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Abstract

The aim of the study is to examine whether classroom teachers' Pedagogical Content Knowledge (PCK) on numbers develops in the period from their university education to their active teaching profession. Cross-sectional comparative study method was used in this research in order to examine what kind of development classroom teachers' pedagogical content knowledge exhibited from their university education to the period in which they actively operate within teaching profession. The sample of the study is composed of 164 pre-service teachers and 53 in-service teachers working in different primary schools in the province. The Mathematics Pedagogical Content Knowledge Test (MPCKT) developed by Kwong, et al. (2007) was used as a data collection tool. According to the results of the study, it was observed that knowledge levels of the prospective teachers in terms of PCK sub-components has not developed as directly proportional depending on the class level and teaching profession.

Introduction

Studies were carried out on teacher's training in many developed countries of the world with the implementation of teaching-related reforms (Cochran-Smith & Fries, 2008; Lin, Wang, Klecka, Odell & Spalding, 2010). In this direction, teacher training and the capabilities they must possess have gained weight (Bolat & Sözen, 2009; Meriç& Tezcan, 2005). When the capabilities of a good teacher are considered, content knowledge comes to the fore (Appleton, 2003; Schempp, Manrooss & Tan, 1998; Tanışlı, 2013). The selection of effective learning activities depends on many teaching activities such as asking productive questions and assessing the learning of the students, the teacher's knowledge on the subjects he/she will teach the students, i. e. a strong content knowledge (Ball & McDiarmid, 1990). It is indicated that teachers who have inadequate content knowledge can transmit their deficient knowledge to their students, may fail to change the misconceptions or mistakes of the students and cannot use written sources critically (Hashweh, 1987; Käpyla, Heikkinen, & Asunta, 2009).

A teacher's content knowledge has a significant role during the teaching process. However the fact that teachers have sufficient content knowledge on a subject does not mean they can teach this subject effectively (Kahan, Cooper, & Bethea, 2003). Teachers must also have adequate knowledge about how to teach a lesson, how to transmit it to their students and to be aware of level of the students (Bator & Bali, 2013). Thus, adequate content knowledge is not sufficient on its own for effective teaching (Tamils, 2013). For this reason, many researchers have focused on how teachers teach a field and reflect their content knowledge during the teaching process in addition to having adequate content knowledge (Cankoy, 2010; Gökkurt, 2014; Gökkurt & Soylu, 2016a, 2016b; Gürbüz, Erdem, & Gülburnu, 2013; Hill, Rowan, & Ball, 2005; Shulman, 1986,1987;Tchoshanov, 2011).

In addition to the adequate content knowledge, teachers must have various types of knowledge that teachers need to know different representations of the concepts, allow students to understand the concepts taught in the best way, and also enable them to understand the mistakes that the students make. This knowledge is defined as a term named Pedagogical Content Knowledge (PCK) created by Shulman (1986). PCK is among the requirement that teachers must have (Kleickman et al., 2013). PCK is a type of knowledge that includes the different representation of concepts, the strongest analogies, examples, descriptions and explanations that will ensure the understanding of a subject (Shulman, 1986).

After pedagogical content knowledge was suggested by Shulman, the researchers have addressed pedagogical content knowledge with a deductive approach and focused on the components of PCK. The studies carried out show that researchers cannot reach settlement on the components of the PCK, and express these components in different manners (Carlsen, 1999; Fermandez-Balboa & Stiehl, 1995; Geddis, 1993; Gökkurt, 2014; Grossman, 1990; Kaya, 2009; Loughran, Berry, &Mulhall, 2006; Magnusson, Krajcik, & Borko, 1999; Marks, 1990; Shulman, 1987; Smith &Neale, 1989; Tamir, 1988). It was seen that Shulman (1987) emphasises the components of *the knowledge of understanding students* and *knowledge of instructional strategies* among the component of PCK. And also many researchers take these two components Shulman (1987) introduced as a subcomponent of PCK in their PCK model. The theoretical structure of these two components is briefly mentioned below as it is thought that they will contribute to the interpretation of the data obtained from the research.

*Knowledge of instructional strategies:*Knowledge of intructionalstrategies is defined as the ways of demonstration and explanation used in the understanding of the concepts and ideas (Shulman, 1987), and includes the teacher's knowledge about the ways of reppresentation of specific concepts and principles in making teaching easier (Davis & Petish, 2005; Magnusson, Krajcik, & Borko, 1999). I. e., it is about answering the question "If it was you, how would you teach?" (Gökbulut, 2010). Within the scope of this explanation, it includes teachers' and prospective teachers' knowledge on teaching strategies and the methods, technical and strategical knowledge used in the teaching domain of numbers.

