic, fracture, light, reflection and angle of incidence, which are interdisciplinary statements, were also among the answers given by students. The structure of the variables analyzed with Matlab in SOM weight positions is given in Figure 3. The areas where the connections are intense indicate the similar statements of the students.

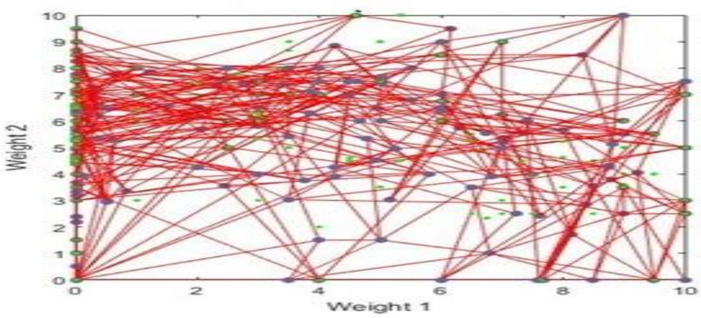

Figure 3. SOM weight positions

SOM-Ward Clustering Analysis

The results obtained from the students with the open-ended conceptual measurement tool were used in SOM-Ward clustering model. Firstly, the answers given in response to the angle concept were collected under 7 groups (Table 1). In defining the groups; the frequency and quality of the answers given by the students were cared for. Similar concepts were merged in certain groups.

Table 1.The themes determined according to the viewpoints of the students

Group no	Group name	Explanation	Sample Answers
1	Angle elements	There are the elements that form the angle, mathematical expressions, and the expressions on the properties on angle are given in this group.	Beam, line, arrow, edge, corner, internal area, external area, point, gap
2	Angle types and measurements	There are expressions that are related with the gap between two beams in this group.	Full angle, acute angle, obtuse angle, straight supplementary angle, explementary angle, 65^0, 90^0, 180^0 and similar angle types
3	Geometrical objects	There are expressions that are associated with geometrical and other shapes in this group.	Triangle, square, circle, prism, pentagon, rectangle, sphere, hexagon, cube, cylinder, parallelogram
4	Daily life	There are expressions that establish relations between the shape and any other properties of the angle and daily life in this group.	Hour, door, football, earth, teacher, panel, scissors, door way, cupboard, sails, frame, house, graph notebook, garden, rainbow
5	Mathematics/ Other sciences	There are expressions that are related with mathematics and the other fields in this group.	Reflection, angle of incidence, number, light, graphic, fraction, area, speed, algebraic expressions, energy, sound, four operations, viewpoint, electricity, natural numbers, rational numbers
6	Symbols/Signs/ Units	There are expressions that depict a thought, quality, concept and object through symbols and the expressions that are used in measuring physical properties are given in this group.	Degree (0^0), equals (=), meter (m), addition (+), volume (v), hectometer (hm), kilometer (km), time (t), second (s), cubic centimeter (cm^3), hand span, area (m^2), temperature (^0C), division (:), multiplication (x)
7	Abstract expressions	There are the expressions that are opposite the abstract one and that cannot be perceived with feelings in this group.	Infinite, success, challenge, intelligence, boredom, excitement, fear, solidarity, mind, love, imagination, dream

Then, a code was defined for each group (Table 2). In the next step, the answers of each students were encoded in order ($S_1,S_2,S_3,...,S_{250}$) and the data obtained were ranked ($R_1,R_2,R_3,\ldots,R_{10}$).

Table 2. The codes of the expressions given by students

Student	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}
S_{122}	2	2	2	2	2	2	6	3	3	2
S_{123}	2	5	4	4	4	2	2	5	5	6
S_{124}	5	4	6	5	4	6	1	1	5	4
S_{125}	2	2	2	2	2	2	1	3	1	3
S_{126}	3	3	5	5	2	2	3	4	5	5
S_{127}	5	3	3	3	3	3	1	5	3	3
S_{128}	5	6	2	2	2	5	3	3	3	2

In the next step, the data that are categorized in Table 2 are standardized, because although there is no difference between the values that symbolize the groups (from 1 to 7), the values in the dataset will mean "1" the smallest and "7" the biggest; and therefore, the result of the operation will be incorrect. In this context, a new organization was made for the numbers that expresses similar meaning for the variables. For this purpose, a new table was formed to calculate the statements of the students on angle concept in which rows. In this context, the statements on angle concept that has the low average rank have been made to become more important compared to the statements that have the high-rank averages. By doing so, the statements on each angle concept in the group were standardized so as to become similar, and the average rank was determined. The average rank was made in the following way: The answers given by the students on angle concept were grouped from 1 to 7, and the average ranks (R_1, $R_2,\ldots,R_7$) were calculated. Then, these values were added for each line and were divided by the total frequency. For example, the theme with code 5 of the student S_{124} in Table 2 was repeated for three times in 1^{st}, 4^{th} and 9^{th} ranks. In this context, the average rank was calculated as (1+4+9)/3=4.66.

