ISSN: 2148-9955

International Journal of Research in Education and Science (IJRES)

www.ijres.net

Developing Students' Mathematical Skills Involving Order of Operations

Ernna Sukinnah Ali Rahman, Masitah Shahrill, Nor Arifahwati Abbas, Abby Tan
University of Brunei Darussalam

To cite this article:

Ali Rahman, E.S., Shahrill, M., Abbas, N., & Tan, A. (2017). Developing students' mathematical skills involving order of operations. *International Journal of Research in Education and Science (IJRES), 3*(2), 373- 382. DOI: 10.21890/ijres.327896

International Journal of Research in Education and Science

Volume 3, Issue 2, Summer 2017 ISSN: 2148-9955

Developing Students' Mathematical Skills Involving Order of Operations

Ernna Sukinnah Ali Rahman, Masitah Shahrill, Nor Arifahwati Abbas, Abby Tan

Article Info

Article History

Received:
29 August 2016

Accepted:
06 March 2017

Keywords

Order of operations
Numerical expressions
Secondary mathematics

Abstract

This small-scale action research study examines the students' ability in using their mathematical skills when performing order of operations in numerical expressions. In this study, the 'hierarchy-of-operators triangle' by Ameis (2011) was introduced as an alternative BODMAS approach to help students in gaining a better understanding behind the concept of the order of operations. The study involved 21 students from Year 9 (or equivalent to 8th Grade in American schooling) in one of the government secondary schools in Brunei Darussalam. Mixed method research design was adopted for this study. Data were collected and analyzed from the students' pre and post-test scores as well as from the interviews. Comparisons of the scores showed positive progress and greater improvement in the students' performance. The interviews were necessary in order to gain the students' feedback in the implementation of the alternative approach. Most of the students who were interviewed responded that it was easier for them to remember the triangle rather than using the mnemonic as a tool to remember the order of operations.

Introduction

The Ministry of Education in Brunei Darussalam has reformed its education system for the 21st Century, known in the Malay language as *Sistem Pendidikan Negara Abad Ke-21* (hereafter known as SPN21). One of the benefits of the reformation is to provide multiple pathways to higher education. It also provides two channels after elementary school levels offering junior high school students to take the General Education Programme or the Applied Education Programme, and introducing the International General Certificate for Secondary Education (or known as IGCSE) as the alternative syllabus other than the General Certificate of Education Ordinary Level (known as GCE O Level). Through these changes, it aims to improve the students' educational performances according to the national standards and also by benchmarking them against the international standards. The SPN21 also benefits the students in acquiring the skills needed in the 21st Century (Ministry of Education, 2013).

Producing highly skilled people requires students to achieve excellently in three core subjects, Mathematics, English Language and also Science. Learning in the 21st Century entails having strong fundamental background of these subjects. One of the examples that a 21st Century learner must master is the fundamental arithmetic operations such as Addition, Subtraction, Multiplication and also Division. To master these fundamental background, there is the need in the understanding of conceptual knowledge. Students should know how to add, subtract, multiply and divide numbers. In these early stages, students should be able to connect the relationship between addition and subtraction, addition and multiplication, and also multiplication and division. However, students are expected to have already learned how to evaluate numbers with different operations independently since their elementary schooling. In the context of Brunei, the mixed operations in arithmetic have been taught from Year 4 of their primary schooling. Meanwhile, the order of operations including brackets and exponents are introduced at the secondary level. Bautista (2012) suggested that when evaluating numerical expressions, it has to 'operate' in an order.

The PEMDAS is an acronym or mnemonic for the order of operations that stands for Parenthesis, Exponents, Multiplication, Division, Addition and Subtraction. This acronym is widely used in the United States of America. Meanwhile, in other countries such as United Kingdom and Canada, the acronyms used are BODMAS (Brackets, Order, Division, Multiplication, Addition and Subtraction) and BIDMAS (Brackets, Indices, Division, Multiplication, Addition and Subtraction). These acronyms have been the common teaching method used to help students in memorization (Headlam & Graham, 2009). Although the acronym given helps the students in remembering the order of operations, it does not develop the concept behind the acronym itself (Ameis, 2011). Generally students have the idea of the acronym, but it took time for them to remember what each letter stands for. Moreover, students have the tendency to depend on the acronym when evaluating numerical expressions (Lee et al., 2013).

Literature Review

According to Vanderbeek (2007), there are two categories in rules, the natural rules and also the artificial rules. The natural rules include the precedence of exponential over multiplication over addition. It is important that students should be able to express the concept behind this precedence. The artificial rules consist of ‘left to right’ evaluation, equal precedence for multiplication and division. The fundamental understanding of arithmetic operation is the relationship of multiplication and addition, addition and subtraction, as well as multiplication and division. The relationship of addition and subtraction is the same as adding a negative number, where 4 – 3 is also consider as 4 + (-3). Meanwhile, multiplication is the repeated addition where 2 x 3 is also consider as 2 + 2 + 2. On the other hand, multiplication of a reciprocal is also the same concept as division (Vanderbeek, 2007).