Knowledge of understanding students (student knowledge): It includes the prior knowledge, learning difficulties, mistakes of the students on the subject, and their underlying reasons (Shulman, 1987). It addresses the conditions required for the students' getting a particular knowledge, and teacher's knowledge of potential learning difficulties of the students regarding the concepts (Özel, 2012). In line with this definition, the knowledge of understanding students was assessed as determining the mistakes that prospective teachers make concerning the learning domain of numbers.

Starting from the importance of these two components, *the knowledge of understanding students and instructional strategies* was taken into consideration in this research in this study it was aimed to determine how the knowledge of these two components of classroom teachers develops in the period from their training at the university to the period of active work as teachers. The selection of the teaching domain of numbers in the study can be justified by including it into Primary School Mathematics Curriculum (Ministry of National Education [MoNE],2015) at the highest rate and extent.
.
In Turkey, the studies about the development of pedagogical content knowledge of the teachers were carried out on science teachers (Karal & Alev, 2016; Özel, 2012) and mathematics teachers (Şahin, et al., 2015). Studies on the pedagogical content knowledge development of classroom teachers were not encountered a lot. Nevertheless, the research of the pedagogical content knowledge of classroom teachers is important. For, primary school students experience difficulties in mathematics lesson, and they especially encounter hardships in many subjects related to the learning domain of numbers such as fractions (Erdem, 2015; Pesen, 2007), and *decimal numbers* (Resnick et al., 1989; Steinle, 2004). Also, classroom teachers have a significant role in the elimination of these difficulties. In this sense, it is considered that the knowledge of understanding students and knowledge of instructional strategies of the teachers are taken to the desired level by taking the necessary precautions in order for the deficiencies of classroom teachers with regard to their knowledge of understanding the students and teaching strategies with regard to the learning domainof numbers in the elimination of these difficulties in this study.Thus, the results obtained from this study are important because they are expected to contribute to teacher education literature, and can fill the gap in this area to a particular extent.

Problem of the Research

The objective of this study is to determine whether the pedagogical content knowledge of classroom teachers develops during the process since their preservice teacher education at the university to the period when they actively work as teachers. In line with this objective, the research questions shedding light on the study are as follows:

1. What is the level of the knowledge of understanding students of the third and fourth-grade prospective classroom teachers and classroom teachers regarding the learning domain of numbers?
2. What is the level of the knowledge about the instructional strategies regarding the learning domain of numbers possessed by the third and fourth-grade prospective classroom teachers and classroom teachers?
3. Is there a statistically significant relationship between the components of the knowledge of understanding students and teaching strategies of teachers and prospective teachers?

Method

Cross-sectional comparative study method was used in this research in order to examine what kind of development classroom teachers' pedagogical content knowledge exhibited from their university education to the period in which they actively operate within teaching profession. A longitudinal study, a group of subject are studied for a long time, but in a cross-sectional comparative study, different groups which are equal examined at the same time (McMillan & Schumacher, 2010). So we used cross-sectional comparative study method in this research because of limitation of longitudinal study. Attitudes, beliefs, opinions and behaviours of two or more groups are compared in cross-sectional studies. These group comparisons may be between students and students, between students and teachers, and between students and families (Creswell, 2012). It is easy to generalize the results to the population as by taking a broader sample can in such studies (McMillan & Schumacher, 2010). Also the correlational method was used for the third sub-problem. Furthermore, quantitative data obtained from the study were supported by direct quotations from the answers of the participants to the PCK test were included in the study.

Sample

The participants of the study were composed of 217 people in total who were 77 third-grade and 87 fourth-grade classroom teachers, and 53 classroom teachers serving in different primary schools in the province. The service time of the teachers was addressed as 5-10 years, 10-15 years and above 15 years. Within the frame of the study ethics, the real names of the preservice teachers and teachers who participated in the study have not been used. The third-grade prospective teachers who participated in the study were assigned with the codes from $T_1$to T_{77}, the fourth-grade prospective teachers from $T_{78}$to T_{164} and the classroom teachers from $T_{165}$to $T_{217.}$ Attention was paid that these groups are equivalent groups by taking into consideration that third and fourth-grade prospective classroom teachers in this sample study at the same university, and that their academic achievement level at the university and their scores of the university entrance exam are similar. On the other hand, it was tried to create equivalent groups by considering that the entrance scores of the teachers into the university that they graduated are similar to scores of prospective teachers and that the environment where they were raised, and their social cultures are similar. Prospective first and second classroom teachers were not included in the sample of this study. The reason for this can be indicated as lessons contributing to the development of pedagogical content knowledge such as *Mathematics Teaching, School Experience and Teaching Practice* are not included in the first and second grades of the Classroom Teaching programme.

