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### **Mixed-reality Learning Environments: What Happens When You Move from a Laboratory to a Classroom?**

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## Mixed-reality Learning Environments: What Happens When You Move from a Laboratory to a Classroom?

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### Abstract

The advent of motion-controlled technologies has unlocked new possibilities for body-based learning in the mathematics classroom. For example, mixed-reality learning environments allow students the opportunity to embody a mathematical concept while simultaneously being provided a visual interface that represents their movement. In the current study, we created a mixed-reality environment to help children learn about angle measurement, and we investigated similarities and differences in learning between students who completed the activity individually during a one-on-one interview and students who observed others complete the activity during whole-class instruction. Pre- and post-assessment results showed that students in both settings learned at similar rates. Additionally, we analyzed the language used during the activity and found that students in the individual setting used more spatial language early in the activity, while students in the whole-class setting used more metaphors. This finding was likely due to differences in the perspectives students have when physically engaging with the angle compared to the perspective when observing someone else engage with the embodied-interaction.

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### Introduction

While the mind and body were once thought to work independently from one another, many theorists and researchers now argue that learning “is rooted in, and shaped by, the body” (Edwards & Robutti, 2014, p. 2). This knowledge, along with the advent of motion-controlled technologies, such as the Wii and Kinect, has spurred educators to create embodied learning environments that take advantage of the interconnections between the mind and the body. In mathematics education research, there is a growing body of literature that shows embodied environments lead to learning (Abrahamson & Sánchez-García, 2016; Author, 2014). However, many of the studies investigating the usefulness of motion-controlled technologies for learning mathematics have taken place in settings where a single or small group of users engage with the task at the direction of a researcher (e.g., Howison, Trninc, Reinholz, & Abrahamson, 2011; Muldner, Lozano, Giroto, Burlison, & Walker, 2013). While these studies have made a significant contribution to the field, our knowledge and understanding about the effectiveness of such activities to transfer to a classroom where large numbers of students may be required to observe someone else engage with the environment is limited. Therefore, our goal in this manuscript is to examine students’ learning outcomes and processes between those students who complete our angle task while physically engaged with the embodied environment and those students who observe someone else engage with the environment as part of whole class instruction.

As part of the current study, we developed a body-based task designed to help students learn about angle measurement using a Kinect for Windows program. During the task, students use their arms to embody a mathematical angle while working to uncover secret rules that are related to important angle measurement concepts. We administered the activity to individual students working one-on-one with a researcher, and during whole-class instruction where students observed someone else engaging with the learning environment. To investigate potential differences in learning among students in these two settings, we administered a pre- and post-assessment designed to measure students’ understanding about angle measurement. Additionally, to investigate whether the processes used by students to make sense of the angle task varied depending on the setting, we examined students’ reasoning by taking a close look at the language students used to describe how they were thinking about the activity. This work was guided by the following research questions, (1) Do students who physically engage with a mathematical concept and those who observe someone else engaging with the concept learn at similar rates? and (2) How does students’ reasoning about the activity vary depending on whether they physically engage with the environment or observe someone else doing so?

## Literature Review

### Embodied Cognition

Our work is informed by an embodied cognition perspective, which argues that cognitive processes are rooted in perceptual and physical interactions of the body with the environment (Wilson, 2002). The same neural systems control both mental thought and physical actions (Seitz, 2000). This is true even for *offline* cognition where environmental input is absent. For example, Hauk, Johnsrude, and Pulvermuller (2004) found that when an individual reads a passage involving a word associated with action, (e.g. to kick) the same areas of the brain are activated as those which are activated when the individual moves his foot. This finding, as well as several other important findings in this area (see for example, Boulenger, Hauk, & Pulvermuller, 2009; Hutto, Kirchoff, & Abrahamson 2015; Solomon & Barsalou, 2001; Zwaan & Yaxley, 2003), lend support to the premise that cognition is grounded in the physical, lived reality (Nunez, Edwards, & Matos, 1999).

Symbols and abstraction are central to mathematics, yet as Abrahamson (2017) argues, these abstractions are related to objects and actions in the physical world. Consider addition, this concept can be thought of as the embodied experience of combining groups of objects. Even scientists studying extremely abstract mathematics discuss using their bodies to help make sense of new concepts (Ochs, Gonzalez, & Jacoby, 1996). The understanding that abstract thinking is grounded in the actions and perceptions of the body has led many mathematics educators to create instruction that makes meaningful connections between physical activity and important mathematical concepts (Lindgren & Johnson-Glenberg, 2013). For example, Abrahamson and Trninic (2014) developed the Mathematical Imagery Trainer for Proportion to allow students to use their hands to engage with the mathematical concept of proportion. Nemirovsky, Tierney, and Wright (1998) use motion detectors to track students' movements and help them make connections to rate of change and distance-time graphs, and Muldner et al.'s (2013) Tangible Activities for Geometry System allows a group of students to walk around a problem space that is projected onto the floor while exploring geometric concepts. When activities like these are used in a whole-class setting, there may not be time for every student to physically engage with the learning environment. Therefore, an important question to consider is whether students who are situated outside the environment as observers will learn as well.

Research on mirror neurons provides some promising results that suggest students may be able to learn through observing the actions of others. Studies with the macaque monkey show that mirror neurons fire both when a monkey performs an action and when he observes another monkey performing the same action (Rizzolatti & Craighero, 2004). Thus, some researchers have suggested that mirror neurons, present also in humans, "appear to account for action-oriented understanding, imitative learning, and synchronized behavior" (Atkinson, 2010, p.604). Rizzolatti and Sinigaglia (2008) claim that when we see someone perform an action, our brains rehearse the action. While it was once believed this process occurred independent of the goals of the action, researchers now believe that it is only in understanding the goals associated with the action that observers' mirror neurons work to rehearse the action (Hickok, 2013; Rizzolatti, Fofassi, & Gallese, 2001). This research suggests that contingent upon understanding the goal associated with an action, observing someone else's actions could lead to conceptual understanding in a way similar to performing the action for oneself. This work on mirror neurons leads to a similar conclusion as the one put forth in Bandura's (1977) Social Learning Theory where he lauds the power of observation to promote learning. Bandura is also clear in his belief that observation is more likely to result in learning when the modeled behavior is associated with a desired outcome. We believe the research on Social Learning Theory and mirror neurons provides motivation to examine whether learning varies between students who observe someone else engage with the embodied environment and for those students who physically engage with the environment themselves.

