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Effects of Animation-Supported Jigsaw Cooperative Learning on Students' Achievement in Geometry: Evidence from a Linear Mixed-Effects Model

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Abstract

Mathematics classrooms in Ghana continue to be dominated by traditional teacher-centered instruction, which often results in low student achievement, particularly in geometry. This study aimed to examine the effect of integrating computer animation with the Jigsaw cooperative learning approach on students' conceptual understanding of circle theorems. The main research question was: "Does teaching circle theorems through Jigsaw cooperative learning integrated with computer animation improve students' academic achievement compared to traditional methods?" A quasi-experimental design with pre- and post-tests was employed, involving 291 senior high school students from three public schools. The participants were distributed into three instructional groups: Traditional Lecture Method (TLM), Jigsaw Cooperative Learning (JCL), and Jigsaw Cooperative Learning with Animation (JCAP). Data were collected using the Circle Theorem Achievement Test (CTAT) and the Perceptual Experience Questionnaire (PEQ). A Linear Mixed-Effects Model (LMM) was used to analyze both individual-level and school-level effects. The findings revealed that instructional strategy and pre-test performance significantly predicted students' post-test achievement. Learners taught with JCL and JCAP outperformed those exposed to TLM by 3.68 ($p < 0.001$) and 5.50 ($p < 0.001$) points, respectively. Gender and perceptual experience showed no significant influence, while age exhibited a weak negative trend. The final model demonstrated better fit indices (AIC = 1665.19; BIC = 1705.60), confirming the robustness of the analysis. Consequently, integrating Jigsaw cooperative learning with computer animation was found to significantly enhance students' understanding of circle theorems by promoting interaction, visualization, and conceptual engagement. These results highlight the importance of embedding technology-supported cooperative pedagogies into mathematics curricula and teacher education programs to improve learning outcomes and align with Ghana's competency-based education framework.

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Introduction

Despite extensive research on innovative pedagogies in mathematics education, traditional, teacher-centered practices continue to dominate classrooms, particularly in developing contexts such as Ghana (Ashiboe-Mensah, 2021; Mwangi et al., 2018). Over the past two decades, scholars in the field have gradually shifted the focus from curriculum content and student performance to the instructional process—specifically, the teacher’s role in mediating learning through interactive and technology-enhanced approaches (Johnson et al., 2024; Lim, 2025). Within this discourse, geometry remains a critical yet persistently challenging component of mathematics education. Its emphasis on abstract reasoning and spatial visualization poses conceptual difficulties for many students, with topics such as circle theorems often yielding poor learning outcomes and limited conceptual understanding (Hershkowitz, 2020; Fabiyi, 2017; WAEC, 2021).

Teaching geometry effectively requires more than the transmission of formulas and theorems; it demands pedagogical innovation that promotes active engagement and conceptual visualization. Traditional lecture methods, which emphasize rote memorization, restrict student participation and hinder the development of deep mathematical reasoning (Hissan & Ntow, 2021). Consequently, there has been increasing advocacy for learner-centered and technology-supported instructional models that integrate collaboration, exploration, and visual representation into the teaching and learning process (Antonio & Castro, 2023). Two complementary strategies that have gained attention in this regard are cooperative learning, particularly the Jigsaw model, and the use of computer animation as a multimedia instructional tool.

The Jigsaw cooperative learning approach fosters peer collaboration by assigning students interdependent roles, enabling them to construct shared understanding through discussion and teaching one another (Aronson, 1978; Johnson et al., 2024). Moreover, computer animation provides a powerful medium for visualizing dynamic geometric relationships and enhancing comprehension of abstract mathematical ideas (Gambari et al., 2014; Asiedu, 2022). Drawing on Paivio’s Dual Coding Theory, animation supports the integration of verbal and visual information, facilitating deeper conceptual processing and long-term retention (Paivio, 2014). While prior studies have demonstrated the individual benefits of cooperative learning and multimedia instruction, limited empirical evidence exists on their combined use in improving students’ understanding of complex geometry concepts such as circle theorems.

To address this gap, the present study investigates the effectiveness of integrating Jigsaw cooperative learning with computer animation as a combined instructional strategy for teaching circle theorems in senior high schools. The study further explores how demographic and perceptual factors—specifically age, gender, and perceptual experience—interact with instructional methods to influence students’ achievement. Employing a Linear Mixed Effects Model (LMM), the analysis accounts for both fixed effects (instructional strategy, test type, demographic factors) and random effects (class-level variation), offering a robust framework for examining nested educational data structures (Li et al., 2024).

