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Abstract

This study investigates the proof skills of gifted 5th-grade students. The study was conducted using the document analysis method, a type of qualitative study. The study included 25 students, 11 females and 14 males, all diagnosed as gifted and attending fifth grade in the center of the city in southern Turkey. The sample of the group was selected by the method of convenient sampling. A sample proof test with 6 items was used as an instrument for data collection. The written and visual data obtained were analyzed using descriptive analysis techniques. Balacheff's proof levels were used to characterize the pupils' responses. According to the findings, 7.33 percent of the students' answers were at Level 4, 13.33 percent at Level 3, 32.66 percent at Level 1, and 46.66 percent at Level 2. The majority of the pupils' answers were deemed to be at Level 2 Crucial Experiment.

Introduction

Reasoning and proof are integral parts of mathematics. It is among the applications that students from pre-school to advanced levels should encounter. Although not directly involved in the curriculum, students reason and prove to justify results and make assumptions (NCTM, 2000; Common Core, 2010). According to research on the function of proof in mathematics education, there is a close relationship between proof and mathematics, the proof is an important factor in the acquisition of mathematical knowledge, and pupils acquire mathematical thinking (Gökyurt, Deniz, Akgün, & Soylu, 2017).With proof teaching, mathematical rules and formulas can become meaningful mathematical concepts for students rather than being just a few symbols. Although the importance of proof and proving is emphasized, studies with proof practices are insufficient in our country. Therefore, there is a need for innovative practices and studies in this field.

What is proof?

The concept of proof is one of the most important concepts which lie at the heart of mathematics and mathematics education (Knuth, 2002; Lee, 2002). Emphasizing the significance of mathematics, some researchers expressed that the name of the mathematics game was proof and if there 'weren't proof, mathematics 'wouldn't exist (Davis & Hersh, 2012). Harel & Sowder (2007) stated that the concept of proof was one of the most important legacies which were left to mankind from the ancient Greek civilization. The foundation of proof tradition is based on Euclid's “Elements” work (Almeida, 2003). It is also defined as a mathematician is the person who proves things about abstract objects such as numbers and geometrical configurations, their relationships and generalizations in the description of a mathematician's job (Garnier & Taylor, 1996).

The teaching of proof is concentrated at the high school and further education levels. Most of the studies that have been conducted in the field of proof teaching do not deal with proof teaching in primary and secondary education (Aylar, 2014). In addition, some studies (Bell, 1976; Fischbein, 1982; Knuth, 2002) indicate that proof in school mathematics is only suitable for students at the upper secondary education level, and secondary school students do not understand and cannot do formal proof.

On the contrary to this approach, the number of studies that propound that teaching of proof should be discussed to start at early age has increased lately (Cyr, 2011; Hanna, 2000; Schoenfeld, 1994). American National Council of Teachers of Mathematics [NCTM] published a book named "Principles and Standards for School Mathematics" in 2000. The book states, "Secondary school curricula must be designed so that all students, from preschoolers to high school students, acquire the ability to understand that mathematical proofs are an important part of mathematics, to draw mathematical inferences and investigate whether those inferences are true, to construct mathematical proofs, to criticize various mathematical arguments, whether or not they provide proofs, and to choose and use various types of proofs.

The Concept of Special Talent

Special talent is defined as a person's high level of performance in abstract thinking and reasoning skills and a person's having an intelligence age higher than normal peers (Gagne, 2004). Renzulli (1978) stated that individuals with special talents have a high level of task awareness and creativity skills, and they also have academic skills above average. Sternberg (2002), on the other hand, expressed that individuals with special talent are those who have analytical intelligence, and creative characteristics, and are practically talented. In addition, he showed that individuals with special talents displayed high performance in all or one of the analytic, creative, and practical fields, and they succeeded by combining all their talents. The high reasoning, analytical thinking, and judgment skills of individuals who were diagnosed as gifted revealed the importance of determining their proof skill levels.

In Turkey, being a gifted individual is defined as learning faster than their peers, being ahead in terms of capacity of creativity, art and leadership, having special academic ability, understanding abstract ideas, and enjoying acting independently in his areas of interest. Gifted students have different learning needs beyond the traditional understanding of education offered in regular classrooms. The nature of these students' abilities requires differentiated learning experiences and opportunities to maximize their potential (MONE, 2016). In the Specially Talented Children and their Education Commission Report of the First Special Education Council (MONE, 1991), which the Ministry of National Education organized, special talent is defined as individuals who are determined to be performing at a higher level than their peers in terms of general and / or special abilities by the field experts.