Table 3. Rank averages of the expressions of the students

Student	x_1	x_2	x_3	x_4	x_5	x_6	x_7
S_{122}	0	4,42	8,5	0	0	7	0
S_{123}	0	4,66	0	4	6,33	10	0
S_{124}	7,5	0	0	5,66	4,66	4,5	0
S_{125}	8	3,5	9	0	0	0	0
S_{126}	0	5,5	3,33	8	6,5	0	0
S_{127}	7	0	5,57	0	4,5	0	0
S_{128}	0	5,5	8	0	3,5	2	0

Despite these processes, the data in Table 3 did not constitute the proper dataset to train SOM, because if one answer from the "angle types and measurements" group is written as the initial statement, it receives "1" as the listing rank. However, this situation causes a calculation problem. For example, the student with S_{123} code wrote theme 6 only in the 10^{th} rank. For this reason, there is the number "10" under this theme. In order for the dataset to become proper for SOM-Ward clustering analysis, it must be organized in a way in which "1" must show the lowest relation, and "10" must show the highest relation. In other words, the answers of the students given in the first order must have higher values than those in the latest order. For this, by subtracting the data in the evaluation set from 11, Table 4 is formed. These data ($x_1, x_2,\ldots,x_7$) were used in training the SOM-Ward model. In addition if the answers written by the students on "angle elements" are not among the 10 statements, they receive 0.00 values (see Table 4, first line). By doing so, the theme that is not mentioned is extremely close to the first and last theme expressed in the dataset.

Table 4 .SOM-Ward clustering entry data

Student	x_1	x_2	x_3	x_4	x_5	x_6	x_7
S_{122}	0	6,57	2,5	0	0	4	0
S_{123}	0	6,33	0	7	4,66	1	0
S_{124}	3,5	0	0	5,33	6,33	6,5	0
S_{125}	3	7,5	2	0	0	0	0
S_{126}	0	5,5	7,66	3	4,5	0	0
S_{127}	4	0	5,42	0	6,5	0	0
S_{128}	0	5,5	3	0	7,5	9	0

SOM-Ward Clustering Analysis Results

The dataset algorithms defined to determine the concept images (cognitive structures) on angle concept are used in order to train SOM-Ward clustering analysis. The Matlab and SOMine programs were made use of in organizing the data obtained from the students (Martinez & Martinez, 2005). However, it is extremely difficult to say how the concept images of the students on angle concept are by considering the clusters obtained by SOM. Especially for the purpose of interpreting the cognitive mapping of students, the hierarchical clustering algorithm, which is given in SOMine program in addition to SOM. The basic justification of the hierarchical cluster algorithm is equalizing the number of the observations and the number of the clusters by taking each student as a cluster. Then, the observation results that are closest to each other are combined with SOM-Ward distance measurement, which results in decreased number of the clusters. The distance of the clusters to each other and their positions on the map are used in new distance measurement.

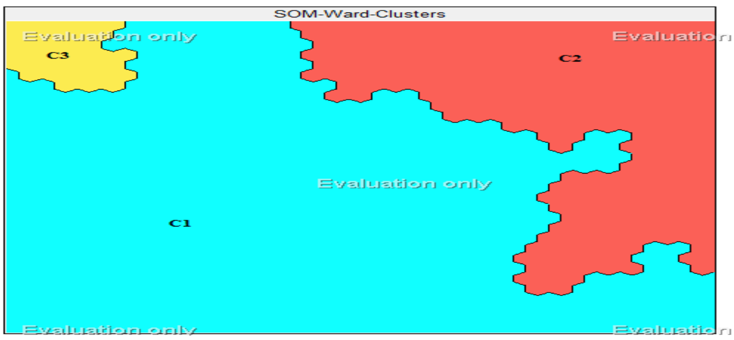

Figure 4. After trained neuron, the SOM-Ward clustering analysis

All of the 10 statements on students, which were obtained from 250 students, were used in forming the clusters. The measurement of the competition between the neurons in the SOM-Ward clustering training is the tension parameter. Usually the tension parameter varies between 0.3 and 2. The tension parameter in this study was found as 0.5. The sets that are formed with the Kohonen layer that appears as a result of the competition of 1000 neurons used in the SOM-Ward clustering analysis are given in Figure 4.