### Mixed-Reality Learning Environments

The invention of motion-controlled technologies has unlocked new ways to create embodied learning experiences in the mathematics classroom. Many of these new environments are what Milgram and Kishino (1994) termed, *mixed-reality environments*. Such environments involve two primary components: first, a student must be situated inside the learning system, and second, the environment must include an interface that represents selected features of the students' movements (Lindgren & Johnson-Glenberg, 2013). The opportunity to physically engage with a mathematics concept, while simultaneously having a selective visual representation of one's movements is unparalleled. There already exists a great deal of research suggesting that mixed-reality environments have the potential to enhance learning (Chang, Lee, Wang, & Chen, 2010; Johnson-Glenberg, Birchfield, & Uysal, 2009; Muldner et al., 2013). However, a major challenge facing educators is

creating environments where teachers have the time needed to allow every student to be physically situated inside the learning environment. In fact, many mixed-reality environments were created for a single-user experience (see for example, Lindgren & Moshell, 2011). Tracking multiple students at a time using current technology is complicated. Johnson-Glenberg, Birchfield, Tolentino, and Koziupa's (2014) SMALLab environment allows four students to be tracked at one time, but that still leaves much of the class observing the action. While using SMALLab, students who are not being tracked observe the participants and interact by completing discussion and whiteboard activities, an experience that is not the same as physically engaging with the environment. The current study seeks to build on this research to investigate the implications, in terms of learning and students' reasoning, between students who physically engage with the environment and those who observe others engaging with the environment.

## Language

In mathematics education research, there is a long tradition of using clinical task-based interviews to gain insight into children's mathematical understanding (Ginsburg, 1997; Goldin, 2000; Piaget, 1965). In this type of interview, the subject works on a mathematical task and shares his/her thinking with the interviewer while completing the task. The language used by the subject provides a pathway into his/her reasoning and understanding about the task (Ginsburg, 1997; Schoenfeld, 2002). In the current study, we wanted to understand how students were making sense of the angle task and how this might vary between the students who physically engage with the learning environment and those who observe someone else engage with the environment. After analyzing the interview transcripts, we decided to focus our analyses on the use of spatial and metaphorical language. Past research provides support for this decision as both spatial language and metaphor have been shown to help students translate bodily experience into abstract mathematical understanding (Sfard, 1994; Wiedenbauer & Jansen-Osmann, 2008). We acknowledge that spatial and metaphorical thinking are not necessarily mutually exclusive and describe later in the manuscript how this was addressed during data collection.

### *Spatial and Metaphorical Language*

The physical action required in our body-based angle task stimulates students to think spatially because action requires the visualization and interpretation of locations, positions, distances, and movements (Sinton, Bednarz, Gersmehl, Kolvoord, & Uttal, 2013). We consider visualization, as suggested by Abrahamson, Lee, Negrete, & Gutiérrez (2014), to be multimodal and conceptualize of a spatial orientation that involves more than just the ability to visualize. As individuals engage with or observe our body-based task, they interpret the movements of the actor in relation to the presumed goals for the activity. Language can be used to describe what happened or to help students create a plan for how specific movements will lead to specific outcomes.

When children are thinking spatially, the use of spatial language can support their thinking by drawing attention to relationships that might otherwise go unnoticed (Gentner, 2003; Levine, Ratliff, Huttenlocher, & Cannon, 2012). Many studies have documented the important role spatial language plays in successful task completion (Pruden, Levine, & Huttenlocher, 2011; Shusterman, 2006). Hermer-Vasquez, Moffet, and Munkholm (2001) found that the researcher's use of the terms *left* and *right* helped students successfully complete a reorientation task. Likewise, Loewenstein and Gentner (2001) found that children who were presented the words *in*, *on*, and *under* performed better on a cross-mapping task than students who were not presented with these words. As Rattermann and Gentner (1998) explain, language provides students with support as their thinking progresses from naïve to sophisticated. In the angle task used in this study, spatial language will likely be an important tool for students to use as they work to formulize their thinking about how to make the screen different colors.

Like spatial language, metaphors are another powerful cognitive tool that can be used to build understanding (Lakoff & Johnson, 2003; Lakoff & Nunez, 2000; Sfard, 1994). Using metaphors is not merely an attempt to make a comparison, but an opportunity to build understanding of a new concept in terms of an existing one (Sfard, 1994). When we encounter something new, "we try to make sense of it by figuring out what it is like" (Abrahamson, Gutiérrez & Baddorf, 2012, p. 75). An example from Presmeg (1992) describes an individual who uses a 'water level' metaphor to help make sense of trigonometric ratios. Metaphors such as this are created and given meaning from the individual's perspective. Schiralli & Sinclair (2003, p.84) call these extraneous metaphors, which are defined as metaphorical mappings that are made "to and from the ongoing experiences (including non-mathematical ones) of the mathematician". For students encountering unfamiliar mathematical concepts, creating metaphors can aid in their development of understanding (Abrahamson,

Gutiérrez & Baddorf, 2012). In the current study, students may use extraneous metaphors to help themselves make sense of the new information presented during the angle task.

As students work to make sense of the angle task, we anticipate they will use spatial language and metaphors as they discuss their thinking during the activity. We hypothesize that depending on the setting used to interact with the activity, physically engaging with the environment or observing someone else engage with the environment, students may opt for one tool more than the other. This may be due to students' spatial frames of reference (O'Meara & Pérez Báez, 2011). Students will either think about the angle task from an egocentric or allocentric frame of reference (Shusterman & Li, 2016).

An egocentric frame of reference means describing the spatial relations from the perspective of one's own body, while an allocentric frame of reference means describing the spatial relations in terms of someone else or something else. We suspect that students who observe someone else engaging with the angle task will be more likely to use an allocentric frame of reference, while the students who physically engage with the task will be more likely to take an egocentric frame of reference. This variation in perspective taking may influence how students' reason as they try to make sense of the task.