The Purpose of This Research

When the related literature was reviewed, it was discovered that there were some studies conducted with teachers of mathematics and pre-service mathematics teachers about the topic of proof (Doruk & Kaplan, 2013; Güler & Ekmekci, 2016; İnam & Uğurel, 2016; Öztürk & Kaplan, 2019), while other studies conducted with primary and secondary school students (Arslan &Yıldız, 2010; Aslan, 2007; Aylar, 2014; Çalışkan, 2012; Knuth & Sutherland, 2004; Zaimoğlu, 2012) and some studies carried out with primary school students (Zack, 1999; Komatsu, 2010; Lampert, 1990; Tall, 1999). And also, in the past two decades, there has been increasing interest in providing gifted and talented students with effective educational programs (Lee, Kim, & Lim, 2021).

There were no research on the evidence skills of gifted 5th graders among the accessible sources. It's crucial to figure out the proof talents of gifted children, a group of pupils who have individual distinctions, because they create big results when their educational demands are met. Furthermore, the research could help raise awareness regarding the value of bright and talented students' proof in mathematics instruction. The goal of this study was to look into the proof skills of gifted 5th-grade children in this context. An answer to the following research question was sought for this reason.

- What is the level of proof skill of gifted 5th grade students according to Balacheff's taxonomy?

Method

The research design, participants, data collection tool, collecting and analyzing data were presented in this part.

Design of the Study

The document analysis method, one of the qualitative research designs, was used in this study. The document review method consists of systematic reviewing scanning or evaluating documents in written or visual form. This method is used both as a complementary method to other methods and as a stand-alone method (Bowen, 2009).

Participants

This study was conducted with a total of 25 gifted students, 11 females, and 14 males, in 5th grade in the center of the city in southern Turkey. The sample of the group was chosen according to the convenience sampling technique. In the convenience sampling technique, the researcher chooses a situation that is close by and easily accessible in order to conduct the study quickly and efficiently. This sampling method is not only widely used, but it also produces results that are less generalizable (Yıldırım & Şimşek, 2018).).

Data Collection Tool

A proof test with 6 items which the researchers developed was used as the data collection tool in the study. The proof test was presented to 3 teachers of English and 2 mathematics educators who were experts in mathematics teaching. There are 3 different question structures in the proof test (PT). The students were asked to choose the closest choice to their answers and explain their reasons for the 1st and 2nd questions. They were asked to decide which student was right in the dialogue given in the 3rd question and explain their reasons. The students were requested to show that the statement in the 4th, 5th and 6th questions was always true.

Data Collection and Analysis

The research data was gathered using a document analysis technique. In this study, the students' proof test responses were used as documents. Individual PT was given to each student, and it was collected in one lesson hour. Furthermore, without providing any advice during the implementation, it was attempted to get them to give distinct replies that were independent of one another.

The research data were analyzed with qualitative and quantitative analysis methods, which are descriptive analysis techniques. Descriptive analysis is summarizing and interpreting the research data according to the themes which had been determined in advance (Yıldırım & Şimşek, 2018). The consistency of the answers given by two mathematics educators and one student currently studying at the doctoral level will be tried to be ensured during the data analysis. The answers given to the data collection tool were not labelled as correct or wrong during the evaluation. The 'students' answers were analyzed qualitatively according to Balacheff's taxonomy. The evaluation criteria about the analysis were presented in Table 1.

Table 1. Coding Schema According Balacheff’s Taxonomy

Levels	**Names of Levels**	**Explanation of Levels**
Level 1	Naive empiricism	Naive empiricism is the first type of evidence we come across this hierarchically. It involves, with a small number of examples, inferring the certainty of the truth of a claim (Balacsheff, 1987).The assumption is considered as correct after students verify that it is valid for a few situations. It is mostly the completion of the proof over a single example.
Level 2	Crucial experiment	Crucial experimentation is the process of verifying a claim (Balacsheff, 1987).The student is confronted with the question of generalization while determining the example to validate the result (Miyazaki, 2000; Simon & Blume,1996). The student tries to prove by asking questions and making generalizations. This level is different from level 1. The students are usually aware of the problem. It is the level in which new conceptual associations are started to set up.
Level 3	Generic example	General example; It consists in explaining the reasons for the validity of a claim as a characteristic representative of a group by performing operations or transformations on an existing object (Balacsheff, 1987). In other words, justification is performed with the help of an example representing all cases belonging to a particular class .The example which was chosen at this level depends on operations or transformations. Students develop arguments by grounding on a general example. Although these arguments depend on special situations, they are not used for special situations. Establishing

Levels	Names of Levels	Explanation of Levels
		conceptual associations over mathematical statements are at the high level. This is the level in which the student reach new information and establish conceptual associations with the help of his old information.
Level 4	Thought experiment	The operations and basic relationships in the proof are shown by mathematical definitions, theorems and inference rules rather than by using existing results, independently from the examples (Balacheff, 1987). Students start to explain the examples considering the intellectual evidences. The student proves by making transitions over mathematical concepts by using questions of why and how in his answer.

