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Examining the Accuracy and Justification of Geometric Constructions Made by Pre-Service Teachers with Dynamic Geometry Software and the Awareness They Gained Throughout the Process

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Abstract

This study examined pre-service teachers' accuracy for geometric constructions with dynamic geometry software, their justification for the accuracy of geometric figures, and their awareness they gained throughout the process. The data come from a sample of 71 elementary grade pre-service teachers activity form completed as a part of geometry course in a large university in Southeast of Turkey. The data were analyzed by using both descriptive and content analysis techniques of qualitative research. Findings revealed that elementary grade pre-service teachers' performance for geometric figure construction and their performance for geometrical justification were limited. Furthermore, only very few elementary grade pre-service teachers noticed the differences and similarities for their geometrical constructions when using dynamic software. Results further showed that elementary grade pre-service teachers need more geometrical construction activities on dynamic software environment in order to make accessible them to improve their conceptual knowledge on geometrical concepts.

Introduction

Geometry is a significant learning field of mathematics which is built upon some actions and postulations. It is a product of human thinking. The part of geometry which is known as plane geometry and based on some postulations is called Euclidean geometry (Smart, 1998). All of the shapes in Euclidean geometry can be constructed by utilizing paper, pencil, straightedge, and compass (Lim-Teo, 1997). Besides, a geometric figure can represent all other varieties of that figure by changing the places of points. For example, when the corner points of a triangle are changed, that triangle can be called right-angled triangle, equilateral triangle or isosceles triangle, or can even lose its triangular properties when all three points are linear. Sometimes, it may be difficult to see such relations all together in a quick way using concrete materials. Therefore, it may be more practical to use dynamic geometry software (Hoyles & Noss, 1994). Studies in the literature showed that dynamic geometry software provided students with the opportunity to better focus on abstract structures compared to concrete materials and paper-pencil activities (Christou et al., 2004; Caglayan, 2016; Empson & Turner, 2006; Harter & Ku, 2008; Hazzan & Goldenberg, 1997). In this way, imaginative power of students is improved. Increased imaginative power in mathematics means increased intuition, increased awareness and opening up new ways of exploration. When new ways are opened up, students can achieve the habits of mind identified by Driscoll et al. (2007) as reasoning with relationships, generalizing geometric ideas, investigating invariants, and balancing exploration and reflection. This is because geometric thinking is a mathematical way of thinking with a specific structure that improves in certain steps based on the process. In this structure, the issues such as thinking and operating with shapes, gaining the skills of establishing spatial relationships and visualization, and understanding geometric concepts and exploring their relationships with each other are important (Clements & Battista, 1992).

In geometry teaching, it is important to identify the knowledge, skills and experiences that students are expected to master and to improve their level of geometric thinking accordingly (Baki & Özpınar, 2007). In this sense, the process should be planned beforehand and teaching should be organized accordingly in order to improve students' geometric thinking. Utilizing dynamic software programs in geometry course provides inquiry-based environment and support students learning experiences, so these properties of dynamic geometry system (DGS) offers new opportunities for teachers to teach geometry (Bokosmaty, Mavilidi & Paas, 2017). In this study, Geometer's Sketchpad (GSP) was used as dynamic geometry software because this computer-based environment allows for examining easily various figures that are hard to examine with paper and pencil. For example, when the required concrete materials in Euclidean geometry are considered, it is seen that they can be made by using the sketchpad instead of paper, the point drawing button instead of pencil, the line/line segment drawing button instead of straightedge, and the circle button instead of compass (Figure 1).