Accordingly, the study was guided by the following research question: What factors influence students’

achievement in circle theorems? The corresponding objective was to investigate the factors that influence students' achievement in circle theorems. This study adds to the literature by integrating cooperative learning and multimedia animation within a rigorous analytical framework to uncover how pedagogical, demographic, and perceptual factors jointly shape learning outcomes in geometry. It advances existing research by combining pedagogical innovation (Jigsaw with animation) with quantitative modeling (LMM), bridging the gap between experimental design and classroom realism. Furthermore, it provides empirical evidence from an African context; an area often underrepresented in mathematics education research; thus enhancing the global understanding of context-sensitive instructional effectiveness. The findings are expected to inform teacher training, curriculum design, and policy formulation aimed at fostering meaningful and equitable learning experiences in mathematics.

Theoretical and Empirical Foundations of the Study

Literature Review

The complexity that innovative pedagogy introduces into mathematics teaching has been well documented (Clark-Wilson et al., 2014). Teaching geometry in particular requires managing cognitive, visual, and collaborative dimensions simultaneously, a challenge that often explains teachers' reluctance to depart from traditional methods (Mwangi et al., 2018; Ashiboe-Mensah, 2021). Studies across diverse contexts indicate that while lecture-based approaches ensure syllabus coverage, they constrain student reasoning and spatial understanding, leading to persistent misconceptions in geometry (Hershkowitz, 2020; Hissan & Ntow, 2021).

Research in mathematics education has progressively shifted from focusing solely on content mastery to understanding how instructional strategies influence conceptual development. Cooperative learning approaches, especially the Jigsaw model, have been shown to foster peer interaction and accountability through structured collaboration (Aronson, 1978; Johnson et al., 2024). In such settings, learners construct shared understanding by assuming both learner and teacher roles, promoting metacognitive reflection and collective responsibility for problem-solving. Yet, empirical evidence reveals that teachers sometimes struggle to facilitate effective interdependence or ensure equitable participation, challenges that can undermine the full potential of the Jigsaw method (Lim, 2025).

Parallel to this development, technology integration—particularly through dynamic visualization—has become a focal point of mathematics instruction research. Computer animation and dynamic geometry software have been found to enhance spatial reasoning, foster conceptual clarity, and sustain motivation by linking abstract ideas to visual representations (Gambari et al., 2014; Asiedu, 2022). Theoretical perspectives such as Dual Coding Theory (Paivio, 2014) and Cognitive Theory of Multimedia Learning (Mayer, 2009) explain that presenting information through both verbal and visual channels enhances cognitive processing and retention. However, as with cooperative learning, studies show that without guided pedagogy and reflection, the use of animation may lead to passive observation rather than active cognitive engagement (Clark-Wilson & Noss, 2015).

Emerging literature on integrated pedagogical models suggests that combining collaborative learning with multimedia instruction can produce synergistic effects on understanding mathematical concepts (Antonio &

Castro, 2023; Li et al., 2024). By encouraging discussion around animated visualizations, students connect formal geometric reasoning with dynamic representations, promoting both procedural fluency and conceptual insight. Yet, despite these promising findings, few studies have explored the interaction between cooperative structures, multimedia design, and learner characteristics such as age, gender, or perceptual experience—factors that may significantly mediate achievement outcomes.

This study builds upon these strands of research by examining how instructional strategies, together with learner characteristics, influence achievement in circle theorems—a topic that encapsulates many of the cognitive challenges of geometry learning. Using a Linear Mixed Effects Model (LMM) framework, the study investigates how Traditional, Jigsaw, and Jigsaw with Animation approaches affect student achievement while accounting for individual and group-level variance. This perspective extends prior literature by quantifying the pedagogical and demographic interactions that shape mathematical learning in authentic classroom contexts.

Theoretical Tools and Framework

The theoretical underpinnings of this study are informed by Vygotsky's Sociocultural Theory (1978), Cognitive Theory of Multimedia Learning (Mayer, 2009), and Cognitive Load Theory (Sweller, 2010). Together, these frameworks explain how collaborative, technology-enhanced environments mediate learning through social interaction, cognitive processing, and mental effort regulation.

Vygotsky's Sociocultural Theory

Vygotsky's Sociocultural Theory emphasizes that learning occurs through social mediation and interaction within the learner's Zone of Proximal Development (ZPD). In the context of this study, the Jigsaw cooperative learning model operationalizes Vygotsky's principles by situating knowledge construction within peer interaction. Each learner's understanding develops through collaboration, scaffolding, and dialogue, which transform social exchanges into internalized knowledge structures. By integrating animation, the cooperative environment provides mediational tools—visual and interactive representations—that scaffold students' reasoning and make abstract relationships in circle theorems more accessible. Thus, the learning process becomes both social and semiotic, shaped by communicative tools and guided interactions.