Data Collection Tool

In this study *Mathematics Pedagogical Content Knowledge Test (MPCKT)* was used as a data collection tool to to determine pre-service mathematics teachers' and pre-service classroom teachers' levels of pedagogical content knowledge on numbers. MPCKT(APP-1) was taken from the study named *‘”Development of Mathematics Pedagogical Content Knowledge in Student Teachers”* prepared by Kwong et al. (2007). Although there are totally 16 questions in the MPCKT, only eight questions are about the learning domain of numbers. So eight item of MPCK regarding numbers was used for the purpose of study. The other eight questions were eliminated because of being related to the learning domain of geometry. The learning domains and objectives of the questions in the mathematics pedagogical content knowledge test were summarized in Table 1 below.

Table 1. Items of the MPCK instrument

Mathematics Pedagogical Content Knowledge Test	**Learning Domain**	
	Natural Numbers	Fractions
Mathematics Knowledge of the Teacher	Question No. 1	Question No. 5
Different Representations of the Concepts	Question No. 2	Question No. 6
Configuration of the Mathematics Knowledge of the Students	Question No. 3	Question No. 7
Detection – Elimination of Student Mistakes	Question No. 4	Question No. 8

The first four questions in Table 1 are about teaching natural numbers. The first question in this test is about measuring teacher's knowledge on natural numbers, the second question is about representing the concepts in different ways, the third question is about how to teach concepts about natural numbers with constructive approach, and the fourth question is about determining and eliminating student mistakes. The last four questions in MPCKT are about teaching fractions. The fifth question is about measuring teachers' knowledge on fractions, the sixth question is about representing the decimal concept in different ways, the seventh question is teaching concepts about fractions with constructive approach, and the eight question is about determining student's mistake about percentages.

The Mathematics Pedagogical Content Knowledge Test was translated into Turkish by a linguistic and mathematics expert. Adapted version of MPCKT was examined by four academicians to determine the validity of the test. According to opinions of experts some changes were made in the test items. MPCKT ıtems were arranged in such a way that they can be understood better by prospective classroom teachers and classroom teachers by taking tl instead of dollars, and 12.5% instead of $12\frac{1}{2}$%, etc. The data collection tool turned into its final version, and it consisted of eight open-ended questions in total. *MPCKT* formed was applied to prospective teachers and teachers in the sample. The data obtained were first scored by the researchers. Then it was scored again by another researcher in order to determine the interscorer reliability. The interscorer reliability was calculated as 84%.

Data Analysis

One Way Analysis of Variance (ANOVA) was used in the analysis of the quantitative findings obtained from MPCKT. The equivalence of the variances of the groups determined was tested with Levene statistics and it was shown that the variances are homogeneous ($p>.05$). In this case, the necessary precondition for performing One Way Variance Analysis (ANOVA) was fulfilled. Correlation analysis was carried out in order to determine whether there is a statistically significant relationship between the Knowledge of Understanding Students (KUS) and Knowledge of Instructional Strategies (KIS) components of the teachers and prospective teachers.

First, a normality test was performed in order to determine whether the data were continuous, and Pearson Product-Moment Correlation Coefficient was calculated upon seeing that the data had normal distribution with $p>.05$ as a result of the test. Furthermore, quantitative data obtained from the study were supported by direct quotations of the participants' answers to MPCKT. While choosing quotations from prospective teachers and teachers, it was taken into account that the prospective teacher and teacher were equivalent individuals, i. e. that their university entrance scores were close to each other, the environments and cultures where they were raised are very close, etc.

While analysing the data, the first, fourth, fifth and sixth questions were examined in the context of the knowledge of understanding students and the knowledge of instructional strategies, which are two components of pedagogical content knowledge. The third and eight questions were examined only in the context of the knowledge of understanding students while the second and seventh questions were exclusively examined in the context of the knowledge of instructional strategies. The scoring categories of each item in MPCKT are shown in Table 2.

Table 2. Scoring categories of MPCKT

Scoring Categories	**Completely True**	Partially True (a)	Partially True (b)	Wrong Answer	No Answer
Score V	4	3	2	1	0

As is seen in Table 2, 4-point rubric (Kwong et al., 2007) was used in the analysis of the answers of the teachers and prospective teachers to questions in MPCKT. These are;

Completely True: It is the answer that includes all of the scientific ideas that are deemed correct about the question.

Partially True (a): These are the answers that are close to all of the scientific ideas that are deemed correct about the question, but those that have minor mistakes.

Partially True (b): These are the answers that are close to incorrect, in which the scientific ideas that are deemed correct about the question are included slightly.

Wrong Answer: These are answers that lack the scientific ideas that are deemed correct about the study, and that are irrelevant to the question.

No Answer: The cases when the preservice teachers cannot give any answer to the ıtems.
According to the rubric used for this test, the maximum and minimum scores that can be taken for each component are shown in Table 3.