An individual who physically engages with the angle will likely focus their attention on the movements or actions that create color changes on the screen. A focus on the process or movements that create each color might prompt them to make sense of the activity by using spatial language to describe the body's movements (e.g. I put my arms *up*). In contrast, someone observing another student engage in the environment may focus less on the movement and more on the outcome, that is, on the body position that creates a specific screen color.

When focusing on the outcome, a student may use spatial language to describe what they see (e.g. their arms are *out to the side*) or they may make a comparison (e.g. they look like the statue of liberty). The potential differences in perspective or frame of reference, may lead students to reason differently during the task. To investigate the type of language students used to make sense of the angle task, we quantified the amount of spatial and metaphorical language used throughout the interviews and noted differences in language between students who physically engaged with the task and those who observed others engaged with the task.

## Method

### Participants

Fifty-two, eight- to ten-year-old third and fourth grade students participated in this study, 28 boys and 24 girls. Thirty-two students participated in a one-on-one structured, task-based interview (Goldin, 2000) with a researcher that lasted approximately 15-20 minutes. The additional twenty students participated in a single, structured whole-class learning session that lasted 18 minutes. In both the individual and whole-class setting, the same structured interview protocol was used (see Appendix A). All students were administered a pre-assessment before the interview began and an identical post-assessment after the interview was completed.

### Design of the Angle Task

The creation of the angle task was guided by embodied views of cognition which emphasize that "cognitive processes are deeply rooted in the body's interactions with the world" (Wilson, 2002, p.625). Our mixed-reality environment used a Kinect for Windows program in which the Kinect sensor bars' skeletal tracking system tracked and recorded the position of the student's arms in space. To engage in the environment, a participant stood in front of the sensor bar and created various angles using their arms. Data on the position of the participant's arms were recorded about three times per second, and these data were used to determine the measure of the angle being formed by the participant's arms.

In front of the student was a large screen used to relay the information about the angle measure back to the student. At the start of the activity, the screen in front of the student changed between four colors based on the measure of the angle formed. An acute angle corresponded with a pink screen, a right angle with yellow, an obtuse angle with light blue, and a straight angle with purple. At first, only the color appeared on the screen, but as the activity progressed additional information about the angle was provided to the student.

## The Interview

In both the individual and the whole-class setting, we used the same interview protocol. To begin the interview, a student was asked to stand in front of the Kinect sensor bar, to keep his or her arms straight, and to move them around while watching the large screen projecting the Kinect program. In the individual setting, the sole participant remained in front of the Kinect sensor bar throughout the interview. In contrast, during the whole-class interview, student volunteers were selected to engage with the Kinect program, each for approximately a three- to four-minute segment of the interview. Only students observing the activity answered the questions from the interview protocol.

The goal of the task was for students to uncover the secret rules that described how to make the screen each color. After being provided with time to experiment with the program, the interviewer asked the student, "How do you make the screen pink?" After the student answered the question, the interviewer frequently repeated the answer and/or asked a clarifying question if it was deemed necessary to do so. The interviewer continued in this manner asking, one by one, how the student could make the screen each of the four colors. In the individual setting, the sole participant answered every question, while in the whole-class setting, after a question was asked, students volunteered to answer by raising their hand, and the interviewer selected specific students to respond aloud while trying to ensure that many different students had an opportunity to speak.

The questions asked about how to turn the screen each color were repeated during each of the four stages of the interview. In Stage 1, nothing but the four colors appeared on the screen. In Stage 2, the interviewer added a pair of arrows forming an angle which corresponded to the angle created by the student's arms. As the student's arms moved, the angle on the screen also moved. In Stage 3 of the interview, a protractor was added to the screen, and the measure of the angle was displayed in the upper right-hand corner. In the final stage, the protractor and degree measure remained on the screen, but the length of the arrows was extended. By extending the arrows, we wanted to challenge a common misconception often held by students about angles, that the length of the line segments that make up an angle are related to the size of the angle (Clements, 2003; Fyhn, 2008).

## Data and Data Analysis

To answer our research questions, we utilized pre- and post-assessment data, video-recordings, and transcriptions from each interview. We used the assessment data to determine if students completing the activity during individual interviews or as part of whole-class instruction showed greater learning gains. The assessment included eight questions designed to assess students' understanding about angle measurement (see Appendix B). The first four questions asked students to estimate the measure of a given angle. In the last four questions, students were given an angle measure and asked to draw the angle. We used a repeated measures ANOVA to determine if students from the individual setting produced significantly greater learning gains than students who observed others engaging with the learning environment.

To answer the second research question, investigating students' thinking during the task, we applied cognitive discourse analysis (CODA). As explained in Tenbrink (2015), CODA involves data collection, data preparation, content analysis, and an analysis of linguistic features. Essential to the data collection process is the opportunity for participants to talk openly about what they are thinking, without being guided towards certain conclusions (Ericsson & Simon, 1993). In the current study, the interviewer's main goal was to encourage the students to freely share their thinking about what made the screen each color. The interviewer was careful to avoid making judgments about students' responses, and the only follow-up questions posed were designed to draw out additional information about how students were thinking about the task, for example "can you tell me more about that" or "I'm not sure what you mean when you say ...". We chose this approach to the interview so that we could better understand the aspects of the task that the students themselves were aware of and wanted to emphasize (Ericsson & Simon, 1993).

Next, we prepared and analyzed the linguistic data. Each interview was digitally recorded and transcribed. To analyze the transcribed data, we used content analysis to identify patterns in the transcripts (Creswell, 2003). First, we read through the transcripts to get a sense of the data and to understand the meaning behind the verbalizations (Tesch, 1990). Next, we read through the transcripts more carefully and assigned codes to words and phrases, and then we examined the codes to identify potential themes (Miles & Huberman, 1994). Two themes or types of language emerged from the analysis, the first involved students' use of spatial language, and the second involved the use of metaphors.