Cognitive Theory of Multimedia Learning

According to Mayer (2009), meaningful learning occurs when learners actively select, organize, and integrate verbal and visual information across dual cognitive channels. The Cognitive Theory of Multimedia Learning (CTML) supports the inclusion of computer animation in this study, as it aids students in visualizing the dynamic relationships among angles, chords, and tangents. Animation facilitates the integration of dynamic and symbolic representations, allowing students to mentally manipulate geometric relationships rather than rely solely on static diagrams. Within the Jigsaw structure, these multimedia elements become collaborative learning objects, prompting dialogue and negotiation of meaning among peers.

Cognitive Load Theory

Cognitive Load Theory (CLT) (Sweller, 2010) complements CTML by addressing how instructional design influences mental effort. Integrating Jigsaw and animation must balance cognitive demand to avoid overwhelming working memory. The segmentation of learning tasks inherent in the Jigsaw model distributes processing load, while animations, when used effectively, reduce extraneous load by providing clear, temporally aligned visualizations. Together, these mechanisms optimize germane cognitive load—the mental resources devoted to constructing and automating schemas—thereby supporting deep conceptual understanding of circle theorems.

Integrative Theoretical Perspective

Synthesizing these frameworks provides a comprehensive view of learning as both a social and cognitive process mediated by technology. Vygotsky's sociocultural lens explains how interaction fosters knowledge construction; CTML elucidates how visual and verbal integration enhances comprehension; and CLT clarifies how instructional design regulates mental effort. The intersection of these perspectives underpins the combined use of Jigsaw and animation as a pedagogically coherent approach to teaching circle theorems. Grounded in this theoretical synthesis, the study explores how cooperative learning structures, multimedia visualization, and learner characteristics interact to influence achievement outcomes. This theoretical triangulation informed the research design and interpretation of results, providing a foundation for understanding not only whether these approaches improve performance, but why and how they do so within real classroom settings.

Methods

Context and Participants

Building on the theoretical perspectives outlined earlier, this study was conducted among senior high school students in Ghana to examine how instructional strategies and learner characteristics influence achievement in circle theorems. The research took place in three public senior high schools within the Kwabre East Municipality, a district known for its emphasis on mathematics and science education. The schools were purposively selected based on the availability of functioning computer laboratories and willingness to participate in a structured instructional intervention.

Participants were second-year elective mathematics students who had already been introduced to foundational geometry concepts in their first year. A total of 291 students took part in the study, distributed across the three selected schools. The sample size was determined using Yamane's (1967) formula for finite populations, ensuring adequate representation while maintaining manageable class sizes. Each school served as one intact instructional group to preserve natural classroom dynamics. Random assignment of schools; rather than individual students; to the instructional conditions minimized selection bias and maintained ecological validity.

Teachers responsible for implementing the interventions were experienced mathematics educators with comparable teaching qualifications. Prior to data collection, they participated in a standardized training workshop

organized by the research team. This training introduced the Jigsaw cooperative learning procedure, the use of the computer animation package (CAP), and the standardized lesson sequence for teaching circle theorems. The aim was to ensure fidelity of implementation and consistency in pedagogical delivery across all groups.

Research Design and Intervention

A quasi-experimental, pretest–posttest, non-equivalent group design was employed to investigate the effects of instructional strategies on students' achievement. This design was appropriate because random assignment of individual students was not feasible within existing classroom structures, yet it permitted comparison of learning outcomes while controlling for initial differences through pretesting (Creswell & Creswell, 2017).

Three instructional conditions were compared:

1. Jigsaw with Animation (JCAP): cooperative learning integrated with a Computer Animation Package developed by the researcher using PowerPoint and Adobe Photoshop to illustrate core circle theorems dynamically;
2. Jigsaw Only (JCL): cooperative learning without the animation support;
3. Traditional Lecture (TLM): conventional teacher-centered instruction following the textbook approach.

The intervention was implemented over five weeks. The first two weeks were dedicated to preparatory activities, including securing administrative consent, installing the CAP software, conducting teacher training, and orienting students to the study procedures. Weeks 3 and 4 involved the delivery of the instructional treatments according to group assignment, while week 5 was reserved for administering posttests and perceptual-experience questionnaires. In the JCAP group, the animated lessons presented theorems such as the angle at the center, angles in the same segment, and cyclic quadrilaterals. The animations simulated movement of chords, tangents, and radii, allowing students to observe relationships dynamically. In the JCL group, the same topics were taught using the Jigsaw model alone, where students were divided into expert and home groups to research, discuss, and re-teach the assigned sub-topics. The TLM group received direct instruction through lectures and board demonstrations. Throughout the intervention, classroom visits were conducted by the research team to ensure adherence to the assigned instructional models. Any deviations were documented and addressed through feedback sessions with teachers. This systematic monitoring strengthened internal validity by ensuring that observed effects were attributable to the instructional strategies rather than procedural inconsistencies.