On the other hand, the U-Matrix has been formed by running the Ward basic agglomerative hierarchical clustering technique and SOM together. The purpose of forming the U-Matrix is to determine the number of the clusters. The dark color in the U-Matrix show the areas where the relation between the neurons is weak and the light color shows the areas where the relation is strung, i.e. where neurons pile up (more). In this respect, the program works with the logic of defining a limit where the relation is weak and forming clusters. Although the issue of how many clusters will exist has been decided on by considering the U-Matrix, it may influence the number of the clusters because the perceptions of students on a concept are considered. Kohonen layer is divided into 3 clusters. The level of the effects of the groups on these clusters is determined with a parameter varying between 0 and 10. As the value of these parameters approach towards 10, their effects on the cluster also rise.

Table 5. Statistical results and clusters

Cluster	Frequency	Attribute 1	Attribute 2	Attribute 3	Attribute 4	Attribute 5	Attribute 6	Attribute 7
C1	62,40%	3,10	4,49	3,18	2,31	5,40	3,95	0,099
C 2	33,60%	1,79	7,02	1,38	3,90	1,12	1,88	0,036
C 3	4,00%	1,97	4,80	4,46	1,45	3,37	1,95	6,350

When Table 5 is examined it is observed that the 62.40% of the students in the dataset are in C1 cognitive structuring. The students in this cluster used mostly the answers in the mathematics/other sciences, angel types and measures, symbol/sign/units groups in the first order. 33.60% of the students are in C2 cognitive structuring. The students in this cluster used the angle types and measurements and daily life themes in the first order in their answers. As the last item, 4% of the students are in C3 cognitive structuring. The students in this group mostly used the abstract expressions, angle types and measurements and geometrical objects themes in their answers in the first order.