### Linguistic Features: Spatial Language and Metaphor

The two key linguistic features from our analysis were spatial language and metaphor. Because our focus in this study was to understand how participants in the individual setting were similar to or different from participants in the whole-class setting, we wanted to quantify the usage of each linguistic feature to give us a picture of how they were used in each setting. It is important to note that the thinking described by students was, at times, both spatial and metaphorical. We accounted for this by coding for both types of thinking. For example, the following statement involves both spatial and metaphorical thinking, “I put one arm *up* and one arm *down*, as if I’m raising my hand in class.” The student uses spatial language (*up*, *down*) to describe the position of the arms, and then elaborates by comparing the arm position to a familiar situation, raising a hand in class.

To quantify the spatial language used by the participants during the task we analyzed the transcripts using a variation of the coding system suggested by Cannon, Lavine, and Huttenlocher (2007). A team of researchers met several times throughout the coding process using a system of discussion and consensus to code the transcripts for spatial language (Harry, Stagers, & Klingner, 2005). We began by creating a definition for how we would identify a spatial word. In our definition, a word was spatial if it described the student’s body in space or the position of the angle’s image in space. We completed a preliminary examination of the transcripts and eliminated several words suggested by Cannon et al. (2007) because they did not fit our definition or were not used by any student in our study. After finalizing our list of spatial words (see Appendix C), we identified all instances of the spatial words in 20% of the transcripts. Two raters examined each spatial word to determine if its use was indeed spatial. For example, in the phrase *my arm is up*, the word *up* would be coded as spatial; however, in the phrase *it’s up to you*, *up* would not be coded as spatial. The two raters were in high agreement (90% accuracy) concerning which words should be considered spatial. Each difference was discussed until agreement was reached. Lastly, a single researcher coded the remaining transcripts. Once all the spatial words were identified in each transcript, we calculated the total number of words spoken, the number of spatial words spoken, and the number of unique spatial words spoken in each interview. We also identified these statistics for each stage of the interview and for each type of angle.

To quantify the use of metaphor we used a slightly different procedure. To begin with, we considered a common definition of metaphor as involving talk about two things at once, one of which is already up for consideration. For the purposes of our study, we considered all attempts to describe how the angle from the task (either the one formed by the student or the one on the screen) compares to an outside object or action, as a metaphor. Although metaphor and simile are technically not the same, we opted to include simile as a type of metaphor. Leino & Drakenberg (1993) argue that metaphor and simile are “functionally the same” (from Soskice, 1985, p.59), but do explain that metaphor is an implicit comparison, while simile is an explicit comparison. For the purposes of this study, we decided to consider all attempts to make a comparison between the new mathematical concepts being investigated and an individual’s prior knowledge as metaphors. For example, the following statements are both considered metaphors, as defined in our study, “My arms are an L.” and “My hands are like a cross.” We developed three different codes to describe the types of metaphors used throughout the task. A metaphor was coded as *new* the first time it was used in the interview. If a student invoked a metaphor that was previously used it was coded as a *repeat metaphor*. Lastly, a *building metaphor* was used to code a metaphor that was similar to, but an extension of a previous metaphor. For example, if a student used a new metaphor, “I could make an A”, the comment “it’s like an A”, would be considered a repeat metaphor. However, if the student later said, “Light blue, it’s much more than an A”, this statement would be coded as a building metaphor because it invokes the same metaphor of being like an A, yet is a clear extension of that metaphor.

Two researchers worked together to code 10% of the transcript data while finalizing our definition of metaphor and the types of metaphor we wanted to code. After agreement was reached, we independently coded an additional 10% of the transcripts. This resulted in 92% agreement in applying the codes. Differences in coding were discussed among the researchers and the codes were refined. At this point, a single researcher coded the remaining transcripts. For each interview, we found the total number of metaphors used and the type of metaphors used. These data were also collected by stage of the interview and by the type of angle discussed.

### Results

In this study, we explored the implementation of our embodied angle task during individual interviews and as part of whole-class instruction. First, we used pre- and post-assessment results to examine learning differences

between students from each setting. Second, we analyzed the language used to reason about the activity among students in the two settings to help us better understand differences in how students made sense of the mathematical concepts explored during the task.

### Learning Gains

We began the analysis by examining whether students' assessment scores in the individual setting and whole-class setting improved from before to after the angle task. As shown in Figure 1, the average assessment scores for students in both settings increased from the pre- to post-assessment. To examine whether these differences were statistically significant we conducted two paired-samples t tests and found that the scores for students in the individual setting and the whole-class setting improved. Next, we conducted a repeated measures ANOVA to determine whether the interaction between setting (individual or whole) and students' assessment scores (pre and post) was statistically significant. The interaction provides valuable information about whether students in one setting or the other improved at significantly greater rates. The interaction effect of the setting on students' assessment scores was not statistically significant,  $F(1, 58) = 1.02, p > 0.05$ . These results support what is seen in the last section of Figure 1, showing that the change in assessment scores was similar for students who physically engaged in with the angle task and those who observed someone else engage with the task. In the whole class condition where students observed someone engage with the task, students' scores improved on average 20.7%, while the improvement was almost identical in the individual setting with scores improving 19.9%.