Instruments

Two principal instruments were used for data collection: the Circle Theorem Achievement Test (CTAT) and the Perceptual Experience Questionnaire (PEQ).

The CTAT consisted of 20 multiple-choice items derived from the Ghana SHS Core Mathematics syllabus and aligned with prescribed learning objectives for circle theorems. The test measured conceptual understanding, application, and reasoning. Content validity was established through expert review by mathematics educators and curriculum specialists. Reliability was determined using the Kuder-Richardson Formula 20 (KR-20), yielding coefficients of 0.75 for the pretest and 0.78 for the posttest, confirming acceptable internal consistency.

The PEQ comprised 20 Likert-scale items capturing students' cognitive, emotional, and engagement-related perceptions of the instructional approaches. Sample items included statements such as "The method of teaching helped me understand the relationships between angles and chords" and "I enjoyed learning through this classroom approach." Responses ranged from 1 (Strongly Disagree) to 5 (Strongly Agree). The instrument's reliability was confirmed through a pilot study, producing a Cronbach's alpha of 0.81.

Procedure

Data collection followed the sequential structure of the intervention. Ethical clearance and institutional consent were obtained prior to fieldwork. In the preparatory phase, teachers were trained, and the CAP materials were installed in the computer laboratories. During the second week, the pretests (CTAT) and baseline PEQs were administered under standardized conditions. After four weeks of instruction, the posttests and final PEQs were administered to capture both achievement gains and perceptual shifts. Throughout the data-collection period, the researcher and trained assistants conducted classroom observations to ensure adherence to procedures and verify the authenticity of the data. Data from completed instruments were screened for accuracy, coded, and entered into IBM SPSS Statistics Version 29 for analysis.

Analytical Strategy

Data analysis proceeded in two stages. First, descriptive statistics—means, standard deviations, and frequency distributions; were computed to summarize overall performance and identify patterns of improvement across instructional conditions. Second, to account for the hierarchical data structure (students nested within instructional groups), a Linear Mixed-Effects Model (LMM) was employed. The LMM was estimated using maximum likelihood procedures to simultaneously assess fixed effects (instructional strategy, test type, age, gender, and perceptual experience) and random effects (group-level variability). This approach provided a more precise estimation of the contribution of individual- and group-level factors to achievement outcomes.

To facilitate interpretation, dummy coding was applied to the categorical predictors. The instructional-group

variable was coded as follows:

$$Group_i = \begin{cases} 1, & \text{Control Group (traditional method)} \\ 0, & \text{Jigsaw with Animation (reference)} \\ 2, & \text{Jigsaw Cooperative learning group} \end{cases}$$

The test-type variable was coded as:

$$Test_j = \begin{cases} 1, & \text{Pretest} \\ 0, & \text{Posttest} \end{cases}$$

This coding structure allowed for direct interpretation of contrasts between instructional approaches and between pre- and post-test performance. Regression coefficients thus represented differences relative to the reference categories. The statistical significance of fixed effects was evaluated using likelihood-ratio tests, and 95 % confidence intervals were computed for all parameter estimates. By distinguishing between within-group and between-group variance, the LMM framework controlled for interclass correlations and improved the generalizability of findings. This modelling strategy was instrumental in identifying the relative influence of

instructional strategies, demographic characteristics, and perceptual experiences on students' achievement in circle theorems.

Validity, Reliability, and Ethical Considerations

Ensuring methodological rigor was central to the study. The Circle Theorem Achievement Test (CTAT) and the Perceptual Experience Questionnaire (PEQ) were reviewed by mathematics education experts to establish content and construct validity and ensure alignment with the SHS curriculum. A pilot study confirmed clarity and contextual appropriateness of all items. Reliability analyses indicated strong internal consistency, with KR-20 coefficients of 0.75 (pre-test) and 0.78 (post-test) for the CTAT and a Cronbach's α of 0.81 for the PEQ, all exceeding accepted thresholds. Ethical approval was granted by the Graduate School of Kwame Nkrumah University of Science and Technology (KNUST) prior to data collection. Participation was voluntary, and informed consent was obtained from both school authorities and students. Respondents were assured of anonymity, confidentiality, and the exclusive academic use of all data.

Results

Overview of the Analysis

This section presents the results of the quantitative analysis examining the effects of instructional strategy, learner characteristics, and perceptual experience on students' achievement in circle theorems. The analysis followed two main stages: (a) descriptive statistics to summarize overall performance patterns across instructional groups and testing occasions, and (b) inferential modeling using a Linear Mixed-Effects Model (LMM) to account for the hierarchical structure of the data, students nested within schools. This approach ensured that both individual-level and group-level variations were adequately modeled, yielding more accurate and generalizable estimates of the factors influencing achievement.