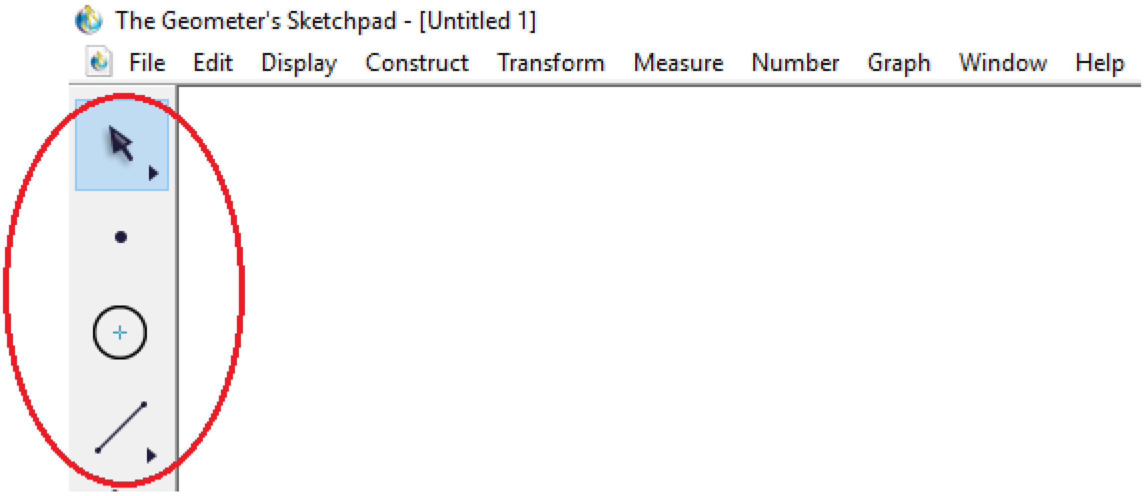

Figure 1. Point, circle and line/line segment buttons in the programs like GSP

From this aspect, it helps students think more deeply about mathematical concepts as intended by the mathematics curricula (SMLP: Secondary Mathematics Lesson Program, 2017). The purpose of the study was to investigate elementary-grade pre-service teachers' accuracy, justification, and raising awareness about the properties of rectangles, squares and isosceles triangles when utilizing GSP. Therefore, the present study examines the following research questions:

1. How accurate are the rectangle, square and isosceles triangle constructions of the participants?
2. What are the justifications of participants for the accuracy of their rectangle, square and isosceles triangle constructions?
3. In what aspects does the process of the construction of geometric shapes raise awareness in participants?

Teachers' knowledge of concepts and operations should be accurate so that they can structure the courses they deliver effectively. Teachers should also make sense of the principles of these types of knowledge in order to frame the course substantially (Ball, 1990; Zulnaidi & Zamri, 2017). Studies show that the explanations and justifications of teachers and pre-service teachers are usually based on the facts they have memorized rather than understood (Chua, 2016; Henningsen & Stein, 1997; Kinach, 2002; Uçar, 2011). To structure their courses effectively, math teachers should know the concept or rule they will teach and the mathematical knowledge and principles this concept is based on (Almeida, 2003). This requires teachers and pre-service teachers to be equipped with mathematical concept knowledge, proof and justification skills (Zazkis & Zazkis, 2016). This study attempts to provide more evidence regarding geometric conceptual knowledge and justification skills and contribute to the relevant literature by enhancing pre-service teachers' competencies and awareness about utilizing dynamic geometry systems.

Method

The study examined how pre-service teachers constructed geometric figures using GSP as well as their reasoning and the awareness they gained throughout the process. In this study, a phenomenological case study was used. The researcher's aims are two-fold: first, to contribute to help participants' question their knowledge about geometric concepts and justification skills; second, to introduce them to utilize dynamic geometry systems while drawing certain geometric constructions such as square, rectangle and isosceles triangle. As known, case study is a way of examining what is existent in the setting, systematically collecting data, analyzing them and drawing conclusions. The product obtained is the understanding about why the case has occurred in that way and what should be emphasized in a more detailed way in further research (Davey, 1991).

Procedure

The study was conducted with the participation of primary mathematics department at a state university. In the first week, the content of the geometry course and geometric thinking skills were elaborated within the context of the geometry course, and the GSP program was introduced. In the second and third weeks, the menus of the GSP software program were introduced in the computer lab during the 3-hour course process, and the participants practiced some applications about the active use of these menus. For instance, they were helped to find a vertical line by drawing a line on the sketchpad via the circle button. After that they were asked to draw a vertical line starting from the mid-point. The data of this study were obtained from the activities carried out in the 4^{th} week of the study. The participants from whom data were collected attended math classes at the computer laboratory (where an interactive whiteboard and computers are available) where they used the GSP dynamic geometry systems. The instructor was available at the classroom to give assistance to those in need of help.