Findings

Findings and Interpretations about the Answers of the Students to the Items in the Proof Test

The data was obtained from 25 students through PF in this study. The distribution of the proof skills levels obtained from the students' answers to the items in PT was presented in Table 2.

Table 2. Distribution about The Proof Skills Levels of The Students’ Answers

Item No	Level 1		Level 2		Level 3		Level 4	
	f	**%**	**f**	**%**	**F**	**%**	**F**	**%**
Item1	5	20	11	44	4	16	5	20
Item2	8	32	12	48	4	16	1	4
Item3	15	60	9	36	1	4	-	-
Item4	1	4	16	64	4	16	4	16
Item5	4	16	14	56	7	28	-	-
Item6	16	64	8	32	-	-	1	4
Total	49	32.66	70	46.66	20	13.33	11	7.33

When Table 2 is considered, it is seen that 7.33% of the 'students' answers were at Level 4, 13.33% of them were at Level 3, 32.66% of them were at Level 1 and 46.66% of them were at Level 2.In this context, it can be said that most of the 'students' answers were at Level 2. Below are some of the students' solutions at different levels. Figure 1 depicted item 3 and S7's response, which was supported by a single example. According to Balacheff's proof taxonomy, the student's response is characterized as level 1 Naive empiricism.

In Figure 1, the conversation between two students, Ali and Mehmet, in question 3 was given. These two students are discussing whether the sum of three consecutive numbers can always be divided by 3 without a remainder. While Mehmet claims that this is always true by trying in a single example, Ali argues that it is not enough to say that it is always true with a single trial and that there are infinite numbers to try.

Item 3. Ali and Mehmet are classmates and both of them love mathematics and doing operations with numbers. One day, they had the following conversation about numbers.

Mehmet : 4, 5 and 6 are consecutive numbers. When we add them to each other, we get 15. We can divide 15 by 3 without a remainder.

Ali: Yes.

Mehmet: Yes, it is true for the numbers 4, 5 and 6. Is it always true for the sum of three consecutive numbers if their sum is divided by three without a remainder?

Two friends did not have enough time to solve this question. That's why they decided to talk about this question the next day at school.

It was break time the next day.

Mehmet: Ali, the sum of three consecutive numbers can be divided by 3 without a remainder. For example; the sum of 6, 7 and 8 is 21. 21 can be divided by 3 without a remainder. The sum of 11, 12 and 13 is 36 and 36 can be divided by 3 without a remainder. It always works.

Ali was not so sure.

Mehmet: I studied hard on this last night. I think we cannot say it always works with a few trials. Maybe it won't work when we try it with larger numbers. I don't think a few sample trials will show that it is true for all numbers.

According to the conversation above;

A. Mehmet is right

B. Ali is right

What would your answer be? Can the sum of three consecutive numbers always be divided by 3 without a remainder? If it can be divided by 3, how do you show this?

The answer of S7:

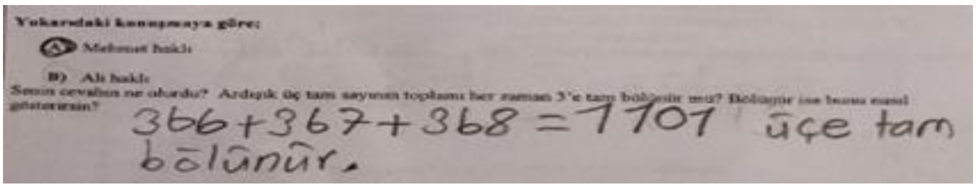
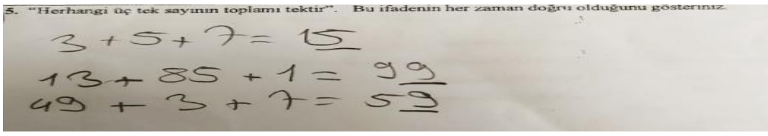


Figure 1. Item 3 and the Answer of S7

The researchers asked gifted fifth-grade kids whether Ali or Mehmet was correct, according to this conversation. Students were also asked to explain their replies and give explanations for their choices. S7 agreed with Mehmet and provided an example to justify his response. As a result, according to Balacheff's evidence taxonomy, S7 was at the level of 1Nave empiricism. According to Balacheff's proof taxonomy, the responses of S10 and S13 were at the level 2 Crucial Experiment in Figures 2 and 3. As shown in Figure 2, the question is ""The sum of any three odd numbers is an odd number"". Please, show that this statement is always true"". S10 made several trials and showed that the result which he found was always an odd number.