A study done by DeLashmutt (2007) investigated the role of mnemonics in learning mathematics. She stated that students who practice mnemonics may remember the math concept and they will be “able to retrieve them at a later date” (p. 2). Although there was not much research on the use of mnemonics in the Mathematics subject area, never the less, mnemonics could be one of the instructional strategies to connect on the new information to the information students already know. With mnemonics, students will be able to remember factual information, answer questions and demonstrate comprehension (DeLashmutt, 2007). However, Kalder (2012) argued that the use of mnemonic such as PEMDAS may be helping the students but at the same time hinder them. It goes back to the way teachers introduce the mnemonic to students (Lee et al., 2013), where “they were taught incorrectly the first time they learned it” (p. 74). Therefore it is vital for teachers to introduce and explain thoroughly what each letter represents until students acquire how to use them appropriately (DeLashmutt, 2007). When PEMDAS is used in the class, the teachers always reminded students about the equal importance between multiplication and division. However, the students misinterpret the mnemonic, obviously M comes before D, and from there they apply multiplication before division (Kalder, 2012; Bautista, 2012). This also implied that the same misconceptions could be developed in the students’ learning as they memorized BODMAS. When the order of operations D comes before M, students always thought that division should be performed first rather than multiplication. Bautista (2012) explained that if there are two operations, example multiplication and division, in the same expression, it should be performed from left to right. In spite of using mnemonics approach in teaching, it does not connect to the conceptual understanding of the order of operations.

A previous study by Headlam and Graham (2009) compared the ability of students in carrying out calculations in arithmetic and the procedure between the Japanese and the British students. Headlam and Graham concluded that Japanese and British students approached each questions differently. From the study, the Japanese students were not exposed to mnemonics but the teaching style in Japan was more on practicing questions that focus on algebra. On the other hand, British students were more dependent on applying BIDMAS to the questions. However, when the students were too dependent on the mnemonic itself, they are incapable to apply PEMDAS when they could not understand entirely about the order of operations. In addition, Headlam and Graham stated, “The principle of the Order of Operations is a cornerstone of the understanding of arithmetic” (p. 37). This indicated that students should have good foundation in the principle of the order of operations before solving any numerical expressions. A stronger foundation in understanding the idea behind the order of operations will lower the possibility of students having problems in solving numerical expressions. According to Joseph (2014), the order of operation is not specified to only just one grade level, but it is required in all mathematics courses from Year 4 onwards (Glidden, 2008). When students are unskilled with the procedure in the order of operations, they will have difficulty in understanding the algebraic structure. Students with less understanding in arithmetic will find it tough for them to learn and understand algebra. Banerjee and Subramaniam (2005) discussed that students need to be aware of the numbers and the operation signs as well as the rules and properties in manipulating algebra expressions.

In a study by Lee, Lickwinko and Taylor-Buckner (2013), the researchers used alternative approach in order for the students to find mathematical reasoning in simplifying numerical expression. Participants from the experimental group were exposed to the ‘Rearranging Numerical Expression’ approach whereas the control group used the ordinary order of operations method. Lee and colleagues aimed to guide students to use mathematical reasoning in simplifying numerical expressions. Instead of applying ‘Left to Right’ rule when performing order of operations, their study connected the basic algorithm properties such as associative and commutative properties to rearrange the expression and follow exactly PEMDAS in order. They found that the experimental group managed to simplify questions by applying the ‘Rearranging Numerical Expression’ method. Furthermore, the experimental group showed that the students understood the method and this could be one of the ways to develop their mathematical reasoning in using the mnemonic, PEMDAS.

Ameis (2011) stated that the “left to right processing is not a mathematical law” (p. 417) and he started to develop the concept to the pre-service teachers. In his study, he introduced another approach for his students to develop a conceptual understanding that overlaps in two aspects, which were the hierarchy-of-operators and any-which-way processing. With this concept he tried to develop for his students where he began with Rocky the Squirrel stories. Jeon (2012) also agreed that using story-writing activities to children would help them in understanding of the order of operation. However, Ameis (2011) concluded by drawing a triangle containing the operations in each level as the priority of each operation decrease with the level. Ameis (2011) then continued the stories to establish about the precedence of multiplication and division than addition and subtraction. In Figure 1 below, the triangle is illustrated as ‘powers’ on top of the triangle, followed by multiplication and division as having the same priority. Finally, addition and subtraction are placed at the bottom of the triangle, which also shared the same priority.