Sample of the Study

The study group of this research comprised 71 pre-service primary mathematics teachers selected through the criterion-based sampling, one of the purposive sampling methods. While determining the participants, the main criterion was the participation in the abovementioned 3 week-training in the background of the study. Due to the limited capacity of the computer lab, the participants were divided into two groups of 34 and 37 people based on the name list.

Data Collection Instrument

In this research, the activity form was used as a data collection instrument. In this form, it was asked 1) to construct a square, rectangle and isosceles triangle by using the GSP software; 2) write how they were convinced about the accuracy of the construction method they used for each geometric figure. For example;

- Draw any line segment using GSP.
- Construct a non-square rectangle, with your segment as one of its sides
- Write your build process step by step.
- Write about how you are convinced that your construction method is accurate.

Similar to the required process for drawing a rectangle, the form asked participants to repeat the same process for drawing a square and isosceles triangle. Then the following question was asked to pre-service teachers in the form: “You created three different figures by providing your reasoning for its accuracy and justification. Now, what have you noticed about this figure when creating them? Could you please compare and contrast the process of drawing of each figure?”. We used Driscoll, Wing DiMatteo, Nikula, Egan, Mark and Kelemanik’s (2008) activity form for this question. A faculty member’s views were taken with regard to the form. Then, the form was administered to six master students in mathematics education, the required corrections were made on the form, and then the form was finalized.

Data Collection Process

During the implementation of the instructional activity, each participant was provided with a computer. The participants worked individually and tired to complete the operations intended on the activity paper by utilizing the GSP software. The participants attempted to draw the figures included in the activity by using the GSP software. Meanwhile, the instructor supported the participants who needed help by walking around in the classroom. There was no time limit for the activity. However, it took about 50 minutes for the participants to complete the process. After the worksheets were collected from the participants, responses were given to the tasks specified in the activity under the guidance of the instructor. During the implementation of the activity, mathematical language and evidence that the participants used in their responses for the activity were elaborated, and they were encouraged to use geometric thinking habits.

Data Analysis Process

In this research, the data were analyzed by using both descriptive and content analysis techniques of qualitative research. The data obtained in the descriptive analysis are summarized and interpreted based on previously determined codes and categories, and the data obtained in the content analysis are summarized and interpreted based on emergent codes and categories during the analysis process (Cohen, Manion & Morrison, 2007). As shown in Figure 2, the geometric figures can be drawn using only circles, points and lines.

Figure 2 shows the sample construction requested from the participants.

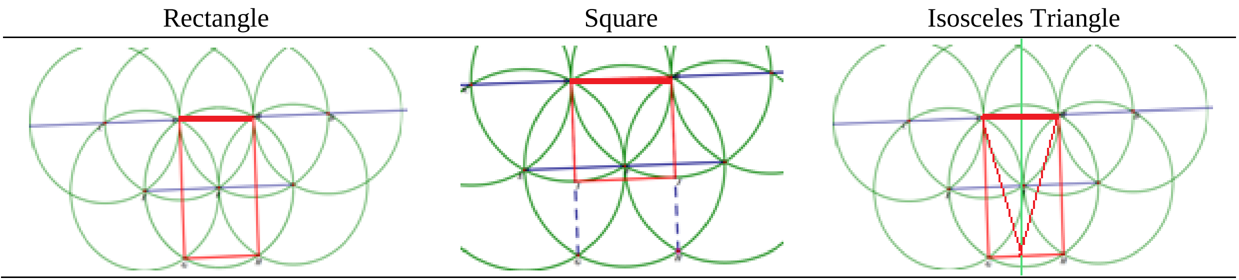

Figure 2. The geometric figures draw using only circles, points and lines

The one shown in Figure 2 is not the only way of construction. Alternative construction ways can be preferred. Within the scope of the study, within the context of descriptive analysis, the responses were categorized into four groups based on their accuracy: blank, wrong, partially correct, correct. Table 1 shows the participants' responses under each category.