Descriptive Statistics

Descriptive statistics provided an initial understanding of students' performance patterns across instructional conditions. Across the three groups—Traditional Lecture Method (TLM), Jigsaw Cooperative Learning (JCL), and Jigsaw Cooperative Learning integrated with Computer Animation (JCAP)—students generally demonstrated improvement from pre-test to post-test.

Table 1 presents the descriptive statistics for students' performance on the pre-test and post-test across instructional groups. The mean pre-test scores across schools ($M = 10.0$ and $M = 12.19$) reflect a relatively moderate baseline understanding of circle theorems prior to the intervention. Following instruction, all groups exhibited improvement, though the magnitude of gains varied across the three instructional conditions. Students in the Traditional group recorded a post-test mean of 10.97, reflecting only marginal improvement over their pre-test performance. This modest gain suggests that teacher-centered instruction facilitated limited conceptual advancement, likely constrained by the passive learning environment characteristic of traditional pedagogy.

Table 1. Descriptive Statistics on the Pre-test and Post-test for the Groups

Descriptive Variable	Pre-test				Posttest	
	School A	School B	School B	Traditional	Jigsaw	Jigsaw with Animation
Minimum	4	2	4	3	7	5
Maximum	18	21	21	21	24	25
Mean	10.0	12.19	13.13	10.97	14.81	16.74
Std. Deviation	3.99	4.99	4.40	4.19	4.38	4.25
Skewness	0.341	0.018	-0.287	.312	-.024	-0.094
Kurtosis	-0.761	-0.907	-0.857	-.648	-1.038	-0.296

By contrast, participants in the Jigsaw cooperative learning (JCL) group achieved a higher post-test mean of 14.81, indicating a more pronounced improvement. This result suggests that collaborative learning enhanced engagement and knowledge sharing, fostering deeper comprehension of geometric relationships. The Jigsaw with Animation (JCAP) group exhibited the greatest improvement, with a mean post-test score of 16.74. This result highlights the pedagogical advantage of integrating computer animation into cooperative learning, as visual representations likely aided the abstraction and retention of complex geometrical concepts such as chord relationships, tangent properties, and angle theorems.

Furthermore, the relatively low standard deviations across groups (ranging from 4.19 to 4.99) indicate consistent performance among students, suggesting that the instructional interventions were broadly effective across ability levels. The near-normal skewness and negative kurtosis values across groups further support the distributional stability of the test scores, with minimal outlier influence.

Model Fitting and Interpretation

Linear Mixed-Effects Model

To investigate the combined and individual effects of the predictors on students' post-test achievement, an LMM was employed. This model incorporated both fixed effects (instructional strategy, test type, gender, age, and perceptual experience) and a random effect (school-level variance). By doing so, it accounted for potential dependency among students within the same school and partitioned variance attributable to between-school differences. The model estimated post-test achievement as a function of the predictors, with pre-test and perceptual experience entered as continuous covariates, and group, gender, and age as categorical variables. Each parameter's coefficient was interpreted as the expected change in achievement associated with a one-unit change (for continuous variables) or relative to a reference category (for categorical variables), holding all other variables constant.

Null Model (Random Intercept Only)

The baseline or null model included only a random intercept for schools, without any fixed predictors. This model assessed whether students' post-test scores varied significantly across schools. The estimated grand mean post-

test score across all schools was 14.10 (SE = 0.78, $p < 0.001$), reflecting the average achievement prior to accounting for any explanatory variables. The between-school variance was estimated at 1.58 (SE = 1.81), while the within-school (residual) variance was 22.95 (SE = 1.91). Using these variance estimates, the Intraclass Correlation Coefficient (ICC) was calculated as:

$$ICC = \frac{\sigma^2_{school}}{\sigma^2_{school} + \sigma^2_{residual}} = \frac{1.5751}{1.5751 + 22.9632} \approx 0.064$$

Thus, approximately 6.4% of the total variance in post-test achievement was attributable to differences between schools, with the remaining 93.6% explained by individual-level factors. This modest ICC justified the inclusion of a random effect for schools in the LMM, ensuring accurate estimation of fixed effects while accounting for the nested structure of the data.

Full Model with Fixed Effects

The full LMM incorporated all predictor variables simultaneously. Table 2 presents the fixed-effect estimates, standard errors, z-values, p-values, and 95% confidence intervals. The intercept ($\beta_0 = 9.18$, SE = 1.42, $p < 0.001$) represents the predicted post-test score for a baseline student—one in the Traditional group, male, aged 10–13 years, with average perceptual experience and a pre-test score of zero—adjusted for random school effects.