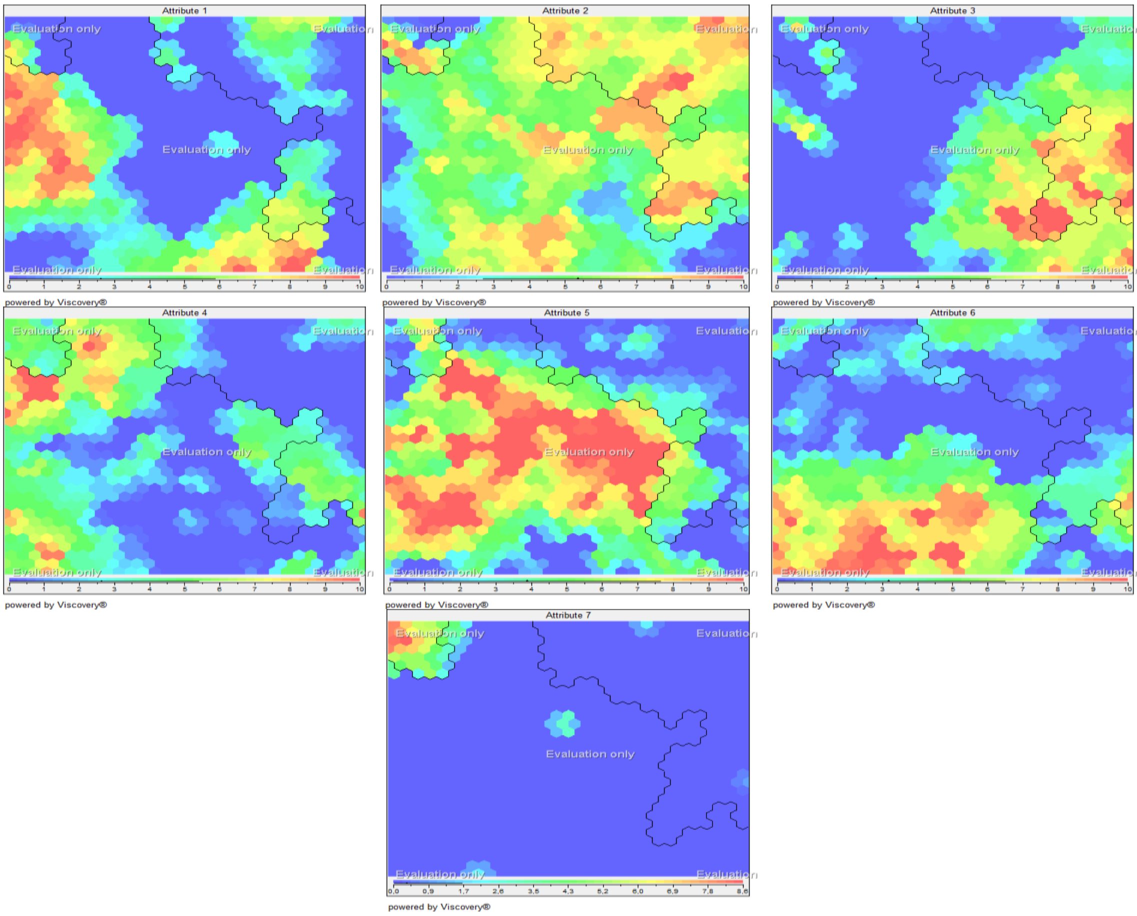

Figure 6. Feature planes for all classes

When the component plains in Figure 6 are examined, it is observed that the expressions of the students about the angle types and measures are given place in the first order, and the contribution of each variable to the clusters is shown on the map with the help of colorful measurement. In the SOM-Ward modeling in which 7 groups are divided into 3 clusters, dark blue color refers to “0” which means there is no relation, and the dark red color refers to “10”, which means the relation is strong. For example in the first map in Figure 6 shows the contribution of the first group (i.e. the angle elements group) to the clusters. According to this map, it may be claimed that the first group affects mostly the C1 cluster, forms the most part of the cognitive structure; it has the red color in certain areas of C2 and C3 clusters, and is included in the first answers given by students in these areas. When the second map is examined it is observed that in addition to the second group being more in C2 cluster, it is also intensely observed in C1 and C3clusters. Especially in the middle areas of the C1 cluster cage, it exists intensely in many areas of C2 cluster and in the lower parts of C3 cluster cage. In the third map, it is observed that the third group intensifies in the right side of the C1 and C3 cluster cages. It is observed in the fourth map that the fourth group exist in the right side of C2 cluster in a relatively lower level and intensify in the upper-left areas in C1 cluster cage. In the fifth map, it is observed that the fifth group is distributed equally in the C1 cluster except for the lowest part of the cage, and in addition to this, it intensify in the middle areas of the cage. In the sixth map, it is observed that the sixth group generally exists in the lower-left corner of the cage and in the C1 cluster. In seventh map, it is observed that the seventh group exists in C3 cluster.

Discussion and Recommendations

The basic purpose of the study is to perform a less-dimensional and detailed visualization process to reveal the images on angle by students. For this purpose, Clustering Analysis was conducted with the help of SOM to the dataset obtained from the students on angle concept. The images on angle concept were obtained by transferring 10 variables obtained from 250 students to 2-dimensional map. The cognitive structures of the students on angle concept were collected in three clusters, which were C1, C2 and C3. In this context, C1 cluster consisted of mathematics/other sciences, symbol/signs/units and angle types and measurement; C2 cluster consisted of angle types and measurements and daily life; and C3 cluster consisted of the cognitive structures on angle concept in which abstract expressions and geometric objects, angle types, and measurements. In addition, the formation of a different color harmony in C1, C2 and C3 clusters according to input variables obtained from the students’ expressions provide us with important clues in understanding the cognitive structure of angle concept. It is observed that the topographic structure of the C1, C2 and C3 clusters overlap with the hypothetical approaches claiming that the concept images mostly consist of cognitive structures and these structures include visual presentations, experiences and the name of the concept in the literature (Rösken & Rolka, 2007; Schwarz & Herskowitz, 1999; Tall & Vinner, 1981). The map designs that constitute the cognitive dimension of the concept image, which are interpreted as the properties and structure of the concept, i.e. the holistic images formed by concept when the name of the concept is heard or seen, which is claimed by Tall and Vinner (1981), support this viewpoint. A concrete justification has been formed especially with the help of the mental imaging maps for the results of many studies, which claim that angle concept is a complex structure for students (Battista, 2007; Keiser, 2004; Mitchelmore & White, 2000). Many study results reported in the literature on how students shape and structure the angle concept in their minds emphasize that students have misconceptions and misunderstandings on angle concept (Clements & Burns, 2000; Keiser, 2004; Mitchelmore & White, 1998, 2000; Munier & Merle, 2009). In this context, visualization of the concept images by using SOM may cast a light on the basic reasons underlying the misconceptions and make us understand them better by ensuring that the characteristic properties of the clusters are observed clearly in mental structuring of the angle concept. It is especially expected that the present study will bring a different viewpoint in examining the studies that deal with the cognitive structuring models of students like geometrical thinking level (Piaget, Inhelder & Szeminska, 1960; van Hiele, 1986), because it presents concrete evidence on the topic.