Item 5. "The sum of any three odd numbers is an odd number". Please, show that this statement is always true.

The answer of S10:



I experimented with different digit numbers. Single digit, two digit, both single and double digit.

Figure 2. Item 5 and the Answer of S10

Item 2. A teacher asks the following question to his students.

Question: "If the sum of the digits of a number is multiple of 3, that number can be divided by 3 without a remainder". Do you think this statement is always true?

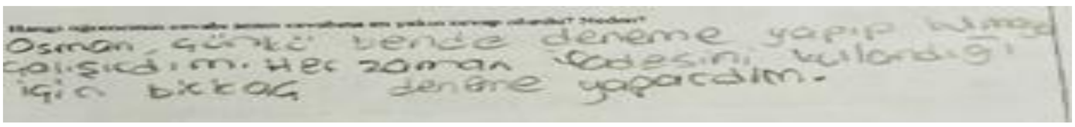
Four students gave following answers to this question.

<table>
<tr><td>Ecrin:
The sum of the digits of number 12 is 3 (1+2=3) and this is multiple of 3. 12 can be divided by 3 without a remainder.

As a result, this is always true for any number.</td><td>Emir:
<u>12</u>=10+2=9+<u>1</u>+<u>2</u>
Number 12 can be divided by 3 without a remainder. The number which is left behind its digits is 9 and it is a multiple of 3. Besides, the sum of its digits is also a multiple of 3.
As a result, this is always true for any number.</td></tr>
<tr><td>Aybüke:
abc=100a+10b+c
=99a+a+9b+b+c
=3(33a+3b)+a+b+c
When we write the number in digit numbers, the sum of the digits and the sum of the numbers which are left behind the sum of the digits are always multiple of 3.
As a result, this is always true for any number.</td><td>Osman:
The sum of digits of number 12 is 6 and it is a multiple of 3. 15 can be divided by 3 without a remainder.The sum of digits of number 48 is 12 and it is a multiple of 3. 48 can be divided by 3 without a remainder.
I tried with two-digit and three-digit numbers.
As a result, this is always true for any number.</td></tr>
</table>

Which student's answer would be close to your own answer? Why?

The answer of S13:

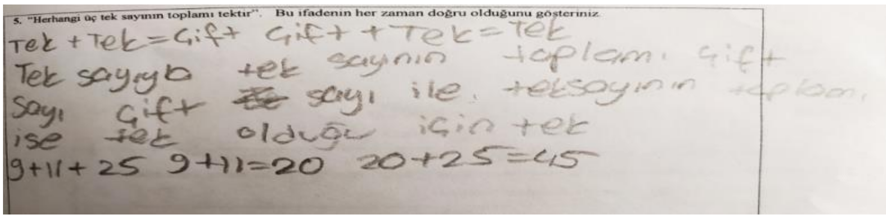


Osman's answer would be close to my answer because I would try the same as he did. I would try several times as the question says "always". (Translated from Turkish)

Figure 3. Item 2 and the Answer of S13

Question 2 contains the responses of four students called Ecrin, Emir, Aybüke, and Osman to the statement ""If the sum of a number's digits is a multiple of 3, that number may be divided by 3 without giving a remainder."" "Do you believe this statement is always correct?" is posed. "Which student's response is the most similar to your own?" The pupils in the study sample were asked, "Why?" "Osman's answer would be quite close to mine," S13 said in response to this question, and he reasoned, "I would try the same way he did." Since the question is "always," I would try multiple times. As a result, according to Balacheff's taxonomy of evidence, S10 and S13 were in stage 2 of the important experiment. Students used many attempts to try to generalize their answers. Figure 4 depicts task 5 and S17's response, which was level 3 of the general example in Balacheff's evidence taxonomy.