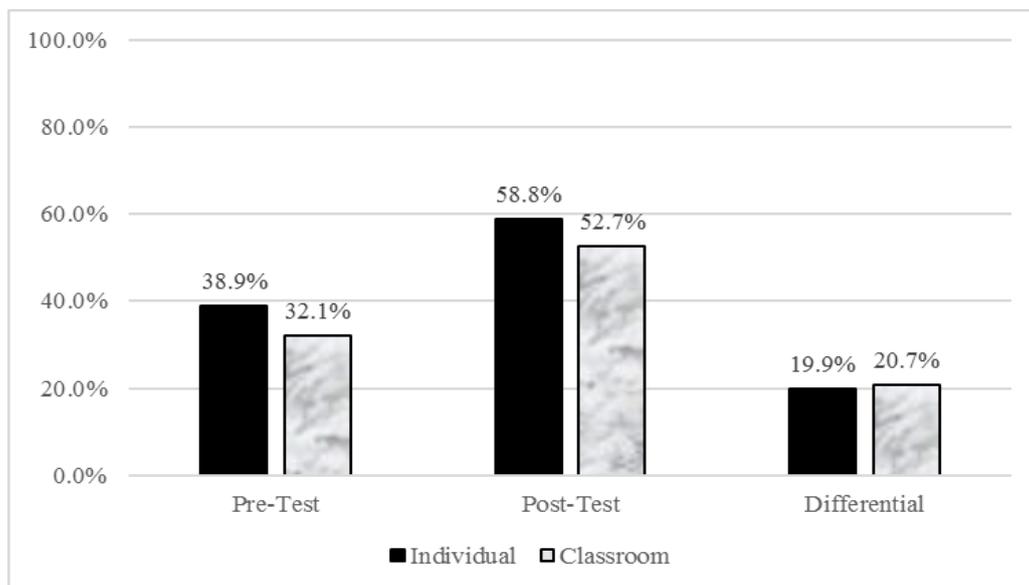


Figure 1. Average assessment scores and improvement by setting

### Students' Thinking as Expressed Through Language

In the second portion of our analysis, we wanted to investigate students' reasoning during the learning task. We identified two types of language students used to make sense of the angle task, spatial and metaphorical. In the sections that follow, we describe how spatial and metaphorical language were used throughout the learning task and identify different usage among students in the different settings. As we discuss the results below, when referencing students in the individual setting their results were averaged across the 37 students who completed the study individually, while the whole-class results describe the language used by the class.

#### *Spatial Language*

First, we investigated the spatial language students used while describing their thinking about how to make the screen each color. As shown in Table 1, for the average student in the individual setting 11.93% of their words spoken were spatial. In contrast, 7.00% of the words spoken in the whole-class setting were spatial. Although the students in the whole-class who observed someone else engage with the learning environment used fewer

spatial words than the average individual who physically engaged with the environment, they used a greater percentage of unique spatial words, 44.74%, compared to 35.25%. This means that students in the whole-class setting who were observing were less likely to repeat a spatial word that was previously used.

Table 1. Descriptive statistics for spatial language by setting

	Individual (averages)	Class
# of Words Spoken	418.67	543.00
# of Spatial Words	47.58	38.00
# of Unique Spatial Words	15.03	17.00
% of words that were spatial	11.93%	7.00%
% of spatial words that were unique	35.25%	44.74%

The most common spatial words used during the task were *straight*, *down*, *a little*, *up*, *to*, and *same*. *Straight* was the spatial word that was used most frequently among students in the individual setting. Students often described their thinking about making the screen purple by referring to a straight line, for example “I put my arms *straight* out.” Among students in the whole-class setting, *to* was the most commonly used spatial word. As an example, one student said, “I think it goes up *to* 90” when she was describing the range for an acute angle.

Next, we examined how spatial language was used differently in the two settings by looking at the amount of spatial language used during each stage of the activity. The pattern observed from the individual interviews was different than the pattern observed in the whole-class interview. As shown in Figure 2, the average student who physically engaged with the angle used considerably less spatial language as the interview progressed. It appears that as more information was provided to students on the screen (the arrows in Stage 2 and the protractor in Stage 3) the need to use spatial language to describe how to make the screen each color decreased. A typical response in Stage 1 involved students describing the location of their arms, for example “you put them *straight out* to the *side*”. While in Stages 3 and 4, students were more likely to use an angle measure to describe how to make the screen a certain color, perhaps saying, “you make it 180 degrees”. The same pattern was not observed among students in the class who observed someone else engaging with the environment. In this case, spatial language was used more frequently in Stages 2 and 4 of the interview. Lastly, the difference in spatial language usage by setting is most significant in Stage 1. For the average student in the individual setting, 38.7% of their spatial language was used in Stage 1, compared to the whole-class setting where only 7.9% of the spatial language was used in Stage 1.

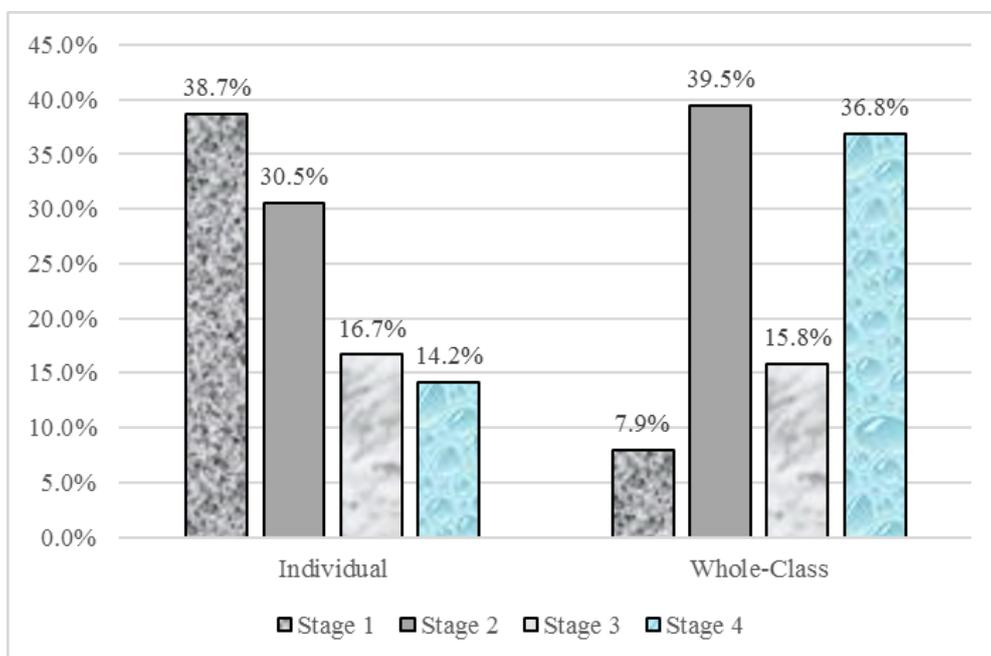


Figure 2. Amount of spatial language used by setting and by stage

When considering spatial language usage across the different screen colors, we noticed that students who were observing used considerably less spatial language when discussing how to make the screen yellow than when discussing how to make the screen each of the other three colors. This pattern did not exist among students who physically engaged with the learning environment, who on average, used similar amounts of spatial language when discussing each screen color. After examining the transcripts, it was clear that students in the individual setting frequently described a right angle by describing the position of their arms, for example, “if I have one arm kind of straight and the other one pointing down.” On the other hand, students who observed someone else physically creating a right angle with their arms were more likely to describe what this looked like using a metaphor, for example “It looks like a person dancing” or “she is saying hi to somebody.”