It is observed that the cognitive structuring of students intensify mostly in mathematics/other sciences in C1 cluster; in angle types and measurements in C2 cluster; and in abstract expressions in C3 cluster. It is expected that this result will contribute to the literature by enabling students to establish cause-result relations between simulating an angle to a sharp and absolute corner, to a point where two lines meet, to the area or the distance between two lines, to the difference between the inclinations of lines, and to a point where two inclined lines meet (Clements & Battista, 1989; Davey & Pegg, 1991). On the other hand, students establish relations most frequently between angle concept and mathematics/other sciences and angle types and measurements. This result supports the interdisciplinary approach and concept associations in concept teaching. One of the strong sides of the study is the fact that the visualization process performed by using SOM clarified characteristic properties of clusters as well as the cognitive structures that form the cluster. For example, the areas where red points are intense in the component dosages of seven different groups show the efficiency level of the winning electron. By optimizing the SOM structure in these areas and changing the number of the output neurons, different color intensities may be obtained. Similarly, the map component plains may also be change by making some changes in learning parameters. This situation may be observed mush clearly when using the variables that are not related with angle. In addition, classification may be made with other datasets instead of the dataset obtained from the 250 students or SOM may be renewed by adding some other datasets even some variables may be removed to increase the performance. Especially by changing the connection weights improvements may be made in the topographical map on angles according to the input designs of trained artificial network. Such applications may be tested with different variables to ensure that the clustering analyses have higher performance. However, the safety of the input dataset is of crucial importance during this process. As a conclusion, with the help of studies conducted on such artificial neural networks, other artificial models may be developed mainly in cognitive field and in anatomy, neurology, neurosurgery, psychophysiology, biopsychology to increase information processing capacities. It is especially expected that the success of SOM in visualizing the mental perceptions will contribute to the solution of different problems.

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Appendix 1. Inputs Data for Training the SOM-Ward Model