Table 2. Fixed Effects Estimates (Full Model)

Variable	Coef.	Std. Err.	z	p-value	95% CI Lower	95% CI Upper
Pretest	0.1682	0.0627	2.68	0.007	0.045	0.291
Group						
Traditional	Ref					
Jigsaw	3.6825	0.6226	5.91	<0.001	2.462	4.903
Animation	5.4955	0.6423	8.56	<0.001	4.237	6.754
Gender						
Male	Ref					
Female	0.753	0.6215	1.21	0.226	-0.465	1.971
Age						
10–13	Ref					
14–17	-1.142	0.8389	-1.4	0.173	-2.786	0.502
18–21	-1.812	0.9565	-1.9	0.058	-3.686	0.063
21+	-0.218	1.1895	-0.2	0.855	-2.549	2.113
PER	0.0898	0.1299	0.69	0.49	-0.165	0.344
Intercept	9.1799	1.4218	6.46	<0.001	6.393	11.967

Pre-test performance emerged as a significant predictor of post-test achievement ($\beta = 0.17$, $p = 0.007$). Specifically, a one-point increase in pre-test score was associated with a 0.17-point improvement in post-test

score, suggesting that students who began with stronger prior knowledge achieved higher outcomes. Instructional strategies also demonstrated substantial and statistically significant differences across groups. Students exposed to the Jigsaw method scored an average of 3.68 points higher ($p < 0.001$) than those taught through the Traditional method, while those in the Jigsaw with Animation group achieved an even greater gain of 5.50 points ($p < 0.001$) relative to their counterparts in the Traditional group. This pattern underscores the effectiveness of interactive, student-centered instructional approaches, with animation-based learning producing the greatest improvement in achievement.

Gender differences were minimal; female students scored slightly higher than males ($\beta = 0.75$, $p = 0.226$), though the difference was not statistically significant. Thus, gender did not emerge as a key determinant of achievement in circle-theorem learning. Age category also showed a non-significant trend, with older students performing slightly lower than their younger counterparts. Compared to the youngest cohort (10–13 years), those aged 14–17 ($\beta = -1.14$, $p = 0.173$), 18–21 ($\beta = -1.81$, $p = 0.058$), and 21+ ($\beta = -0.22$, $p = 0.855$) scored marginally lower. This pattern suggests that younger students may be more receptive to cooperative and multimedia-based instructional methods, possibly due to higher adaptability and engagement levels.

Finally, perceptual experience, which captures students' cognitive and emotional engagement during learning, was positively but not significantly related to achievement ($\beta = 0.09$, $p = 0.490$). While students generally reported favorable learning experiences, these perceptions did not directly translate into improved performance. Collectively, the findings indicate that instructional strategy and pre-test achievement were the strongest predictors of learning outcomes, with cooperative and animation-based pedagogies exerting the most substantial positive effects on student achievement.

Random Effects and Variance Components

Table 3 summarizes the random-effect estimates for the full model. The between-school variance decreased from 1.58 in the null model to 0.74 (SE = 0.99) after including the fixed predictors, while the residual (within-school) variance decreased to 16.88 (SE = 1.43).

Table 3. Random Effects Estimates (Full Model)

Random Effect	Estimate	Std. Error	95% Confidence Interval
School: var(_cons)	0.7378	0.9884	[0.053, 10.192]
Residual	16.8802	1.4263	[14.304, 19.920]

This reduction in school-level variance yielded an adjusted ICC of 0.0417, indicating that only about 4.2% of total variance in post-test achievement was attributable to differences between schools after accounting for fixed effects. Compared to the null model (ICC = 6.4%), this suggests that part of the variation previously linked to schools was explained by the included instructional and demographic factors. Most of the remaining variability thus reflects individual differences among students rather than institutional context.

Model Comparison and Diagnostics

Model Fit Evaluation

The fit of the null and full models was compared using the log-likelihood, Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). Table 4 presents the model fit statistics comparing the null model, which includes only random intercepts, against the full model comprising all fixed effect predictors, including Test, Group, Gender, Age, and Perceptual Experience (PER). The models were evaluated using Log-Likelihood, Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). The null model exhibited a Log-Likelihood of -870.72 with 3 estimated parameters, whereas the full model achieved a substantially higher Log-Likelihood of -821.60 with 11 parameters. This appreciable difference in Log-Likelihood values indicates that the full model provides a significantly better explanation of the observed data, given that higher (less negative) Log-Likelihood values denote improved model fit. Further evidence supporting the superiority of the full model is found in the AIC and BIC values. The full model's AIC (1665.19) and BIC (1705.60) are markedly lower than those of the null model (AIC = 1747.43, BIC = 1758.45). Since both AIC and BIC apply penalties for model complexity, these reductions suggest that the increase in the number of predictors is justified by the improvement in fit. This balance between goodness-of-fit and parsimony confirms that including covariates substantially enhances the model's ability to capture variation in student post test scores. Consequently, these model comparison results demonstrate that the full model is preferable to the null model for subsequent analyses, as it provides a more accurate and informative representation of the factors influencing student achievement in this study.