Table 1. Evaluation of the construction of geometric figures

	Rectangle	Square	Isosceles Triangle
Blank	-	-	-
Wrong	*I took a circle. Then I drew 4 random points on the circle and drew lines from one point to another. I drew another line from A to B and from C to D. (#8)*	*If we draw a circle in a way to intersect with a line, and then use this line as a reference and draw other line in a way to cover the circle, we get a square (#28)*	*Two circles are drawn with a line as their diameter. The points they intersect are connected. A point is chosen on the connecting line. The point chosen is connected to the endpoints of the first line. Then we get a isosceles triangle (#3)*
Partially correct	*A line is drawn. 2 congruent circles are drawn from both endpoints of this line. Another circle is drawn with the intersection of the other circles as its center. The intersection points below are connected. The figure in the bottom part is a rectangle. (#7)*	*I drew a line. Then I drew two congruent circles with the endpoints of this line as its center. Then I get a square when I connected the centers of these circles (#37)*	*We draw two circles from both endpoints of a line. Then we connect the centers of these circles. We draw lines from the centers of a circle to the intersection points of the circles (#29)*
Correct	*First, I drew a line. Then I drew a circle with the line as its radius. Then I drew a little bigger circle using the same center. I drew a line tangent to the big circle and parallel to the radius. I connected the point of tangent and center. Then I drew a line parallel to the this line and tangent to the little circle and I constructed a rectangle (#22)*	*I drew a line and marked its endpoints. Then I drew two congruent circles with this line as their radius. I continuously drew congruent circles using the points at which two circles intersect. I got two chords in equal length. I connected them and got a rectangle. I divided it into two right at the center and I got a square (#44)*	*I drew a line and then two congruent circles with the endpoints of this line as their centers. I marked the intersection points of these circles and created a beam to these points. I marked a point on the beam and connected it with the endpoints of the line. That's how I got a isosceles triangle (#11)*

Regarding the participant performance on the given task, blank, wrong, partially correct and correct responses were coded as 0, 1,2 and 3 respectively. Thus, the score of each participant was determined. These scores were used to compare the two groups.

Then, in the content analysis, codes were created for the participants' justifications regarding the accuracy of the figures, and frequency of these codes were analyzed. These codes and their frequencies are given in the section "findings". Finally, codes were assigned to the participants' assessment of what they realized when they thought about the construction process in general. These codes were also analyzed.

To ensure reliability of the data analysis, 20% of total data were analyzed by two researchers independently using the content analysis method. The inter-coder reliability was found to be 88% for the data obtained in the descriptive analysis. The degree of agreement (reliability) among the coders was found to be 82% for the data obtained in the content analysis. The remaining data were also analyzed. Such an analysis method ensured the reliability of the themes (Green & Gilhooly, 1996). The codes on which two coders did not agree were analyzed again to reach an agreement. The rest of the data was analyzed by another researcher. The reliability analysis was completed and all of the data were coded. Frequencies of the codes were shown in a tabular display.

Findings

Based on the data obtained in the research, the findings regarding the accuracy of the responses and whether there was a significant difference between groups are presented. After that, the participants' convincing justifications related to the accuracy of the geometric figures are given. Lastly, the findings about what the participants realized given the construction of these figures in general are provided. Table 2 shows the frequencies and percentages of the accuracy of rectangle, square and isosceles triangle constructions of the participants in the GSP environment.

Table 2. The frequencies and percentages of the accuracy of rectangle, square and isosceles triangle constructions of the participants

	Rectangle	Square	Isosceles Triangle
Blank	38 (53%)	40 (57%)	4 (6%)
False	9 (13%)	13 (18%)	17 (24%)
partly correct	13 (18%)	13 (18%)	21 (30%)
correct	11 (16%)	5 (7%)	29 (41%)

The findings in Table 2 show that no answer was received from more than half of the participants (53%) about the construction of rectangle, from about two thirds of the participants (57%) about the construction of square and from 6% of the participants about the construction of isosceles triangle. 16%, 7% and 41% of the participants gave correct answers regarding the constructions of rectangle, square and isosceles triangle, respectively. Table 3 shows the findings about whether there is a significant difference between the accuracy of the participants' explanations about their constructions of rectangle, square and isosceles triangle in the GSP environment.

Table 3. Comparison of the correctness of the construction of geometric shapes desired by participant groups

		N	Mean	SD	t
Rectangle	1. Group	34	1.00	1.128	.291
	2. Group	37	.92	1.21	
Square	1. Group	34	.53	.79	1.945*
	2. Group	37	.97	1.18	
Isosceles Triangle	1. Group	34	1.91	1.03	1.248
	2. Group	37	2.19	.85	

*: p<,05

The findings given in Table 3 show that there is a significant difference at the level of 0.05, only between the accuracy of explanations about square construction (t=1.945; p=.056). Such difference is in favor of the participants in the second group. However, participants' justifications for their geometric constructions were addressed in general without taking account of the significant difference observed in square construction.