Table 3. Minimum and maximum scores for each component of PCK

PCK Components	Questions	Min	Max
Knowledge of Understanding Students	1. 3.4.5.6. 8.	0	24
Knowledge of Instructional Strategies	1.2.4.5.6.7.	0	24

Limitation of the Study

In this study, the point whether there is a development in KUS and KIS, which are the two subcomponents of the pedagogical content knowledge was examined using the cross-sectional research method. However, no interviews were held for obtaining data on why the pedagogical content knowledge levels of teachers do not change. Accordingly, longitudinal studies that examine the behaviours of the teachers in the long-term and investigate the development processes and the classroom activities they perform in all development processes through observation and interviews are necessary. We chose the cross-sectional research method because of limitation of longitudinal studies such as time, cost data loss. So limitation of study can be summarized as a lack of interview, observation and using cross-sectional research method instead of longitudinal studies.

Results

In this section, data obtained from MPCKT were presented in the form of a table in order to determine whether the PCK levels of the teachers on *the knowledge of understanding students* and *the knowledge of instructional strategies* developed from the third grade to the period when they worked actively as teachers. Furthermore, quantitative data were supported by direct quotations related to the answers of the participants to the questions. And also percentage values about the components of PCK were included for each question. Data obtained from MPCKT were analysed concerning the component of the knowledge of understanding students, and the results of this analysis were presented in Table 4.

Table 4. Descriptive statistics of KUS

		N	$\bar{x}$	**Sd**	**Std. Error**	**Min.**	**Max.**
KUS score	**3rd-grade prospective teachers**	77	12.08	3.523	.402	3	23
	4th-grade prospective teachers	87	11.93	3.409	.366	2	20
	Teachers	53	12.04	3.710	.510	6	21
	Total	217	12.01	3.509	.238	2	23

When the scores obtained from MPCKT concerning the component of KUS in Table 4 are examined, it is seen that the pedagogical content knowledge of all three groups is not at the sufficient level. However, it is seen that the PCK levels of the prospective classroom teachers studying in the third grade on the component of the knowledge of understanding students are higher when compared to the other two groups ($\bar{x}$ =12.08). The line graph shown in Figure 1 shows this clearly.

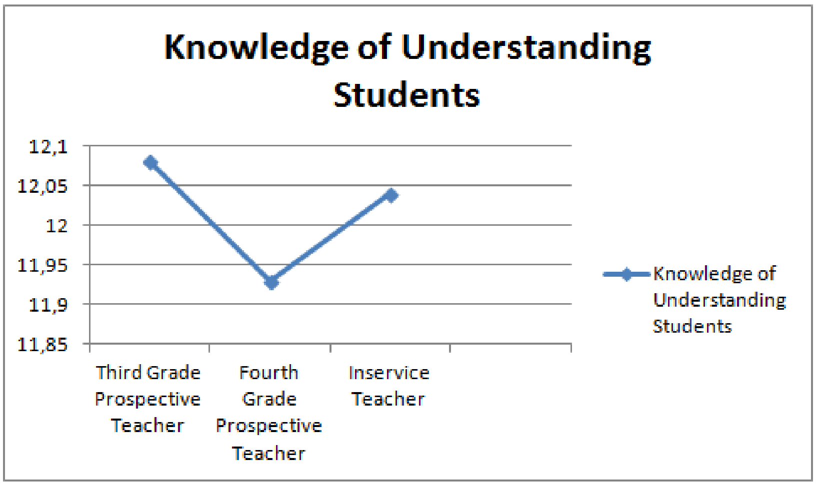


Figure 1. Development of knowledge of understanding students

Also, whether the differences between the average scores taken by these groups from MPCKT are significant was tested with one-way variance analysis (ANOVA), and ANOVA scores are shown in Table 5.

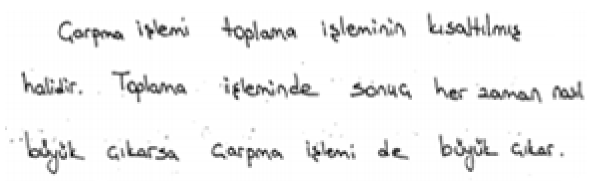
Table 5. ANOVA results of KUS

Source of the Variance	Sum of squares	Sd	Sum of squares	f	p
Between-groups	.938	2	.469	.038	.963
Intro-groups	2659.043	214	12.425		
Total	2659.982	216			

When Table 5 was examined, it was seen that the PCK levels of the teacher concerning the component of the knowledge of understanding students did not develop from the third grade of the university to the time when they worked actively as teachers. In other words there is no significant difference between groups ($F_{(2\text{-}214)}$= .038, p >.05). The quotations below clearly show that both the prospective teacher T_{32} and the teacher T_{168} are not aware of the mistake made by the student.

"Question 1: A pupil tells you that when you multiply two numbers together, the product is always larger than either of the two numbers. How do you respond to the pupil?