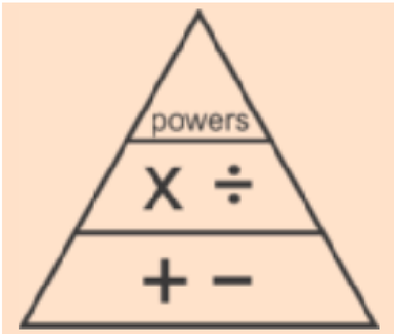

Figure 1. Hierarchy-of-operators triangle by Jerry Ameis (2011)

The ‘Rearranging Numerical Expression’ approach could not be used in this present study as it needed time to revisit the topic, integer and fractions. Moreover, due to the ability of the students, they may not be able to connect the relationship of addition and subtraction as adding a negative number, and also the relationship of multiplication and division as multiplying a reciprocal. Although the study in Ameis (2011) was for pre-service teachers, this hierarchy-of-operators triangle approach was also applicable to the sample in this study.

The Study

In the IGCSE textbooks used in Brunei, when performing operations on the number, students must apply the BODMAS rule. Even though the topic on order of operations has been introduced to the students since they were in the primary school, specifically at Year 4, it may still be difficult for them to apply the knowledge when they are in their secondary school years. This is especially difficult for those who are weak in Mathematics; they may have forgotten how their teachers previously taught them. Vanderbeek (2007) suggested that when teaching the order of operations, instead of memorizing the mnemonic device using BODMAS, it should be focusing on the basic fundamental mathematical principle. However, there are some students who understood the mnemonic used and knew which order to perform first, but they still have difficulty in manipulating and solving expressions.

Consequently, this present study was based on the observations of Year 9 (or equivalent to 8th Grade in American schooling) students simplifying mathematical expression containing multiple operations by using the ‘Left to Right’ rule, without applying the order of operations. Hence, the purpose of this study was to investigate how Year 9 students develop their mathematical skills when evaluating the numerical expressions. The mathematical skills referred to the skills in understanding the relationships between the operations and also the skills in manipulating the expressions. Carpenter and Lehrer (1999) suggested that the importance of learning skills with understanding is to avoid rote application. When students do not understand the skills they try to acquire, they will subsequently attempt it and just continue without understanding the skills. As the concept of the order of operations is not specified to one grade level only (Joseph, 2014), it is essential knowledge to all levels of students and not just limiting it at the Year 9 level.

One of the purposes of this study was to enhance the basic skills needed before students are exposed to the topic of algebra. Acquiring these skills will help students in the algebra topic, which includes manipulating algebraic expressions (Yahya & Shahrill, 2015). Therefore in this study, another approach was introduced apart from the mnemonic. The focus is on the ability of students performing the order of operations when evaluating numerical expressions. Students were exposed to the hierarchy-of-operators triangle (Ameis, 2011) and more to the conceptual knowledge of the order of operations. The objectives were for the students to have the conceptual understanding of the order of operations and use their mathematical skills in the process of evaluating the problems. This study is guided by the following research questions: Were there any improvements when the students performed the order of operations during the evaluation of numerical expressions? And what were the students' perspectives when using the alternative approach in recalling the order of operations?

Methodology

Research Design

An action research approach was used as the research design in this study. According to Kemmis and Mc Taggart (1992) and Cohen, Manion and Morrison (2007), action research is an approach to improve education by changing and starting with small cycles where problems are identified and the researcher(s) plan and implement the intervention. Yasmeen (2008) mentioned that in educational system, action research is conducted in classroom not only to gain student response but also the teaching strategies over the sessions. In this way, it can extend the teaching skills and understand about the classroom and students (Nelson, 2013). Furthermore, mixed methods as both quantitative and qualitative methods (Brannen, 2007) were used to answer the research questions for this present study.

The Sample

This study was conducted in one of the government secondary schools in the Brunei-Muara district. Initially, there were 25 students enrolled in this study but only 21 students completed this study due to absenteeism. The students consisted of 11 male and 10 female, ranging from 14 to 15 years old. The sample was from the Year 9 class (or equivalent to 8^{th} Grade in American schooling) registered in doing Mathematics at the International General Certificate for Secondary Education level (or known as IGCSE Mathematics). The students in this study were categorized as having low ability in Mathematics because their current examination results (at the time of study) were below the pass range of 50%.