In accordance with the second research question, the codes about pre-service teachers' justifications emerged from their responses to the given questions are presented in the following tables. Table 4 shows the justifications of participants for their rectangle constructions.

Table 4. Justifications of participants for their rectangle constructions

Justification	N
Blank	49
Interior angles are 90°.	4
It looks like a rectangle.	3
Adjacent sides are of different length.	3
The sides meet at a right angle.	3
Opposite sides are parallel and angles are 90°.	3
Opposite sides are of equal length.	2
The lines are perpendicular and the opposite sides are of equal length.	2
Radii are the short sides of rectangle and they are of equal length.	1
Since there is a distance between two lines	1

Table 4 shows that 15 participants out of those who did not leave the question blank (including those answering with a shape) emphasized that the interior angles were right or 90°. 8 participants indicated that the opposite sides were of equal length. There were three participants who accentuated that the figure they drew likened to a rectangle. Table 5 shows the justifications of participants for their square constructions.

Table 5. Justifications of participants for their square constructions

Justification	N
Blank	47
Using the properties of a circle	7
Sides are of equal length.	5
Sides are of equal length and the interior angles are 90°.	7
Interior angles are 90°.	4
Drawing a shape.	4

Table 5 shows that 15 participants out of those who did not leave the question blank (including those answering with a shape) emphasized that the interior angles were right or 90°. 16 participants indicated that the opposite sides were of equal length. Seven participants stated that they used the features of the circle. In addition, four participants drew figures as persuasive justifications. Table 6 shows the justifications of participants for their isosceles triangle constructions.

Table 6. The justifications of participants for their isosceles triangle constructions

Justifications	N
Blank	21
Radii on both sides	19
Because the intersection points of the circles are connected on the same line	9
The straight beam drawn divides the line into two	5
Drawing a shape	4
Because the intersection points of the circles are connected on the same line; two sides of an isosceles triangle should be of equal length	3
Two sides of an isosceles triangle should be of equal length	3
The inscribed angle subtended by a diameter is 90°; the straight beam drawn divides the line into two; the altitude cuts the base in half.	1
The lines drawn are symmetrical	1

Table 6 shows that 31 participants out of those who did not leave the question blank (including those answering with a shape) emphasized that the sides were of equal length. 20 participants indicated that a line drawn from a corner cut the base in half. Furthermore, four participants drew figures as persuasive justifications.

In line with the third research question, the codes about the awareness the participants' gained throughout the process are presented in Table 7.

Table 7. The participants' assessment of what they realized during the construction process

Justifications	N
Blank	46
Figures can be drawn using circles	10
After drawing a figure, others can be created using it.	5
Various two dimensional figures can be drawn using a line.	3
Various figures can be drawn using a circle, point and line.	2
Radius can be used in construction	2
Difficulty in expressing	1
Figures can be drawn using a single line	1
Common points are used	1
A line can be considered as a radius	1
The process requires effort	1

Table 7 shows that 10 participants realized that circles could be used while drawing a geometric figure, 5 participants realized that a geometric figure could be used to construct another one, 3 participants realized that various two dimensional figures could be drawn using a line and 2 participants realized that radius could be used in construction. 1 participant pointed out the difficulty in expressing (answer of the participant no. 1: *I have realized that all figures can be drawn when you work hard. But I cannot explain it here, which is saddening*) and 1 participant indicated that many figures could be drawn using a line. Moreover, 1 participant indicated the use of common points, 1 pointed out the use of a line as a radius and 1 expressed that the process required exerting efforts.