Table 4. Model Fit Comparison

Model	Log-Likelihood	Parameters	AIC	BIC
Null Model	-870.72	3	1747.43	1758.45
Full Model	-821.60	11	1665.19	1705.60

Diagnostic Checks

Visual diagnostic checks were conducted to assess the adequacy of the linear mixed-effects model. The Q-Q plot of residuals Figure 1 showed that the residuals followed an approximately straight line, indicating that the assumption of normality was reasonably satisfied.

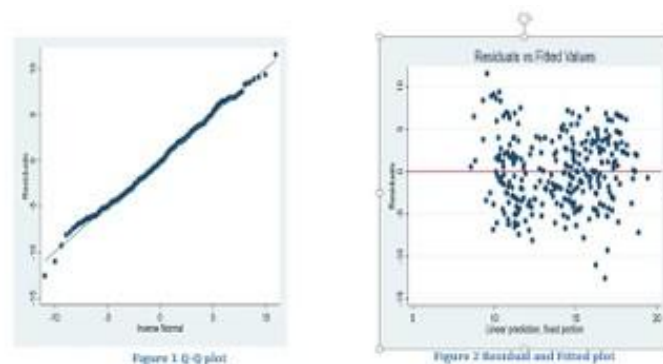


Figure 1. Q-Q Plot of Residuals from the Linear Mixed Model

Additionally, the residuals vs fitted values plot 1 exhibited a random scatter of points around the horizontal axis with no discernible pattern, suggesting that the assumption of homoscedasticity (constant variance) was not violated. These visual diagnostics confirm that the model's residuals are approximately normally distributed and exhibit constant variance. Taken together, these findings support the overall adequacy of the specified linear mixed model in capturing the underlying data structure and indicate that the key assumptions required for valid statistical inference have been met.

Model Equation and Interpretation

To represent the hierarchical structure of the data, the fitted Linear Mixed Model (LMM) is expressed as:

$$\hat{Y}_{ij} = \beta_0 + \beta_1(\text{Test}_{ij}) + \beta_2(\text{Group}_{ij}) + \beta_3(\text{Age}_{ij}) + \beta_4(\text{Gender}_{ij}) + \beta_5(\text{PER}_{ij}) + \mu_j + \varepsilon_{ij}$$

where μ_j denotes the random effect for school j and ε_{ij} represents the individual-level residual error.

Substituting the estimated coefficients yields:

$$\hat{Y}_{ij} = 9.18 + 0.17(\text{Test}) + 3.68(\text{Jigsaw}) + 5.50(\text{Animation})$$

This predictive model reflects the combined influence of instructional and learner-level factors on student achievement. The intercept corresponds to the expected score for a male student aged 10–13 years in the Traditional group at pre-test, while deviations represent adjustments for alternative categories. The positive coefficients for Jigsaw and Animation clearly demonstrate the learning advantages of interactive and technology-enhanced methods.

Discussion and Conclusions

This study examined the effects of three instructional strategies—traditional lecture, Jigsaw cooperative learning, and Jigsaw cooperative learning integrated with computer animation—on students' achievement in circle theorems. The findings highlight the effectiveness of integrating collaborative and multimedia-enhanced instruction in improving students' conceptual understanding of geometry. Cross-analysis of pretest and post-test results revealed significant gains for students exposed to the Jigsaw and Jigsaw-with-animation approaches compared to those taught through conventional methods. The Jigsaw-with-animation group recorded the highest post-test mean score, confirming the synergistic value of combining cooperative learning with visual animation to enhance engagement and cognitive processing. These findings provide empirical support for learner-centered pedagogies that utilize technology to facilitate meaningful mathematical understanding.

The Linear Mixed-Effects Model (LMM) analysis provided a robust framework for understanding the

determinants of student performance. By incorporating both fixed and random effects, the model accounted for the nested structure of the data, where students were grouped within schools. The full model significantly improved explanatory power over the null model, as shown by reduced AIC and BIC values, indicating that the fixed predictors meaningfully contributed to explaining variability in achievement. Instructional group emerged as the strongest predictor, with students in the Jigsaw and Jigsaw-with-animation conditions outperforming those in the traditional group by 3.68 and 5.50 points, respectively ($p < 0.001$). The post-test effect ($p = 0.007$) confirmed measurable learning gains, while the decline in intraclass correlation (ICC) from 6.4% to 4.2% demonstrated that differences between schools were partially explained by the instructional interventions. In contrast, gender and perceptual experience (PER) were not statistically significant, and age showed a marginally negative effect, particularly among the 18–21 age group ($p = 0.058$). These findings align with previous research emphasizing that once instructional quality is optimized, demographic variables exert less influence on academic outcomes (Hattie, 2023).