Data Collection Procedures and the Instruments Used

Permissions were sought from the relevant agencies before conducting this study. Additionally, consents from parents and guardians were also obtained. The participants were informed of their confidentiality in this study, and where possible, pseudonyms were used to protect the students' identity. Before the intervention lessons, the students were given the pre-test. The pre-test acted as the first step to identify the problem. In the first cycle of action research, the first author completed the intervention lessons in three sessions where one session was around 50 minutes. During this intervention, similar stories referencing to that of Ameis (2011) and Jeon (2012) were conveyed to enhance the conceptual understanding of the reasoning behind the precedence of multiplication than addition. And this is followed by introducing the hierarchy-of-operators triangle by Ameis (2011). She concluded the intervention lessons by giving practice questions in performing the order of operations when evaluating numerical expressions.

Immediate post-test were given to the students after the completion of the first intervention lessons. From the results of the first post-test, we felt that the students had not acquired the skills in performing the order of operations. By comparing the results in the first post-test and the pre-test, it was necessary to design another cycle for this study. In the second cycle, the students were placed into groups of two and the groupings were dependent on the scores they achieved in the first post-test. During the subsequent intervention lesson, an activity was set up for the groups. Each group completed a task where one of the members had to evaluate an expression. Meanwhile, the other member corrected the work and explained to the partner about performing the order of operations. The task consisted of questions ranging from simple expressions to more complex and harder expressions. Upon completing the intervention lesson in two sessions, another post-test were given to check the students' development.

Mathematics Achievement Test

Scores from pre-test and post-tests were collected to assess the students' mathematical skills in evaluating numerical expression involving order of operations. The students were given only 30 minutes to answer ten items in the test paper. During the test, the students were not allowed to use the calculators. The reason for not using the calculators was to check their ability in computing simple mathematics calculation especially addition and multiplication. Table 1 below listed the problems in each item in the test for different types of operations.

Table 1. Problems for each item and the types of operations included

Item Number	Problem structure	Problem	Types of operations included
1	Multiple choice question	$9 \div 1 + 3 \times 0$	Division, addition and multiplication
2	Multiple choice question	$8 + 2 \times 2 - 6 - 1^2$	Addition, multiplication, subtraction and exponent
3	Multiple choice question	$9 + 3 \times (6 - 2) - 8 \div 4$	Addition, multiplication, bracket, subtraction and division
4	Multiple choice question	$3 + 9 \div 3 + 1$	Addition and division
5	Multiple choice question	$14 - 3 \times 2 - 5$	Subtraction and multiplication
6	Multiple choice question	$(3 + 2) \times 2 \div (1 + 9)$	Brackets, addition, multiplication and division
7a	Inserting brackets	$3 \times 4 + 5 - 15 = 12$	Multiplication, addition and subtraction
7b	Inserting brackets	$4 + 8 - 2 \div 2 + 3 = 10$	Addition, subtraction and division
8	Justification of answer	9 + 2 x 6 = 66	Addition and multiplication
9	Comparison	$5 \times 2 - 2 \times 4$ or $5 \times (2 - 2) \times 4$	Subtraction, multiplication and bracket

Each problem in the test papers has different types of operation included. There were four different structures of problems in all ten items. For example, items 1 to 6 were designed as multiple choices questions. Various operations were included in the six items and students were required to find solutions for each answer by selecting one of the options in the answers provided and must show all the workings. However, there were two problems in item 7 where students were required to insert bracket(s) in each of the problem to make the answer true.

The students were required to provide their justifications in answering item 8 where they had to show whether it is true for the following answer. This problem focused on the ability of the students in explaining the validity of the expression. Meanwhile for item 9, it was designed to compare which of the two following expressions were bigger compared to the other. Therefore the students had to show their workings for each expression and circle the correct answer.

For the marking scheme, the students were required to evaluate each of the problems in a number of steps. Each item was scored based on a given rubric, on a 0-2 point scale. It was essential for the students to show their workings and have correct answers with the appropriate use of the correct order of operations to earn the two points. However, one point was given only when they showed acceptable workings even though the answers were not exact. Otherwise, the students will get zero point for incorrect answers or no attempt in the problem.

Audio-Recorded Interview

A semi-structured interview was conducted with selected participants from the study sample. The purpose of this interview was to extract more information, ideas from the interviewee, and interviewing is also appropriate to explore detailed insights from the participant (Gill et al., 2008). The use of audio-recording interview was to avoid biasness and it remained as a permanent record. This also helps to make notes after the interview and when analyzing the data (Gill et al., 2008).

The selected students for the interview were the ones who had improved in their pre-test and post-test. The interview, conducted by the first author, was around 10 to 15 minutes for each participant and the students were interviewed after they sat for the second post-test. During the interview, the students were asked whether they had encountered the order of operations before Year 9. The students were also asked what they understood during the intervention lessons and how their skills developed after the intervention. Throughout the interview, references were made to the pre- and post-tests of the participants, and asked about how they attempted the questions in the tests. The students were also questioned the difficulty in remembering two approaches, the mnemonic they had learned before and the recently hierarchy-of-operators triangle. In the interview, the students were also requested to point out the easiest and hardest problems.