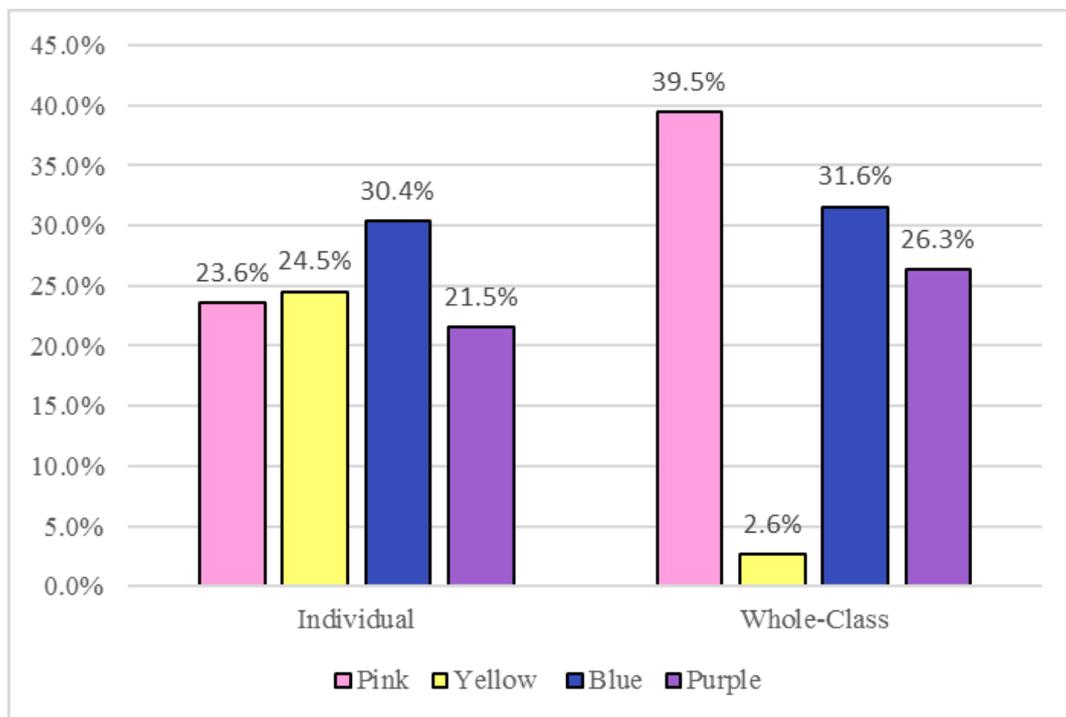


Figure 3. Spatial language used by setting and angle

### Metaphor

Next, we examined our second linguistic feature, metaphor. Students who completed the interview individually while physically engaging with the angle used, on average, 4.58 metaphors, while students in the whole-class who observed others engaging with the angle used 20 metaphors during the interview. Only one student in the individual setting used more than 20 metaphors, while 80% used fewer than eight metaphors. Students in both settings used more new metaphors than repeat or building metaphors. Students in the individual setting were more likely than students in the whole-class setting to repeat a metaphor that was said previously. On average, 25% of the metaphors used during individual interviews were repeat metaphors compared to 10% of metaphors being repeated in the whole-class setting. The infrequent use of repeat metaphors among the class may indicate that students were working hard to make sense of the activity in their own way or were working to think of new ways to describe what was happening in the activity that had not been previously shared. Lastly, metaphors that built on a previous metaphor were used infrequently in both settings.

Looking at the use of metaphorical language during each stage and for each type of angle the results were similar by setting with two exceptions, students in the whole-class setting used more metaphors during stage 1 and when asked how to make the screen yellow. The results for the amount of spatial and metaphorical language used at these two significant segments of the interview are shown in Figure 4. In both Stage 1 and when asked how to make the screen yellow, students in the whole-class who were observing someone else engage with the environment used more metaphors, while students in the individual setting who physically engaged with the angle used more spatial language. To better understand the significance of these differences in language, Table 2 provides sample statements made by students in each setting during Stage 1 and when asked to make the screen yellow.