From a pedagogical perspective, the superior performance of the Jigsaw and animation-supported groups underscores the effectiveness of cooperative learning and multimedia instruction in mathematics education. This supports the assertions of Slavin (2022) that collaborative learning structures enhance social interaction and shared cognition, fostering deeper understanding. Similarly, Mayer's (2024) Cognitive Theory of Multimedia Learning suggests that integrating verbal and visual information channels enhances comprehension by reducing cognitive overload. The results of this study confirm these theoretical assumptions, showing that animation can help concretize abstract geometric ideas, particularly when combined with peer collaboration. The absence of a significant gender effect corroborates findings from Hattie (2023) and other meta-analyses, which argue that high-quality instruction benefits all learners irrespective of gender. However, the no significant role of PER differs from studies such as Tan et al. (2023), who emphasized the importance of spatial reasoning in geometry learning. One possible explanation is that the strong scaffolds provided by animation compensated for differences in perceptual ability, making spatial reasoning less decisive under multimedia-enhanced instruction.

The modest but nontrivial ICC indicates that although instructional methods explain much of the variation in student outcomes, contextual factors such as teacher expertise, school resources, and classroom environments still matter. As Hattie (2023) and Mayer (2024) note, pedagogical success depends not only on method but also on implementation fidelity. This implies that schools and teacher education programs should prioritize continuous professional development that equips teachers to design, deliver, and evaluate cooperative and animation-based instruction effectively. In Ghana, where the National Pre-Tertiary Curriculum Framework promotes competency-based learning, these findings are timely. They suggest that integrating animation-supported cooperative methods can advance curriculum goals related to problem-solving, collaboration, and digital literacy.

From a methodological standpoint, the use of the LMM strengthened the study by accurately partitioning variance at the individual and group levels. The reduction in between-school variance after including instructional predictors demonstrates that multilevel modeling can provide more precise estimates than single-level analyses. Nevertheless, future research could extend this by including random slopes for instructional strategies to assess whether treatment effects vary across schools. Moreover, cross-level interactions could be explored to determine

whether factors such as age or PER moderate the impact of cooperative or animation-based learning. While residual diagnostics confirmed model adequacy, employing simulation-based residuals or Bayesian approaches could provide even more robust inference.

The study, however, is not without limitations. It focused exclusively on circle theorems, which may restrict generalizability to other mathematical topics. Prior research (Gambari & Yusuf, 2016) suggests that instructional effects can differ across domains; therefore, replication across algebra, trigonometry, and statistics is recommended. Additionally, the operationalization of PER could be refined through validated spatial reasoning tests or dynamic visualization assessments to capture its multidimensional nature. The relatively low between-school variance may reflect the homogeneity of participating schools or the standardized nature of intervention delivery. Incorporating longitudinal designs with multiple assessment points would enable examination of long-term retention effects and learning trajectories.

From a theoretical viewpoint, this study reinforces the complementary roles of social constructivism and multimedia learning in fostering deep mathematical understanding. The cooperative learning framework promotes peer interaction, dialogue, and shared reasoning, while animation serves as a cognitive scaffold that visualizes abstract relationships. The convergence of these frameworks enhances both motivation and comprehension, affirming the value of integrated pedagogical innovation.

Overall, this study contributes to the expanding literature on technology-enhanced cooperative learning by providing empirical evidence from a Ghanaian context. The findings confirm that combining Jigsaw cooperative learning with computer animation significantly improves students' achievement in circle theorems compared to traditional instruction. Although demographic factors such as gender, age, and PER did not significantly predict achievement, the results emphasize the overriding influence of instructional design. These insights have practical implications for policy and curriculum reform, highlighting the need for mathematics teacher education programs to include structured training in technology-supported cooperative pedagogy.

Finally, future research should explore how teacher-level variables—such as pedagogical content knowledge, digital literacy, and classroom management—mediate the effectiveness of cooperative multimedia instruction. As suggested by Mayer (2024) and Çeken and Taşkın (2022), the quality of multimedia teaching depends as much on design and delivery as on content. The integration of quantitative and qualitative findings in this study reinforces the power of a mixed-methods approach: while statistical modeling identified significant instructional effects, teacher and student reflections contextualized how these approaches enhanced engagement and conceptual understanding. Taken together, these results confirm that blending collaboration and visualization represents a powerful pedagogical pathway for deepening mathematical understanding and fostering a more interactive and equitable learning environment.

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