Item 5. “The sum of any three odd numbers is an odd number”. Please, show that this statement is always true.
The answer of S17:

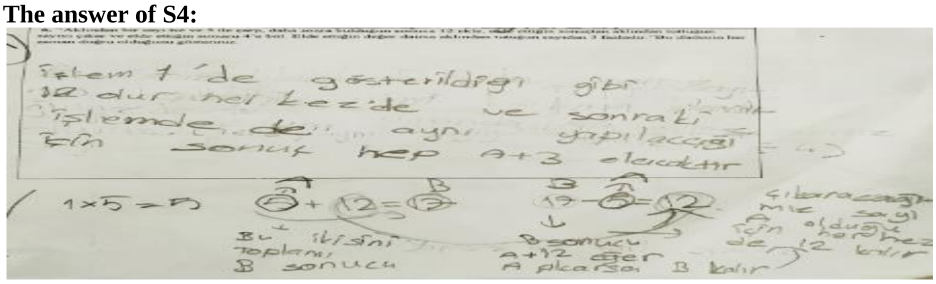


Odd+Odd=Even   Even+Odd=Odd
The sum of two odd numbers is even. The sum of an even number and an odd number is odd so the sum of any three odd numbers will be odd.
9+11+25   9+11=20   20+25=45
(Translated from Turkish)

Figure 4. Item 5 and the Answer of S17

In Figure 4, Question 5 asks, "The sum of any three odd numbers is odd." Please show that this statement is always true." S17 explained his answers by establishing a conceptual association between his existing knowledge of "The sum of two odd numbers is even." The sum of an even number and an odd number is "odd" and the situation of "the sum of three odd numbers is odd". The student built a conceptual association with the help of his previous knowledge. Therefore, it can be said that S17 was a Level 3 Generic Example according to Balacheff's proof taxonomy. In Figure 5, Item 6, and the answer of S4, who was at Level 4 Thought Experiment according to Balacheff's proof taxonomy, was presented.

Item 6. Keep a number in your mind. Multiply it with 5. Add 12 to what you found. Subtract the number in your mind from the number you last found. Divide that number by 4. The number you get is always 3 more than the number in your mind. Please, show that this statement is always true.
The answer of S4:



As it is shown in Operation 1, the answer would be 12. As it would be the same for all and the next operation, the result will always be A+3.

	A B	*B A*	*As the number we will*
1x5=5	*5+12=17*	*17-5=12*	*subtract is A, everybody*
	The sum of these	*Result B is A+12*	*will get 12.*
	two is B	*If we subtract A, we get B*	

(Translated from Turkish)

Figure 5. Item 6 and the Answer of S4

In Figure 4, Question 6 asks "Keep a number in mind. Multiply that by 5 to get the answer. Add 12 to the answer you came up with. Subtract the number you're thinking of from the last number you found. Divide the result by four. The number you get is always three times higher than the number you think of. Please demonstrate that this assertion is always true." In fact, S4 misinterpreted the query and produced an inaccurate answer by doing wrong operations. However, the student attempted to clarify that no matter what number he used, the issue would remain the same. The student demonstrated this by transitioning between mathematical ideas, such as why and how, in his answer. As a result, S4 can be classified as a level 4 thought experiment.

Conclusion and Recommendations

This research investigated the proof skill levels of gifted 5th grade students through data obtained from 25 students. The results revealed that 7.33% of the students' answers were at Level 4, 13.33% were at Level 3, 32.66% were at Level 1 and 46.66% were at Level 2. It was found that most of the students' answers were at Level 2 Crucial Experiment. In this study, the level of proof of the students was the second level, showing that they have difficulty generalizing. This situation may be because the students have not structured mathematical knowledge in their minds and perform operations based on memorization. In the studies of Güler and Ekmekçi (2016) and Birinci (2010), both students and teacher candidates could not reach the generalization level. The fact that students are more often confronted with this type of question may make it easier for them to move from patterns to algebraic expressions and to make generalizations.

Similarly, in the studies of Arslan (2007), Aylar (2014), Sağır (2013), and Polat (2018) conducted with typically developing students, it was found that students tend to perform operations based on memorization. It can be said that the reasons for students' memorization-based operations stem from the fact that the knowledge they use in the proof-making process is not learned in a meaningful way. Thus, it can be said that this research finding is similar to the literature findings. The concept of proof is at the centre of mathematics. With proof teaching, mathematical rules and formulas can become meaningful mathematical concepts for students, rather than being just a few symbols. In this context, it may be suggested to the teachers to make the students find the rules and prove them while teaching mathematics. Furthermore, this research may be repeated with regular and gifted children at various grade levels.

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