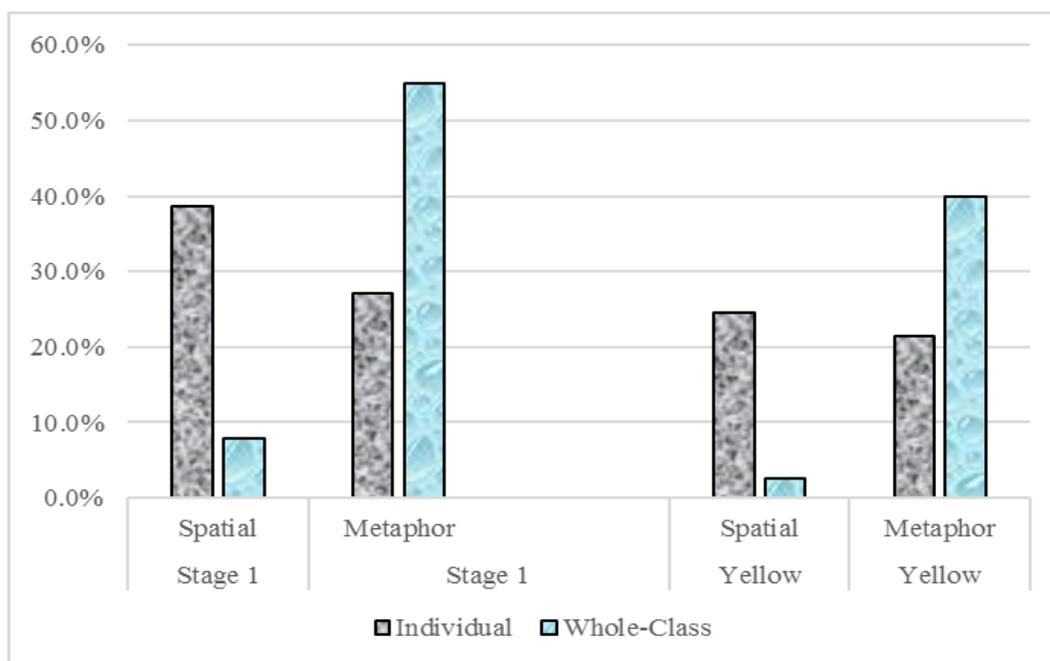


Figure 4. Differences in language usage by setting

Students in the individual setting almost always used spatial language at these points in the interview to refer to the position of their body in space. For example, when one student during an individual interview was asked how he could make the screen yellow he said, “One hand is *above* my head, *straight*. And the other is *to the side*.” In contrast, students in the whole-class setting used very little spatial language during these stages and instead explained their thinking through metaphors, for example, “It’s like the statue of liberty.” A similar difference was observed when students discussed their thinking in Stage 1. One student in the individual setting said, “I move my hands *up a little bit higher*.” Notice the use of spatial language here, and how the student describes the change in movement that created the new color. We can compare this with a comment made by a student in the whole-class setting, “It’s like a soccer player screaming, goal!” In the latter example, the student is using a metaphor to describe the ending body position of the student engaging with the environment that created the screen color. This difference in spatial and metaphorical language use is likely related to differences in the frame of reference used by students in the two settings.

Table 2. Sample statements made by students in each setting

Individual Setting	Whole-class Setting
Stage 1: Only the color appears on the screen	
Example 1 “Your arms go up, it’s like when you go down in the middle, it switches to blue.”	Example 1 “It’s like a soccer player screaming, goal!”
Example 2 “I move my hands up a little bit higher.”	Example 2 “He was like an airplane.”
How do you make the screen yellow?’	
Example 1 “One hand is above my head, straight. And the other is to the side.”	Example 1 “You put your hands up. Like the clock hand is at five forty-five.”
Example 2 “Putting my hand out kinda to the side.”	Example 2 “It’s like the statue of liberty.”

## Conclusion

In the current study, we compared the learning gains between students who physically engaged with the embodied learning task and those who observed someone else engage with the task and found that students in both settings learned at similar rates. This result is promising as it indicates that a mixed-reality, embodied learning environment that was developed for a single user, not only has the potential to help the individuals who are physically engaging with the environment learn but can also help students who are observing others engage with the environment learn. The implications of this result are likely significant for classroom teachers who do not have the resources, time, or support to provide an opportunity for every student to engage in an embodied learning task. For example, in our study, a space of approximately 10 feet by 10 feet was needed for the Kinect, screen, and user. There would not be space available, and maybe not the resources available to purchase multiple Kinect devices so that every student could physically engage with the learning environment at the same time. Although there are shortcomings when using this technology, this study showed there is tremendous potential to reach every student in a math class, whether or not they physically engage with the environment.

It is important to note that we intentionally constrained how the activity was implemented in the whole-class setting because we wanted to use the same protocol used in the individual interviews to assist us in making comparisons between the two settings. Without this constraint, we believe we could create an even better learning experience for students who are observing others engage with the learning task. First, students who are observing someone else engage with the embodied environment could be asked to use their bodies as well. One way to incorporate this would be to ask students who answer a question about how to make the screen a given color, to show how this is done with their bodies as they provide a verbal description. Although such students would be missing the immediate and visual feedback provided by the Kinect program, the work of Susan Gerofsky (2008, 2011) demonstrating the importance of *being*, rather than just *seeing* the graph, suggests using their bodies to act out the angle would increase their understanding. A second way to improve the learning experience for students who are observing would be to provide time for them to discuss the answers to each interview question with a partner or in a small group so that every student has an opportunity to share his or her ideas aloud. Lastly, the interviewer was purposely limited as to the questions she could ask, however the interviewer could potentially increase learning by asking questions that promote deeper and more complex thinking. For example, they could ask if there is more than one way to make the screen each color in an attempt to bring students' attention to the fact that there are multiple ways to make the screen each color.

When trying to understand why students who physically engage with the angle and those who observed someone else engage with the angle learned at similar rates, we looked at the language students used as they described their thinking during the task. The people observing were more likely to rely on the use of metaphor to describe their thinking. A metaphor was useful in this instance as the observers were likely trying to make sense of the outcome, the position where the body ended up when making a certain color. On the other hand, the students physically engaging with the environment were more likely to use spatial language to describe their thinking. This likely was a result of their efforts to make an action plan to describe how their bodies moved while creating a certain color. Even though students in the two settings engaged with the activity in different ways, both were able to use language in a way that helped them learn about angle measurement.

A limitation of this study was not accounting for the prior language capacity of the participants. As a result, we must consider whether the results about differences in the language used to reason about the angle task were due to prior differences in the language capacity of the participants. In future work in this area, students' prior language capacity should be taken into account and controlled for in the analyses. Another limitation of this study involved the type of questions asked on the assessment. Students were only asked to name or create static angles. Perhaps our finding that the setting (individual or class) was not predictive of assessment score was due to the types of questions asked on the assessment. We wonder whether the students who physically engage with the environment would be better able to think about the dynamic nature of angles, that is to understand that the size of an angle is related to the amount of turn between the straight lines of the angle (Clements & Battista, 1990; Mitchelmore & White, 1998). While investigating this was beyond the scope of the current project, future research using an embodied learning environment to study angles should explore students' thinking about dynamic and static angles.

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## Appendix A. Interview Protocol

*Turn on the Kinect program. Make sure the program registers the student. The screen will likely be pink.*

Ok, now we are going to do an activity using the Kinect. I need you to stand right about here for the activity (place an x on the floor 6-8 ft from the Kinect sensor with masking tape, if needed).

In this activity, you only move your arms, and you only move them out to the side, with your elbows and wrists straight, your body stays still and your arms move around, you can move your arms across your body—close into your body. Do you see how the screen is changing colors? (*Demonstrate for a relatively long time, move your arms, one at a time, both at the same time etc. Ask the student to try moving their arms around the way you are describing. Remind them not to bend their elbows or move their arms in front of their body.*) Verify that the student understands.