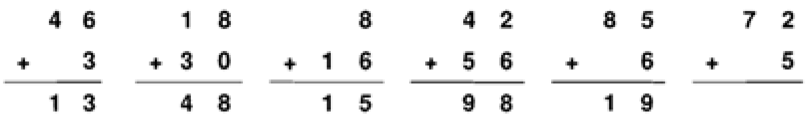
Table 6. Answers of the prospective teacher T_{32} and teacher T_{168} concerning the first question

Participant	Answer to Item 1	Translation of Quotation
T32		*Yes correct. When two numbers multiplied, Product will be greater than these two numbers.*
T168		*The multiplication is abbreviated form of addition. Product is always greater than factors like addition.*

When the instructional explanation in Table 6 was taken into consideration, it was seen that both the prospective teacher and the teacher had difficulty in understanding the mistake of the student. It is seen that both participants agree with and approve the idea of the student rather than expressing his mistake by saying that the multiplication of the two numbers is always greater. However, both participants failed to think of the possibility that the results cannot be greater than zero and one when these numbers are multiplied. Thus, both explanations were assessed in the *wrong answer* code. The only difference between these explanations is that the teacher expressed the reason for her answer when compared to the prospective teacher.

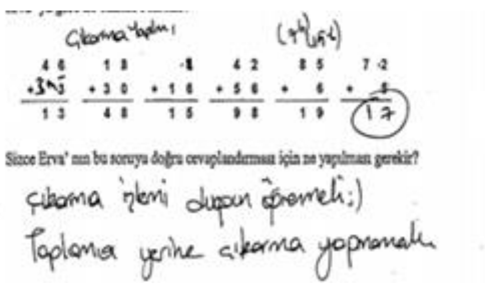
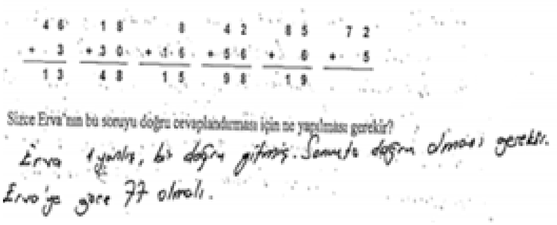
Similarly, both the prospective teacher and the teacher gave similar explanations for the fourth question concerning the addition in natural numbers. The prospective teacher studying in the fourth grade and the teacher that serves actively failed to determine correctly the mistake of the student, and provided an incorrect instructional explanation that is not related to the mistake of the student. The quotations given in Table 6 represent this case in the best way.

"Question 4: Erva gets some of his addition questions correct, but gets some of the simplest ones wrong. Here are five of the questions he did. If Erva makes the same mistake with the sixth question below, in the answer you think Erva would have got.



Give one suggestion that might help Erva get his questions correct."

Table 7. Answers of the prospective teacher T_{81} and teacher T_{175} to the fourth question

Participant	Answer to Item 4	Translation of Quotation
T81		*Student has to learn subtraction properly. Student must not make subtraction instead of addition.*
T175		*Erva gave a wrong answer than a true answer in order. In this respect result should be right. According to Erva ıt should be 77.*

As is seen in Table 7 both the prospective classroom teacher and the classroom teacher failed to understand the mistake the student made when summing up the natural numbers of two digits and one digit, and making addition according to the number value rather than the digit value, and made incorrect explanations that are irrelevant to the mistake. Thus, the answers they gave were assessed with the incorrect code. Table 8 shows the percentage values of the answers given by the teachers and prospective teachers to MPCKT concerning the component of the knowledge of understanding students.

Table 8. Percentage values of the scores of the knowledge of understanding students

	Prospective teachers (3rd-grade)					**Prospective teachers (4th-grade)**					**Teachers**				
			(%)					(%)					(%)		
Score / Item	**4**	**3**	**2**	**1**	**0**	**4**	**3**	**2**	**1**	**0**	**4**	**3**	**2**	**1**	**0**
I1	64.9	2.6	5.2	27.3	-	66.7	5.7	1.1	24.1	2.3	71.7	9.4	-	17	1.9
I 3	38.9	20.8	5.2	35.1	-	40.2	12.6	1.1	42.5	3.4	43.4	9.4	5.7	39.6	1.9
I 4	46.7	-	2.6	33.8	16.9	45.9	1.1	1.1	36.8	14.9	52.8	-	-	30.2	17
I 5	7.8	1.3	-	72.7	18.2	5.7	2.3	2.3	75.9	13.8	7.5	1.9	1.9	77.4	11.3
I 6	5.2	27.3	42.9	14.3	10.4	-	39.1	34.5	16.1	10.3	7.5	13.2	49.1	18.9	11.3
I 8	2.6	1.3	3.9	80.5	11.7	4.6	2.3	5.7	63.2	24.1	1.9	1.9	1.9	56.6	37.7

When Table 8 is examined, it is seen that both the teachers and prospective teachers are the most successful in understanding the student's mistake in the first question, but they have hardship in understanding the student's mistake in the eighth question about fractions. The scores taken by teachers and prospective teachers from MPCKT regarding the component of the knowledge of instructional strategies were analyzed, and the results of this analysis are provided in Table 9.