Discussion

This study examined how pre-service teachers constructed geometric figures using GSP, their justifications for the accuracy of geometric shapes and the awareness they gained throughout the process. The findings of the study revealed that about two thirds of the participants were unable to give an answer regarding each construction. Besides, the participants were observed to have less difficulty in constructing an isosceles triangle than constructing a square and a rectangle. It is especially remarkable to see that more than half of the participants left the rectangle and square construction questions blank. Based on these findings, we can say that the participants had difficulty in constructing the geometric figures using correct strategies. The use of software requires following a specific algorithmic order for the construction of a geometric figure. Following an algorithmic order requires an education process focusing on the teaching activities related to the production processes of geometric knowledge. Both conceptual and procedural knowledge is required during the problem solving and strategy development processes (Silver, 1986). Participants were expected to use the conceptual definitions of the relevant geometric figures and to take the necessary procedural steps to draw the figures meeting these definitions so that they could do the things they were asked to do. Van de Walle (2007) indicates that having conceptual knowledge will help using the structure of a problem and the concepts in it instead of memorizing the previously known solution. Participants' inability to find the correct answer might be the result of their lack of conceptual knowledge or the lack of an education period that requires taking necessary procedural steps to reach conceptual knowledge.

One of the most important conclusions inferred from the findings is that participants have low-level geometric thinking ability. Therefore, they remained weak in terms of presenting convincing justifications regarding the accuracy of the geometric figure they drew. Moreover, some participants were detected to have drawn figures in the spaces provided for the written explanations regarding their justifications. The main requirement of geometric thinking is the accurate description of the properties of a geometric figure. For example, one should first have knowledge of the basic properties of a rectangle to answer the question "How can I construct a rectangle?" A rectangle has 3 basic properties: (1) opposite sides are of equal length, (2) interior angles are 90^o, (3) it is a quadrilateral. The participants were expected to predicate their justifications for how they constructed a rectangular using the GSP on these 3 basic properties. However, they had difficulty in meeting this expectation. A plenty body of research has revealed that geometrical drawing activities have a positive impact on learners' geometrical concept knowledge, justification and thinking skills (Gür & Kobak Demir, 2017; Tieng, & Eu, 2014; Lee, 2015). With this object in mind, there is still a need for studies on creating geometrical figures with the support of various materials such as dynamic geometry software.

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According to the findings, some of the pre-service teachers recognized that they can interact with geometric figures to move on to the next level. This finding is crucial to indicate Driscoll et al.'s (2007) learning outcome for the relating figures and developing geometrical reasoning. However, the number of participants using such statements was quite low. The accurate drawings provided in Figure 2 were also given in accordance with this geometric thinking. Participants thought each geometrical drawing separately. For example, they did not make use of drawing square or rectangular to draw an isosceles triangle. Researchers foresee that participants solved the items which assess their geometrical content knowledge to get into the teacher education program. Thus, it is important to note that there is a need for a design of an education program to improve students' geometrical thinking and justification skills.

Conclusion

The participants were pre-service mathematics teachers; therefore, we can assume that they answered similar questions testing their geometrical knowledge before in order to be placed in the teaching program. It is clear that an education process that will improve geometric thinking and justification skills should be designed and implemented beginning from the elementary and secondary education level. To do this, activities that require the use of 3 basic materials (straightedge, pencil, compass) on which Euclidian geometry is based should be incorporated in the education process. In the software programs like GSP, point, circle and line/line segment buttons can be used instead of these materials (Figure 1).

Carpenter (1986) indicated that strategy use of a math teacher to solve a problem is directly proportional to his/her conceptual knowledge. Therefore, learning environments should be designed in a way to help students improve and use their conceptual knowledge. In addition to teaching strategies or rules, helping students develop their own ways of solution, discuss them and see different points of view would provide them with the opportunity to think about and assess their thinking. If students are not able to associate problem solving strategies to their conceptual knowledge, they may not able to adjust the problem solving strategies to different situations, just like memorized order of operations. Activities that require pre-service teachers to use their conceptual knowledge, express themselves and discuss and interpret solutions using different points of view should be further used in teacher training.

Some dynamic geometry software such as GSP can be used as a tool in such activity-based studies. Research conducted on dynamic geometry tools and the use of these tools in geometry has contributed to realizing change and stability in geometric environments (Bokosmaty, Mavilidi & Paas, 2017; Ware & Stein, 2014). DGS helps users to notice properties of geometrical shapes (Hoyles & Jones, 1998; Kaur, 2015); for instance, when used reflection, rotation, transition, or dilation on DGS, users easily recognize the differences and similarities on geometrical shapes dynamically on DGS. This feature and other features of dynamic geometry software imply that new technologies should affect school geometry significantly (Schwartz, 1999; Yuan-Hsuan, Waxman, Wu, Michko & Lin, 2013).

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