*There are not that many rules to this game. The rules are:*

1. *Your body stays straight*
2. *Your arms are straight at elbows and wrists*
3. *Move arms all around, keeping them out to your side. Like you are pressed against a big glass window, so they can't go in front of you.*
4. *You can move one arm across your body, but you have to keep them tight to your body with your elbows and wrists straight.*

*Use the colors that the student names, for example purple might be used instead of pink.*

*(As the student is moving their arms wait until the screen is some color other than pink, then ask the first question—is possible. If you ask about Pink and they can't find it, ask them what colors they see.)*

Q: Will you tell me how you can make the screen Pink?

*(The student will likely make the screen pink and provide some verbal response.)*

*If the student says "like this" or the response is unclear follow-up with a question – I don't understand. Can you tell me more?*

*Level of persistence for questioning: Move on once you feel satisfied that the student has explained their thinking to the level of depth they are capable of at that moment.*

Q: Can you find another way to make the screen Pink? (*The student will likely make the screen pink in a new way.*) *If the student does not give an explanation, ask a follow-up question. I don't understand. (Can you tell me in another way?) Only ask them one time to make pink a different way.*

*Note to the interviewer: Try to move slowly through the interview. Be patient and encourage the student to talk as much as possible, particularly early on. Give them plenty of time to explore the activity space. Provide positive feedback as necessary. For example, oh, I see what you mean or oh, I understand. (Aim for comments that are less about right or wrong and more about the fact that you understand the students' thinking.) Also, particularly if the student is shy you may want to try repeating back what the student said.*

Q: Can you tell me what you would do to make the screen yellow?

*(The student will likely make the screen yellow and provide some verbal response.)*

*If the student says "like this" or the response is unclear follow-up with a question. I don't understand. (Can you tell me in another way?)*

*Other potential follow-up questions: Can you tell me a little bit more about that? I'm not sure I understand or I'm not sure what you mean. Can you describe what you are doing in a different way?*

Q: Can you find another way to make the screen yellow? (*The student may make the screen yellow in a new way.*) *If the student is unable to find another way to make the screen yellow you can move on. If they find another way but don't explain what they did, ask a follow-up question. What did you do make the screen yellow that time? Only ask them one time to make yellow a different way.*

*Repeat these steps for light blue and dark blue.*

Use follow-up questions for understanding and ask for a second way to make the screen the given color for each of these.

I'm going to add an angle to the screen that will help you build on your ideas. Each time we add more information, we're going to ask you about your thinking. Ok, now try moving your arms around again. *(Add the angle with the smallest arrows)*

Q: What do you notice about the angle? *(Pause to give them time to experiment).*

Q: Can you explain to me how to make the screen pink?

*During this stage you should be asking follow-up questions:*

- Can you tell me a little bit more about that? I'm not sure I understand. What is the angle doing?
- I'm not sure what you mean. Can you describe what you are doing in a different way? What is the angle doing?
- *What do you mean by like this? What is the angle doing?*
- *Could you tell (the other person in the room) how he/she could make the screen pink? What is the angle doing?*

*Do not ask about a second way to turn the screen a color.*

Repeat these steps for yellow, light blue, and dark blue.

Have you used a protractor before? This is a protractor *(turn on the protractor)*. A protractor is used to describe the size of an angle. This number *(point to the angle measure on the screen)* is called the angle measure and tells you the size of the angle. We would say this angle is 45 degrees *(or whatever angle measure appears on the screen at this point)*.

I have to go check on something quickly in the hall. While I'm gone play around with the protractor and see what you find out. *(Give them about 30 seconds)*

Q: What did you notice about the protractor?

Q: Can you explain to me how to turn the screen Pink?

*During this stage you should be asking follow-up questions:*

- *What do you mean by like this? Can you describe how to make the screen pink using the angle measure?*
- *Could you tell (the other person in the room) how he/she could make the screen pink? Can you describe how to make the screen pink using the angle measure?*
- Can you tell me a little bit more about that? I'm not sure I understand. Can you describe how to make the screen pink using the angle measure?
- I'm not sure what you mean. Can you describe what you are doing in a different way? Can you describe how to make the screen pink using the angle measure?

*During this stage you should NOT ask them to show you another way to make the screen a certain color.*

Repeat these steps for yellow, light blue, and dark blue.

We are going to add one more thing to the screen.(add the longer arrows) What did we do?

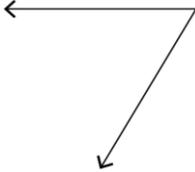
Ask about each color.

Ok, you did a great job explaining your thinking during the activity. I have one last question. If we invited a first or second grader into the room and asked you to explain this, what would you say?

## Appendix B. Pre-assessment

One question was presented to students at a time.

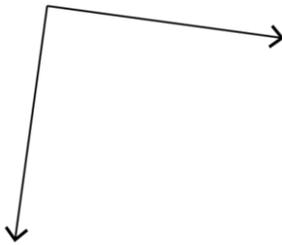
1. Degrees tell you the size of an angle. Draw an angle that is 90 degrees.
2. Degrees tell you the size of an angle. Draw an angle that is 30 degrees.
3. Degrees tell you the size of an angle. Draw an angle that is 150 degrees.
4. Degrees tell you the size of an angle. Draw an angle that is 180 degrees.
5. Estimate the size of this angle.



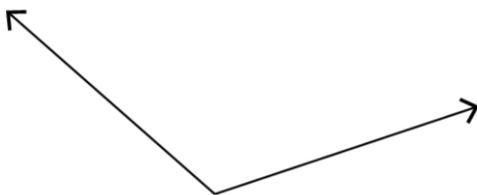
6. Estimate the size of this angle.



7. Estimate the size of this angle.



8. Estimate the size of this angle.



**Appendix C. Spatial Words used in the Study**

a lot	further	position
above	half(way)	rhombus
across	high	same
apart	left	shape
below	a little	side(ways)
between	less	small
big	low	square
bottom	middle	straight
by	more	to
center	much	together
circle	near	top
close	opposite	triangle
down(ward)	out(ward)	turn
far	over	under
flat	part	up(ward)
from	past	wide
	point	