Table 9. Descriptive statistics of KIS

		N	$\bar{x}$	**Sd**	**Std. Error**	**Min**	**Max.**
KIS score	**3rd-grade prospective teachers**	77	14.22	3.275	.373	7	21
	4th-grade prospective teachers	87	14.22	3.658	.392	6	22
	Teachers	53	13.75	3.817	.524	3	20
	Total	217	14.11	3.557	.241	3	22

When Table 9 is examined, it is seen that the levels of PCK concerning the component of the knowledge of instructional strategies of classroom teachers did not change from university third grade to the period when they worked actively as teachers ($F_{(2\text{-}214)}$= .340, $p > .05$). When the averages are considered, it is seen that the instructional strategy knowledge of prospective teachers is higher when compared to teachers. The line chart in Figure 2 shows that there is no development of the instructional strategy knowledge of the teachers, on the contrary, the scores they took from KUS decrease even more.

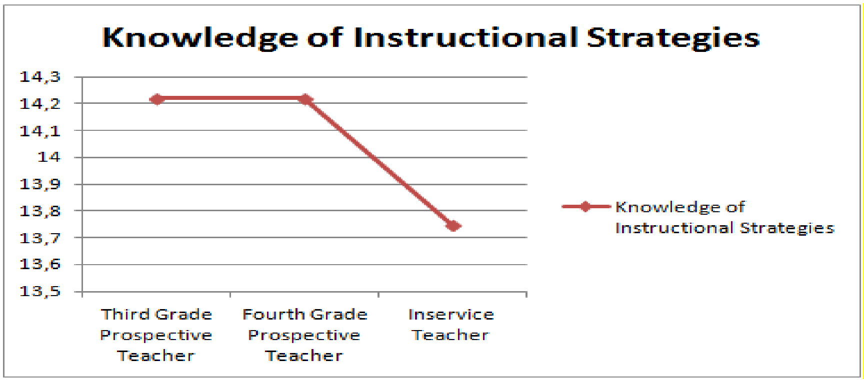


Figure2. Development of knowledge of instructional strategies

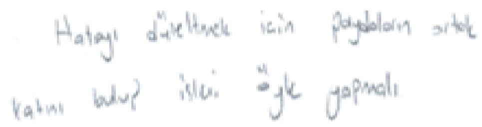
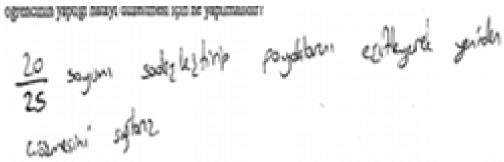
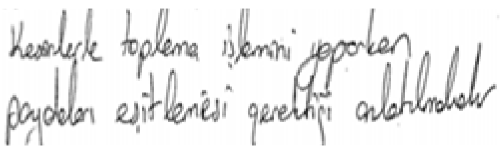
When the written declarations of the participants are examined in detail, it is seen that both prospective teachers and teachers generally make similar statement. The quotations from Table 10 show how inadequate knowledge of prospective teachers and teachers about instructional strategies concerning the elimination of student's mistake is.

"Question 5: A child approaches you with her examination results. She scored 11 out of 15 for paper 1, and she scored 20 out of 25 for paper 2. She writes both results as fractions, as usual, and carries out the following computation:

$$\frac{11}{15} + \frac{20}{25} = \frac{31}{40}$$

She says that she is puzzled because she showed this calculation to her elder brother who said that it was incorrect, yet nevertheless she was given 31 out of 40 on her report card. How do you resolve this situation for the girl?

Table 10. Answers of the prospective teachers T_{37}, T_{95} and teacher T_{205} to the fifth question

Participant	Answer to Item 5	Translation of Quotation
T37	 Hatayı düzeltmek için paydaların ortak katını bulup işlemi öyle yapmalı	*Student has to find common denominator to eliminate mistake.*
T95	 $\frac{20}{25}$ sayısını sadeleştirip paydaları eşitleyerek yeniden cevabını	*I can make student solve this problem by finding common denominator after simplifying $\frac{20}{25}$.*
T205	 Kesirlerle toplama işlemini yaparken paydaların eşitlenmesi gerektiği anlatılmalıdır	*It has to be taught that Finding common denominator is compulsory when adding fractions.*

When Table 10 is examined, neither the prospective teacher nor the teacher made any logical explanation concerning the correction of the student's mistake. The mistake in the question was that instead of transforming the expressions that are not of the same unit into fractions, the student proportioned the total number of correct questions to the total number of questions and found $\frac{31}{40}$. However, neither the prospective classroom teacher nor the classroom teacher was able to understand the real reason underlying the mistake of the student. They focused on denominator equalization process that the student failed to do. Based on this, they put forth inappropriate presentation information by resorting to incorrect instructional explanations.