Results and Discussions

The results of the pre- and post-tests of the students were analyzed quantitatively using the Statistical Package for the Social Science (SPSS v20) tool. Meanwhile, the information collected during the interview was combined and analyzed for any emerging patterns.

The results between the participants' pre- and post-tests scores were compared. A paired t-test was used to determine the significance at the 0.05 level. Table 2 shows that the students greatly improved during the second post-test where the mean of the second post-test (13.43) was greater than the mean of the pre-test (6.14).

Table 2. Mean and standard deviation of the achievement tests

	Mean	Standard deviation
Pre-test	6.14	4.767
Post-test 1	8.71	7.072
Post-test 2	13.43	6.408

Using paired sample t-test, the p-value of 0.000 supports the fact that there is a significant difference between the second post-test and the pre-test of the students ($p = 0.000 < 0.05$). Although the results in Table 2 showed that the mean of the first post-test (8.71) was greater than the mean of the pre-test, the results did not show any statistically significant difference since the p-value of $0.127 > 0.05$.

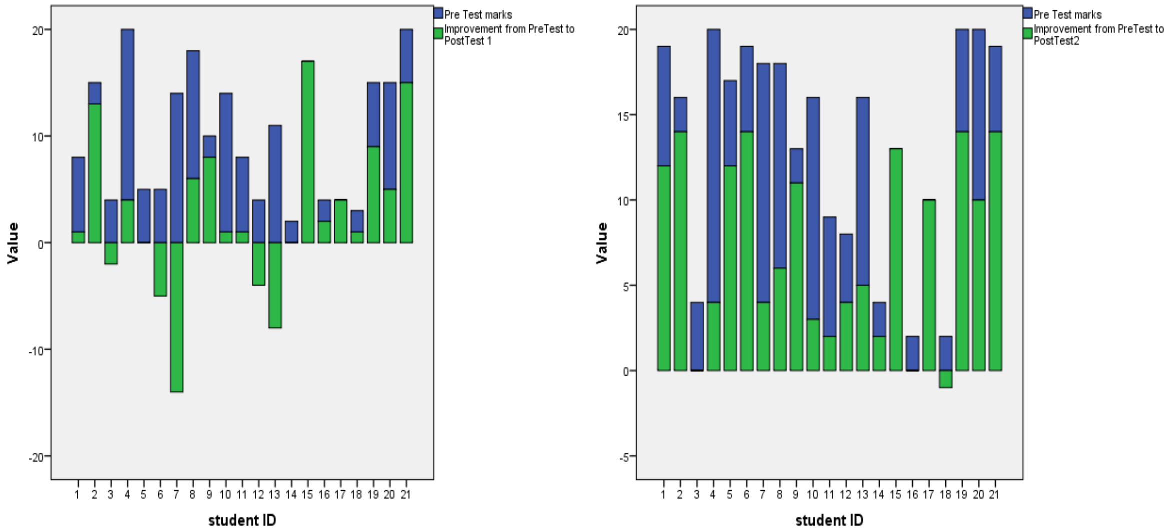

Figure 2. Graphs of students' marks obtained during pre-test and the improvements for the post-tests

Figure 2 above shows the graphs between pre-test of the students and the improvements achieved for the first and second post-tests. There were greater improvements in the second post-test compared to the first post-test. Additionally, three students did not improve during this study. The analyses from the students' pre-test revealed several types of common mistakes made by most of the students, and these are given below.

Mistake No. 1: Evaluating the numerical expression by performing the operations from left to right.
The common mistakes the majority of students made during the pre-test were performing the arithmetic operations for all items from left to right. An example for item 4, most of the students chose the answer 5 to the problem, $3 + 9 \div 3 + 1$, whereas the correct answer is 7. This shows that most of the students did not remember how to perform the order of operations. Students performed the addition of 3 and 9 first, then divide the answer by 3 and finally add the answer by 1.

Mistake No. 2: Multiplying any number by zero.
The focus of item 1 was to check whether students were able to master the multiplication table without the use of calculator. The problem for item 1, $9 \div 1 + 3 \times 0$, was one of the tricky question where it tested the order of

operation and also the multiplication of any number by zero. Most of the students who attempted this question in the pre-test arrived at the answer of either 12 or 0. Only a few of them got 9, which is the correct answer. Some students chose 12 and 0 as their answers either evaluated the expression from left to right basis or multiplying any number by zero and having the number itself as the answer (refer to Figure 3). In this case, the students were limited to the multiplication table and with no access to the calculator to help them in calculating the numbers.