Students	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Students	x_1	x_2	x_3	x_4	x_5	x_6	x_7
S_{1}	0	7.5	0	2.5	0	0	0	S_{126}	0	5.5	7.66	3	4.5	0	0
S_{2}	0	8	3.4	0	10	4	0	S_{127}	4	0	5.42	0	6.5	0	0
S_{3}	3.5	8.66	3.5	0	0	8	0	S_{128}	0	5.5	3	0	7.5	9	0
S_{4}	0	1	6	0	0	0	0	S_{129}	0	7.5	0	4	5.5	2	0
S_{5}	0	6.5	0	0	1	5	0	S_{130}	0	0	6.5	0	0	1.5	0
S_{6}	6.5	5.5	5	0	4	5.5	0	S_{131}	3	5.85	0	2	0	9	0
S_{7}	0	7.5	10	0	2.5	3.33	0	S_{132}	1	6.42	0	0	0	4.5	0
S_{8}	8.5	0	0	2.5	6.4	1	0	S_{133}	8.5	2.5	7	0	0	0	0
S_{9}	7.5	4	1.5	0	5	6.5	0	S_{134}	0	5	5	0	9	5.5	0
S_{10}	0	3.5	7	0	0	6	0	S_{135}	0	3.5	0	9.5	7	8	0
S_{11}	0	7	0	6.83	1.5	4	0	S_{136}	3	6.2	0	3.5	4	10	0
S_{12}	0	3	0	6.66	8	7	0	S_{137}	0	5	0	8.5	10	1.5	0
S_{13}	3.5	5.42	0	0	10	0	0	S_{138}	0	3.33	0	7.5	6	3	0
S_{14}	0	4.5	10	5.33	2	9	0	S_{139}	6.5	2.5	0	4.33	9	7.5	0
S_{15}	2	6.5	0	0	4.33	7	0	S_{140}	6.75	2.33	7	0	0	0	0
S_{16}	9	5.5	0	2.5	8	5.5	0	S_{141}	0	6.5	1.5	0	7	6	0
S_{17}	3.5	8	0	0	10	0	0	S_{142}	6	6.5	0	3	6	2	0
S_{18}	3.5	8	0	0	3	2	0	S_{143}	0	6.5	0	1.5	0	0	0
S_{19}	6	6	10	0	2	3.5	0	S_{144}	0	7.2	4.5	0	1	0	0
S_{20}	0	6.4	0	3.5	9	0	0	S_{145}	0	5.88	0	0	2	0	0
S_{21}	4.5	4.6	8	0	10	5	0	S_{146}	7.6	0	0	7	1.5	0	3.5
S_{22}	0	5	0	0	10	0	0	S_{147}	0	5.4	0	3.5	10	5.5	0
S_{23}	0	8	3	0	0	0	0	S_{148}	0	6.4	0	2	6	4.5	0
S_{24}	4.5	7	0	4.5	0	4.5	0	S_{149}	0	5.25	10	0	3	0	0
S_{25}	0	6.6	4.4	0	0	0	0	S_{150}	0	5.75	4	0	5	0	0
S_{26}	0	9	0	5	0	5.25	0	S_{151}	0	0	0	5	0	10	0
S_{27}	0	0	4	4.5	8	7	0	S_{152}	5.5	6.8	0	0	4	2	0
S_{28}	6	6.75	7	3	0	4	0	S_{153}	3	7	7	4.25	0	0	0
S_{29}	2.5	6	6	0	0	8	0	S_{154}	0	6.5	10	0	0	2	0
S_{30}	0	7.5	7	1.5	6.5	4	0	S_{155}	0	9	5.5	0	2	2	0
S_{31}	0	8.25	0	3.4	0	5	0	S_{156}	0	7	0	5	0	2.66	0
S_{32}	8	3.5	6	0	5.66	1	0	S_{157}	7.66	0	4	10	3.33	0	0
S_{33}	5	7.5	0	2.5	0	10	0	S_{158}	7	9	4	3	0	0	0