Correlation analysis was carried out in order to determine whether there is a statistically significant relationship between the components of the knowledge of understanding students and the knowledge of instructional strategies of the teachers and prospective teachers. As a result of the analyses carried out, the correlation coefficient between these two subcomponents was calculated as [r=0.56; p= <.05], and a positively significant relationship was found. The quotations in Table 10 support this and show that there is a relationship between the two subcomponents of KUS and KIC. For it is clearly seen that the prospective teacher and teacher who failed to determine the mistake of the student also failed to determine the right strategy to eliminate the student's mistake. Table 11 represents the percentage values of the answers given by teachers and prospective teachers to MPCKT concerning the component of the knowledge of instructional strategies.

Table 11. Percentage values of the scores of the knowledge of instructional strategies

	Prospective teachers (3rd-grade) (%)					**Prospective teachers (4th-grade) (%)**					**In-service Teachers (%)**				
Score / Item	**4**	**3**	**2**	**1**	**0**	**4**	**3**	**2**	**1**	**0**	**4**	**3**	**2**	**1**	**0**
I 1	7.8	57.1	6.5	28.6	-	4.6	63.2	4.6	25	2.3	7.5	58.5	17	17	-
I 2	29.9	59.7	5.2	-	5.2	39.1	45.9	11.5	-	3.4	43.4	47.2	1.9	1.9	5.7
I 4	37.7	7.8	16.9	3.9	33.8	27.6	20.7	21.8	-	29.9	30.2	11.3	22.6	5.6	30.2
I 5	2.6	3.9	7.8	74	11.7	6.9	2.3	9.2	71.3	10.3	5.7	3.8	1.9	79.2	9.4
I 6	64.9	3.9	5.2	1.3	24.7	57.5	5.7	5.7	1.1	29.9	64.2	7.5	1.9	-	26.4
I7	7.8	59.7	22.1	7.8	2.6	5.7	64.4	19.5	6.9	3.4	5.7	33.9	22.6	22.6	15.1

When the findings in Table 11 are examined, it is seen that the knowledge of instructional strategies levels of all three groups is better in question two and six. Nevertheless, it strikes out that both teachers and prospective teachers are more successful in choosing the suitable methods, techniques and strategies regarding question five about fractions when compared to other questions.

Discussion and Conclusion

The development of pedagogical content knowledge regarding the learning domain of numbers of classroom teachers was examined in this study. In this context, important and interesting results were achieved based on the results obtained. One of these is that the knowledge of understanding students of participants concerning the learning domain of numbers was not at a sufficient level. Similarly, in many studies it was found that the knowledge of understanding students level of prospective teachers and prospective teachers concerning many mathematical concepts is not at a sufficient level (Ball, 1990a, 1990b; Gökkurt et al., 2015; Koçak, Gökkurt, & Soylu, 2014a; 2014b; Lubinski, Fox, & Thomason, 1998; Ma, 1999; Nagle &McCoy, 1999; Tanisli & Kose, 2013; Tirosh, 2000). In addition, the knowledge of understanding students did not develop from the third grade of the university to the time they work as teachers. In contrast to this study, knowledge of understanding students has developed as directly proportional depending on the class level and teaching profession (Şahin et al., 2015; Şahin, 2016).

One of the important results of the study is that the knowledge of instructional strategies of participants is not at a sufficient level, and this knowledge did not make any development from the third grade of the university to the time they work as teachers. Direct quotations related to the answers of the participants to MPCKT also support this result. Consequently, prospective teachers are advised by the teachers working in the schools where they attend the *Teaching Practice* lesson requiring practice and included in the teacher education programme to make a lot of practices for the development of instructional strategies. The fact that classroom teachers with different professional experiences did not make any development in any of the two components when compared to their knowledge during university years came out to be quite a striking result. These results show that the PCK knowledge of the teachers in service should be developed. From this result, it can be said that the periodical assessment of the PCK knowledge of teachers as in their training years at the university is required for the emergence of a more qualified teacher profile. On the other hand, the obligatory effort and time allocated to central examinations made in Turkey through multiple choice tests can be shown as the reason for this consequence. Consequently, it can be said that teachers try to turn students into machines who solve tests rapidly rather than teaching them the concepts through different teaching methods. With another expression supporting this judgment, it is possible to encounter many researches that central examinations encourage exam-oriented teaching (Ayres, Sawyer, & Dinham, 2004; Black & Wiliam, 1998; Crooks, 1988; Dori & Herscovitz, 1999; Erdem, 2015; Stiggins, 1999).