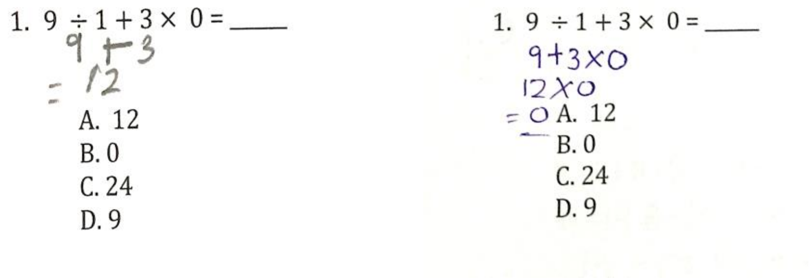

Figure 3. The responses by student A and student B in their pre-test

Mistake No. 3: Exponents.

The focus of item 2 was to check for the students' ability in evaluating expressions involving exponents. We predicted that there may be few students considering 1^2 as 2. However, some of the students did not manage to express 1^2 as 1 x 1 but rather thought as 1 + 1. From Figure 4, student C did not write 1^2 as 2, but from the answer written, it is obvious that student C subtracted the 2 from 14.

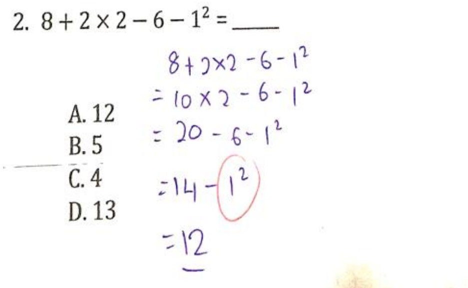

Figure 4. The response by student C when evaluating exponents for item 2

All of the students did not manage to answer item 8 correctly as it required for their justifications. The students may be having difficulty in understanding the question or they thought that 'how Adam arrived to that answer' so they showed the working instead of proving whether the answer is correct or incorrect. Due to careless errors by the students, they lost some of the marks in the test. Shown in Figure 5 is an example of a careless error student D made during the test when subtracting 5 from 8 as shown in the working.

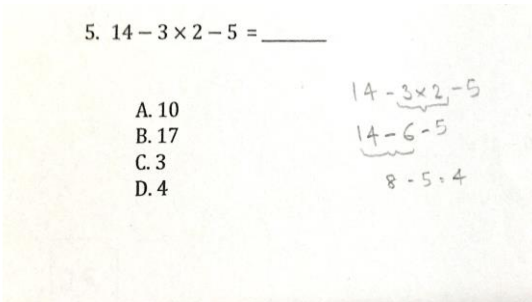

Figure 5. Simple mistake made by student D in performing subtraction for item 5

From the analyses of the post-tests, most of the students had better improvements in writing down the workings for each problem. Student E (see Figure 6) drew the triangle just to help him when evaluating the numerical expression. In Figure 6 also, student E underlined the numbers he wanted to operate first before the other. Meanwhile, student F also did the step-by-step workings while performing the order of operations (refer to Figure 7).

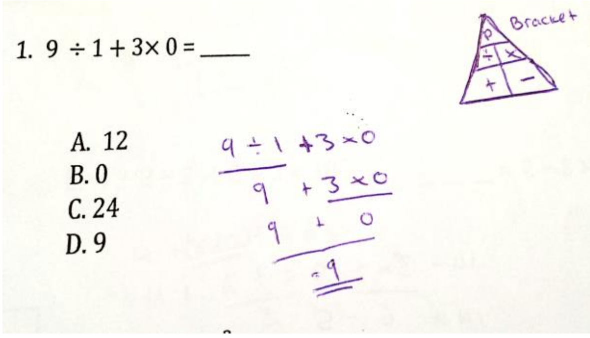

Figure 6. Student E workings for item 1

During the interview, the students were asked if they had come across the topic order of operations before entering Year 9. Most of the students could not remember whether they were taught the order of operations before, but they managed to recall when the mnemonic BODMAS was mentioned, and this concur to the study by DeLashmutt (2007). The students also took their time in explaining what each letter in BODMAS stood for. The students were also asked whether they understood what was taught during the intervention lessons and how their skills developed after the intervention. Some of the students were not able to concentrate during the lessons due to several distractions, such as students making noises at the back of the class. All of the interviewees responded that they remembered the hierarchy-of-operators triangle better than the mnemonic, BODMAS.