S_{34}	5	7.66	0	3.33	6	0	0	S_{159}	0	0	7	2	2	7	0
S_{35}	6	6.4	0	1	7	4.5	0	S_{160}	4.6	10	7	0	1	0	0
S_{36}	0	6.5	0	0	0	1.5	0	S_{161}	6	6	0	0	1	0	0
S_{37}	1	3	0	6.33	10	5.5	0	S_{162}	4	2	10	1	6.5	7	0
S_{38}	0	7.5	3.5	0	10	2.66	0	S_{163}	9.5	0	7	5	3	3.66	0
S_{39}	0	4.66	6.5	0	0	2	0	S_{164}	0	4.33	6	0	0	0	0
S_{40}	4	7.5	0	0	2	2	0	S_{165}	6	9	0	5.5	0	1	5
S_{41}	6	7	0	0	0	3.5	2	S_{166}	0	5	0	5.71	0	0	0
S_{42}	1	8	4.4	0	0	0	0	S_{167}	5	3.5	0	5	9.5	7	0
S_{43}	8	4.62	0	10	0	0	0	S_{168}	6	0	0	4.6	10	4	0
S_{44}	0	4	7.5	0	2	0	0	S_{169}	9.5	0	0	5	3	5	0
S_{45}	0	7	1.5	0	8	4.5	0	S_{170}	0	0	0	5.22	0	0	8
S_{46}	5	7.4	0	0	2	7	0	S_{171}	0	6.5	0	5.16	3	0	8
S_{47}	0	0	10	5	0	0	0	S_{172}	8.5	4.5	0	10	7	1.5	0
S_{48}	0	4.6	6.33	3	10	0	0	S_{173}	0	0	0	5.33	0	7	0
S_{49}	0	6	0	0	3.75	10	0	S_{174}	0	8.5	0	0	2.5	5.5	0
S_{50}	9	6	2	0	0	10	0	S_{175}	4.6	7.5	0	0	0	2	0
S_{51}	4	8	0	4	2	1	0	S_{176}	5	6	0	2.5	7	0	0
S_{52}	5.8	4	0	0	10	0	0	S_{177}	0	5	0	5.5	9	0	0
S_{53}	0	5.14	0	4	10	5	0	S_{178}	0	6.5	0	6.5	1	2	0
S_{54}	7	5.2	0	3.5	10	5	0	S_{179}	5	9	2.5	0	0	6.5	0
S_{55}	8	5.66	0	1	10	2	0	S_{180}	0	8.5	0	0	3	3.75	0
S_{56}	0	8.66	3.25	4	8	0	0	S_{181}	5.33	10	5	4	0	0	0
S_{57}	0	8.66	0	2	8	3.8	0	S_{182}	4	7.5	0	2.5	0	1	0
S_{58}	0	3.66	0	7.66	10	0	0	S_{183}	0	3	8	10	0	6	0
S_{59}	0	5.5	0	0	5.5	0	0	S_{184}	3	5.33	6	0	0	0	0
S_{60}	0	8.66	4	0	3	8	0	S_{185}	10	7	2.75	6	0	0	0
S_{61}	0	7	4	0	1	0	0	S_{186}	0	0	0	5.5	0	0	0
S_{62}	0	5.5	10	0	1	5.5	0	S_{187}	6	8.5	7	1	6	4	0
S_{63}	0	9.5	8	4.2	0	6	1	S_{188}	7	2.5	9	0	5	6	0
S_{64}	6.5	5.71	2	0	0	0	0	S_{189}	6	8.5	1.5	4	0	4	0
S_{65}	10	0	0	4.5	9	0	0	S_{190}	0	7.66	8.5	3	0	3	0
S_{66}	0	5.12	0	0	7	0	0	S_{191}	0	6.25	4	0	0	1	0
S_{67}	3.5	5	0	5.2	10	7	0	S_{192}	0	8.5	4.5	1.5	0	0	0
S_{68}	0	5.5	0	1	10	0	0	S_{193}	0	4.42	0	9	0	7.5	0