In this sense, activities for the development of the components of understanding students and instructional strategies of classroom teachers concerning the learning domain of numbers can be carried out in *Teaching Mathematics* lesson in the teacher education program. For this purpose, it may be ensured that prospective teachers gain insight about student mistakes regarding the learning domain of numbers by using the micro-teaching method. Furthermore, it may be ensured that prospective teachers in *Teaching Practice* lesson are constantly in contact with students by having them make a lot of practices regarding lectures. For, teachers must have professional experience and be aware of how they understand the concepts and their learning difficulties for their pedagogical content knowledge to develop (Jong & Driel, 2004). In addition to this, it may be ensured that they understand the thinking styles of the students by using the clinical interview method on student mistakes with prospective teachers. For example, as a result of the study carried out with six prospective teachers with the aim of determining the role of structured interviews on the development of student knowledge of prospective teachers, Jenkins (2010) indicated that the clinical interview method is effective for prospective teachers to understand the preliminary understandings, misconceptions and thinking styles of the students.

Another aspect that strikes out in the findings obtained from the study is that prospective teachers and teachers have the most difficulty in forming the representation knowledge that is suitable for determining student mistakes and eliminating these mistakes in the fifth and eighth questions about fractions. The result of the study is consistent with the study of Gökkurt, Şahin, Soylu and Soylu (2013). They concluded that the pedagogical content knowledge of prospective classroom teachers regarding the correction of student mistakes on fractions is not at a sufficient level. Relevant researches show that the instructional explanations of prospective teachers concerning the instructional strategies component of PCK in geometry and mathematics are not at the desired level (Gökkurt, Şahin, & Soylu, 2016; Gökkurt, Şahin, Soylu, & Doğan, 2015; Şahin, Gökkurt, & Soylu, 2016). In this context, it is important for prospective teachers to develop their knowledge of instructional strategies that allow determining the appropriate strategies in prospective teachers' understanding the difficulties that students may encounter especially concerning the fraction concept, as well as overcoming these difficulties.

In this study, the point whether there is a development in KUS and KIS, which are the two subcomponents of the pedagogical content knowledge was examined using the cross-sectional research method. However, no interviews were held for obtaining data on why the pedagogical content knowledge levels of teachers do not change. Accordingly, longitudinal studies that examine the behaviours of the teachers in the long-term and investigate the development processes and the classroom activities they perform in all development processes through observation and interviews are necessary. For, when it comes to teacher education, the teachers' pre-service education is emphasized in our country (Azar & Çepni, 1999). Nevertheless, it is seen that the qualifications that teachers must have and their pedagogical content knowledge and its subcomponents are not discussed in teacher education.

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Appendix 1. Items of the MPCK Instrument (Kwong et al., 2007)

Problem 1:
A pupil tells you that when you multiply two numbers together, the product is always larger than either of the two numbers. How do you respond to the pupil?

Problem 2:
One way to illustrate the meaning of division by 3 using *equal sharing* is to tell a story: *"Benny had* 12 *grapes and distributed them equally among his three sons. How many grapes did each son receive?"* Write a story to illustrate the meaning of division using *repeated subtraction.*

Problem 3:
Here are two problems. Do not solve them.
(a) Ali is selling melons at 3 for $5. How much would 9 melons cost?
(b) Leni is selling melons at 3 for $6. How much would 9 melons cost?
Do you think that children in P4 would find these two problems *equally?*
Difficult, or is one *easier* than the other? Explain your answer carefully.

Problem 4:
Timmy gets some of his addition questions correct, but gets some of the
Simplest one's wrong. Here are five of the questions he did. If Timmy makes the same mistake with the *sixth question* below, Jill *in the answer* you think Timmy would have got.

4 6	1 8	8	4 2	8 5	7 2
+ 3	+ 3 0	+ 1 6	+ 5 6	+ 6	+ 5
1 3	4 8	1 5	9 8	1 9	

Give one suggestion that might help Timrny get his questions correct.

Problem 5:
A child approaches you with her examination results. She scored 11 out of 15 for paper 1, and she scored 20 out of 25 for paper 2. She writes both results as fractions, as usual, and carries out the following computation:

$$\frac{11}{15}+\frac{20}{25}=\frac{31}{40}$$

She says that she is puzzled because she showed this calculation to her elder brother who said that it was incorrect, yet nevertheless she was given 31 out of 40 on her report card. How do you resolve this situation *for the girl?*

Problem 6:
When 23 is divided by 4, three possible answers are given
(a) 5.75
(b) *5$*
(c) 5 with remainder *3.*
For each of them, write one story problem for which that answer is most appropriate.

Problem 7:
If you were introducing how to convert a decimal to a fraction, and had to use the following three decimals: 0.2, 0.03, and 0.23, for your introduction which of them would you use first, which second, and which third? Explain your choice.

Problem 8:
A pupil showed the following working to convert $\frac{1}{8}$to %$12\frac{1}{2}$

$$\frac{1}{8}=0.125\times 100$$
$$=\%12.5$$

Do you see anything wrong in what the pupil wrote? If so, explain what it was.