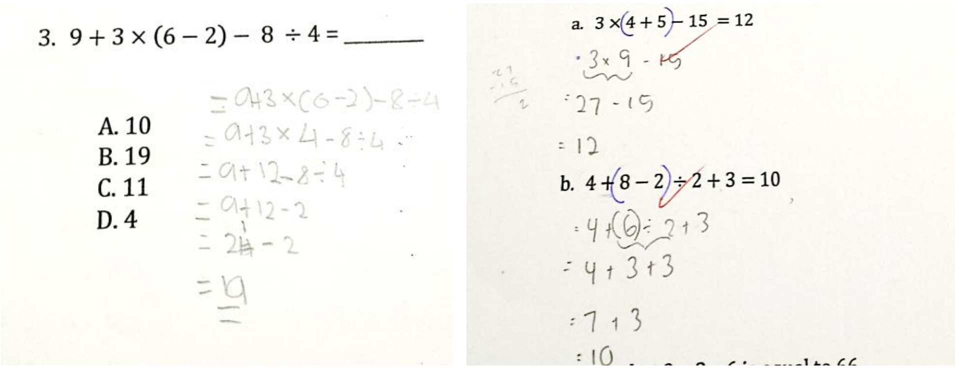

Figure 7. Student F's responses extracted from the post-test

Conclusions

To answer the first research question, there were improvements in the students' results from the pre- and post-tests in the study. The students could perform better when evaluating numerical expressions involving the order of operations. It was observed that the students also improved in the way they wrote their work solutions as it was clearer and easier to understand. Upon completing the study, around 80% of the students followed the hierarchy-of-operators triangle. Some of the students still struggled in computing simple calculations. Meanwhile, for the second research question, most of the interviewed students responded that it was easier for them to remember the triangle rather than using mnemonic as a tool to recall the order of operations. In addition, the students were also able to develop conceptual knowledge when applying the triangle to evaluate numerical expressions.

The common mistakes made by the sampled students when performing the order of operations were also similar mistakes detected in other mathematical problems identified in previous literature. Concurring with Headlam and Graham (2009), the students with strong foundation in conceptual knowledge of order of operations may have the potential to reduce the misconceptions within the topic itself. Reducing misconceptions in this case will help students be more confident in solving algebraic expressions involving the order of operations. For every level, it is important that students be given continuous practice and consistent review of the topic in order to avoid forgetting the concept. Hence, by exposing this approach to students, it may give them clear explanations on how the multiplication or division has more precedence than the addition or subtraction.

Limitations of the Study

One of the limitations for this study is the ability of the participants. Having a low ability and weaknesses in Mathematics would take longer time to solve mathematical problems or equations if they were weak in integers and fractions. Topics such as integers and fractions are important as it relates to the associative and commutative properties and weaker students may not be able to connect the relationship of addition and subtraction as the associative properties. The other limitation is that most of the participants in this study have difficulties in understanding and expressing in English Language, since it is their second language (refer also to Pungut and Shahrill, 2014). So understanding the story telling approach in this study will be very limited since English language was used as the medium to convey the story line.

Recommendations

Since this study was only conducted within a short period of time, it was not sufficient to see the developmental progress of the mathematical skills by the individuals. The mathematical skills include not only the skills in performing the order of operations, but also their skills in manipulating the expressions and their pre-algebra skills. As a recommendation, this approach may work if mnemonic was exposed and taught first to the elementary students. This approach may also develop the students' conceptual understanding and it will continue as long as the procedural and conceptual are continually practiced (Law et al., 2015; Tan & Shahrill, 2015). Carpenter and Lehrer (1999) stated that, "development of understanding is ongoing and continuous process" (p. 31). This implied that a study to investigate the development of students' skills must be long term and it must be continuous in order to check on their progress. Students' learning styles and strategies in studying varies (Matzin et al., 2013; Shahrill et al., 2013; Othman et al., 2016) and some students may not be able to grasp the ideas quickly, but if they were given more time they may be able to do so.

References

Ameis, J. A. (2011). The truth about PEDMAS. *Mathematics Teaching in the Middle School*, *16*(7), 414-420.

Banerjee, R., & Subramaniam, K. (2005). Developing procedure and structure sense of arithmetic expressions. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, (pp. 121-128). Melbourne.

Bautista, G. (2012). PEMDAS and the common errors in the order of operations. Retrieved from http://mathandmultimedia.com/2012/08/26/pemdas-common-errors/

Brannen, J. (2007). Mixing methods: The entry of qualitative and quantitative approaches into the research process. *International Journal of Social Research Methodology*, *8*(3), 173-184.

Carpenter, T., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema, & T. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp.19-32). Mahwah, NJ: Lawrence Erlbaum Associates.

Cohen, L., Manion, L., & Morrison, K. (2007). Research methods in education. Madison Avenue, NY: Routledge.

DeLashmutt, K. (2007). A study of the role of mnemonics in learning mathematics. Summative Projects for MA Degree. University of Nebraska-Lincoln.