S_{69}	0	6.66	3.75	0	0	0	0	S_{194}	5.5	8	0	3	1.5	7	0
S_{70}	1	8	3.5	0	0	0	0	S_{195}	0	4.5	0	8.5	0	1.5	0
S_{71}	0	6.83	3.5	0	0	0	0	S_{196}	10	2.5	9	5	5.5	0	0
S_{72}	0	5.37	0	0	6	0	0	S_{197}	3	5.87	0	0	0	5	0
S_{73}	0	4.4	5.5	6	10	0	0	S_{198}	10	3	6	7	4.5	4	0
S_{74}	4.5	7	0	4.2	10	8	0	S_{199}	0	6	0	0	1	0	0
S_{75}	0	5.37	0	10	2	0	0	S_{200}	0	1.5	4	0	7	9.5	0
S_{76}	9	3.5	7.5	4.33	10	1	0	S_{201}	9	4	0	7.25	6	1.5	0
S_{77}	3	6	2.5	6.33	10	0	0	S_{202}	0	9	3.75	0	6	1	0
S_{78}	0	7.5	2.5	0	0	0	0	S_{203}	6.66	4.83	0	0	0	6	0
S_{79}	10	0	0	4.5	9	0	0	S_{204}	1	6	0	0	0	0	0
S_{80}	0	7	6	4.57	10	0	0	S_{205}	0	0	7	4.5	0	0	0
S_{81}	0	6.4	0	4	7	0	0	S_{206}	0	6.25	0	5.8	0	1	0
S_{82}	3	6.28	4	0	0	0	0	S_{207}	6.5	3.5	0	6.33	5.5	5	0
S_{83}	0	5.12	7	0	0	0	0	S_{208}	10	5	4.66	0	0	6	0
S_{84}	0	8	2.5	0	0	5	0	S_{209}	2	4.25	0	6.5	0	10	0
S_{85}	0	5.75	6.33	6	0	3.5	0	S_{210}	7	3	0	0	0	6	0
S_{86}	3	6.5	0	1	10	9	0	S_{211}	2.5	8	0	0	5	2.5	0
S_{87}	7.66	2.5	0	5	8	9	0	S_{212}	0	8.5	5.5	2	10	0	0
S_{88}	0	0	6	0	0	3.5	0	S_{213}	0	8	0	4.2	10	5.33	0
S_{89}	0	1.5	5.5	0	10	9	0	S_{214}	0	0	8.5	3.5	0	0	0
S_{90}	0	7	6	5	4	3.5	0	S_{215}	0	8.5	4.33	4.5	7	0	0
S_{91}	0	9	5.5	0	0	2	0	S_{216}	0	0	0	4.85	7.5	0	6
S_{92}	5	7.5	2.5	0	0	10	0	S_{217}	0	8.5	3	3.6	0	0	0
S_{93}	0	6.5	4	0	0	0	0	S_{218}	4	7	8	4.4	7	0	0
S_{94}	2	7	5	0	3	0	0	S_{219}	0	0	0	7.5	1.5	3	4
S_{95}	3.33	8	0	4	5	10	2	S_{220}	0	1.5	6	0	7.5	5.33	0
S_{96}	0	6.25	0	1	0	4	0	S_{221}	7	6.25	6	0	3.33	0	0
S_{97}	4	7.5	0	1	2	10	0	S_{222}	3.5	7	9	6	5.66	3	0
S_{98}	7.5	2.5	0	5	10	7.5	0	S_{223}	0	6.5	1	4	2.5	9.5	0
S_{99}	2.5	5	6.25	0	10	0	0	S_{224}	3	4	0	2	8.5	6.5	0
S_{100}	0	6	6	2.5	8	6	0	S_{225}	5.33	4.5	6.5	0	5	7	0
S_{101}	0	0	0	4.62	8	10	0	S_{226}	0	1	9	5.66	6.5	4.5	6
S_{102}	0	8	9	4	10	0	0	S_{227}	2	8	0	3.5	5	10	0
S_{103}	0	7.5	2.5	5	0	0	0	S_{228}	0	0	0	5.77	0	0	3
S_{104}	0	7	7	2	0	2	0	S_{229}	7.33	4.5	6	0	3.66	7	0
S_{105}	2	7.4	3	4.33	0	0	0	S_{230}	3.5	9	10	6	4.33	4	0
S_{106}	9.5	4	0	8	0	0	0	S_{231}	2.5	7.5	0	1	5	7	0
S_{107}	0	5.25	0	0	10	0	3	S_{232}	1	7	2.5	5	7	9	0
S_{108}	0	0	0	2	7.66	4	5.5	S_{233}	0	5	1	6.5	10	3	0
S_{109}	4.66	6	0	5.5	0	4	9	S_{234}	0	5.5	3.5	0	9.5	0	0
S_{110}	9	10	0	3.66	0	6	8	S_{235}	0	3	0	1.5	7.5	4	0
S_{111}	0	8	4	1.5	0	0	0	S_{236}	9	3.6	3	0	7	0	0
S_{112}	0	0	7.75	4	0	0	0	S_{237}	0	5.5	10	3	3.66	9	0
S_{113}	9.5	5.5	5.25	1	0	3	0	S_{238}	6.5	5.25	0	0	0	0	0
S_{114}	0	8.5	0	3	4	0	5	S_{239}	10	7.5	0	4	4.25	5	0
S_{115}	7	5	0	5.33	5.5	0	0	S_{240}	5	1.5	0	9	6.25	8	0
S_{116}	0	3.5	8	0	10	0	0	S_{241}	9	5	0	4	8	5.5	0
S_{117}	7	9	10	1.5	5.2	0	0	S_{242}	0	6	0	2	0	5.5	0
S_{118}	0	7.5	6	2	6.33	9	0	S_{243}	0	0	5.5	6	4.2	8.5	0
S_{119}	0	5	7	4	4.33	0	0	S_{244}	0	7	5.5	4	5	0	3
S_{120}	0	5.33	5.75	0	0	0	0	S_{245}	8	5.22	0	0	0	0	0
S_{121}	0	5.5	0	0	0	0	0	S_{246}	2.66	7.75	6.5	0	3	0	0
S_{122}	0	6.57	2.5	0	0	4	0	S_{247}	4	1.5	0	7	8	10	0
S_{123}	0	6.33	0	7	4.66	1	0	S_{248}	4.5	4.5	0	0	6.66	8	0
S_{124}	3.5	0	0	5.33	6.33	6.5	0	S_{249}	3	3	0	0	8	8	0
S_{125}	3	7.5	2	0	0	0	0	S_{250}	0	8.5	4	3	2	0	0