Gill, P., Stewart, K., Treasure, E., & Chadwick, B. (2008). Methods of data collection in qualitative research: Interviews and focus groups. *British Dental Journal, 204*(6), 291-295.

Glidden, P. L. (2008). Prospective elementary teachers' understanding of order of operations. *School Science and Mathematics*, *108*(4), 130-136.

Headlam, C., & Graham, T. (2009). Some initial findings from a study of children's understanding of the order of operations. In M. Joubert (Ed.), *Proceedings of the British Society for Research into Learning Mathematics (37-42).*

Jeon, K. (2012). Reflecting on PEMDAS. *Teaching Children Mathematics, 18*(6), 370-377.

Joseph, K. (2014). *College students' misconception of the order of operations*. Unpublished doctoral dissertation, State University of New York at Fredonia, Fredonia, New York.

Kalder, R. S. (2012). Are we contributing to our students' mistakes? *The Mathematics Teacher*, *106*(2), 90-91.

Kemmis, S. & McTaggart, R. (1992) The action research planner (2nd ed.). Geelong, Vic.: Deakin University Press.

Law, F. F., Shahrill, M., & Mundia, L. (2015). Investigating students' conceptual knowledge and procedural skills in the learning of college level trigonometry. *Research Journal of Applied Sciences, Engineering and Technology*, *9*(11), 952-962.

Lee, J. K., Lickwinko, S., & Taylor-Buckner, N. (2013). Exploring mathematical reasoning of the order of operations: Rearranging the procedural component PEMDAS. *Journal of Mathematics Education at Teachers College, 4*(2), 73-78.

Matzin, R., Shahrill, M., Mahalle, S., Hamid, M. H. S., & Mundia, L. (2013). A comparison of learning styles and study strategies scores of Brunei secondary school students by test anxiety, success attributions, and failure attributions: Implications for teaching at-risk and vulnerable students. *Review of European Studies*, *5*(5), 119-127.

Ministry of Education. (2013). The national education system for the 21st century: SPN21 (Revised ed.). Ministry of Education, Brunei Darussalam.

Nelson, D. (2013). Action research: An appropriate research paradigm for practitioners. *Language in India*, *13*(10), 183.

Othman, R., Shahrill, M., Mundia, L., Tan, A., & Huda, M. (2016). Investigating the relationship of student's ability and learning preferences: Evidence from year 7 mathematics students. *The New Educational Review*, 44(2), 125-138.

Pungut, M. H. A., & Shahrill, M. (2014). Students' English language abilities in solving mathematics word problems. *Mathematics Education Trends and Research*, [Online] 1-11.

Shahrill, M., Mahalle, S., Matzin, R., Hamid, M. H. S., & Mundia, L. (2013). A comparison of learning styles and study strategies used by low and high math achieving Brunei secondary school students: Implications for teaching. *International Education Studies*, *6*(10), 39-46.

Tan, J. P. H., & Shahrill, M. (2015). Students' learning of college level calculus. In C. Vistro-Yu (Ed.), *In Pursuit of Quality Mathematics Education for All: Proceedings of the 7th ICMI-East Asia Regional Conference on Mathematics Education* (pp. 303-312). Quezon City: Philippine Council of Mathematics Teacher Educators (MATHTED) Inc.

Vanderbeek, G. (2007). Order of operations and RPN. *MAT Exam Expository Papers*, 46.

Yahya, N., & Shahrill, M. (2015). The strategies used in solving algebra by secondary school repeating students. *Procedia – Social Behavioural and Sciences*, *186*, 1192-1200.

Yasmeen, G. (2008). Action research: An approach for the teachers in higher education. *TOJET: The Turkish Online Journal of Educational Technology*, *7*(4), 46-53.

Author Information

Ernna Sukinnah Ali Rahman
Sultan Muhammad Jamalul Alam Secondary School, Ministry of Education
Jalan Sekolah, Mabuhai, BA 1311, Bandar Seri Begawan, Brunei Darussalam

Masitah Shahrill
Sultan Hassanal Bolkiah Institute of Education, Universiti Brunei Darussalam
Tungku Link Road, Gadong, BE 1410, Bandar Seri Begawan, Brunei Darussalam
Contact e-mail: *masitah.shahrill@ubd.edu.bn*

Nor Arifahwati Abbas
Sultan Hassanal Bolkiah Institute of Education, Universiti Brunei Darussalam
Tungku Link Road, Gadong, BE 1410, Bandar Seri Begawan, Brunei Darussalam

Abby Tan
Faculty of Science, Universiti Brunei Darussalam, Brunei Darussalam
Tungku Link Road, Gadong, BE 1410, Bandar Seri Begawan, Brunei Darussalam