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Seda Sahin[1], Muhammed Fatih Dogan[1], Zeynep Cavus Erdem[2], Ramazan Gurbuz[1], Ali Temurtas[1]
[1]Adiyaman University
[2]Mehmet Akif Ersoy Secondary School

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Abstract

This study examined the criteria used by prospective mathematics teachers in determining whether a problem was a mathematical modeling problem. Two data collection tools were used in this case study. The first was a written evaluation form which consists of four problems developed by 27 prospective teachers individually that enrolled in a Mathematical Modeling course. Among them, only one was a mathematical modeling problem and the others were traditional problems with some features of mathematical modeling problems. The other data collection tool was a semi-structured interview form which was used in the interviews with 12 prospective teachers. The data were analyzed using open and axial coding. The findings showed that pre-service teachers had some incorrect or incomplete information about mathematical modeling. The criteria used by teacher candidates were grouped under three categories: content characteristics, formal characteristics and outcome-based characteristics of the problems. The most common criteria used by the prospective teachers in evaluating the problems was the suitability of real life, being complex or thought-provoking and being interesting. In addition, it was determined that teacher candidates did not adopt standard criteria and that some of the criteria were shaped according to the problems.

Introduction

Mathematical Modeling

Mathematical modeling is the process of associating real life and mathematics using mathematical skills (Lesh and Doerr, 2003) and starting with a situation that reflects a problem in real life. The problem solver simplifies, constructs and uses appropriate conditions and assumptions to reflect this real-life situation in a more understandable way (Blum and Niss, 1991). Mathematical modeling requires real problem situations and an underlying relationship between these situations and their mathematical representations (Blum and Niss, 1991; Niss, 1989). In other words, the mathematical model is the external representation of the modeling process. (Lesh and Doerr, 2003). Blum and Niss (1991) call the process of transforming a real life situation to mathematical language *mathematizing*. Since the modeling is a complex process, it should not merely be perceived as the simplification of the real life situation. Mathematical modeling has also an individual reality that reflects the individual's knowledge, perspective and purpose about the situation (Blum and Niss, 1991). While the main objective of modeling problems is to encourage students to solve the real life situation mathematically, another objective is to help students gain independent thinking and working skills (Meier, 2009). Therefore, mathematical modeling is a subjective process and involves an individual's effort to reach his/her own reality. An individual's learning process is one of the most important features that mathematical modeling problems distinguish from traditional problems. In traditional problems, students do not feel in stuck and try to reach a certain result by applying the appropriate operations expected from them according to the correct procedure. However, mathematical modeling activities create a feeling of helplessness and insecurity in the person who solves the problem and modeling can only be learned by individual efforts (Kaiser, Schwarz and Buchholtz, 2011). There is no certainty in mathematical modeling. Mathematical modeling requires that the person who solves the problem to make assumptions and choose the ones that he/she thinks are important among the necessary variables for the solution. Therefore, rarely identical models emerge and for the same reason models cannot represent the real world perfectly (Gould, 2016). However, no matter how different, all models are acceptable as long as the assumptions and estimates are reasonable and the solution process is logical (Mathematical Association of America [MAA], 1972). This characteristic of mathematical modeling challanges the perception of *'there is only one result and all the information necessary to achieve this result is given in the problem'* about traditional problems.

Another feature that separates mathematical modeling from traditional problem solving is the process of mathematization involved in mathematical modeling. In traditional problems, everything is usually presented in a mathematical form and association is built with numbers, symbols and/or graphics. This is not the case for mathematical modeling. It is the responsibility of the person who solved the problem to develop the mathematical forms from complex real life situations or phenomena. Moreover, the process of creating a model is a cyclical and iterative process. Best problem solvers can continuously improve their models. However, when a solution is achieved in traditional problems (and after checking the accuracy), the task is considered complete (Gould, 2016). Briefly, the general characteristics of mathematical modeling activities can be listed as follows (Bliss ve Libertini, 2016; Borromeo Ferri, 2018; Gould, 2016; MaaB, 2007): being suitable to real life, being open-ended, complex or thought-provoking and being able to be solved according to the modeling process.

As it is understood from the definition of mathematical modeling, it is the first feature of the mathematical modeling problem that it includes a *real life situation*. Being *open-ended* means that modeling activities are based on assumptions and estimates, and therefore different and unique solutions may emerge. The characteristic of *being complex or thought provoking* is that it creates a feeling of helplessness when faced with the problem at the beginning. In other words, mathematics is implicit in mathematical modeling activities. Once the problem situation is determined, it should not evoke the idea of achieving the solution by using mathematical operations and formulas according to the procedure. The characteristic of *being able to be solved according to the modeling process* refers that the problem could be solved in accordance with the steps in the modeling cycle. Instead of using common mathematical formulas and operations, here, making assumptions, determining variables, creating a mental model, model building, following the solution and evaluation stages of the model iteratively, and finally reporting the solution are emphasized. All these properties of mathematical modeling can also be used as criteria for distinguishing mathematical modeling problems from traditional problems. However, having one or a couple of these features is not sufficient for a problem to be a mathematical modeling problem. Not all problems that reflect real life situations are problems of mathematical modeling. Similarly, the problems that require complex mathematical processes to achieve the result may not be mathematical modeling problems as well.

Teacher Competencies in Teaching Mathematical Modeling

Mathematical modeling is an important part of the mathematical competencies of all students (Ministry of National Education [MNE], 2018). Consequently, all teacher educators need to consider the knowledge and skills that teacher candidates and teachers need to acquire about mathematical modeling. Teachers who have not been trained in mathematical modeling, or who have never worked with modeling activities, are unlikely to be able to teach mathematical modeling to their students. Research studies on teachers' development of their knowledge of mathematical modeling are limited (Borromeo Ferri, 2018; Zbiek, 2016). Borromeo Ferri and Blum (2009) stated that teachers should have four basic competences in teaching mathematical modeling and Borromeo Ferri (2014, p. 29) classified these competencies as follows: 1) Theoretical dimension, 2) Task dimension, (3) Teaching dimension, and 4) Diagnostic dimension. The sub-skills under these dimensions are shown in Figure 1.

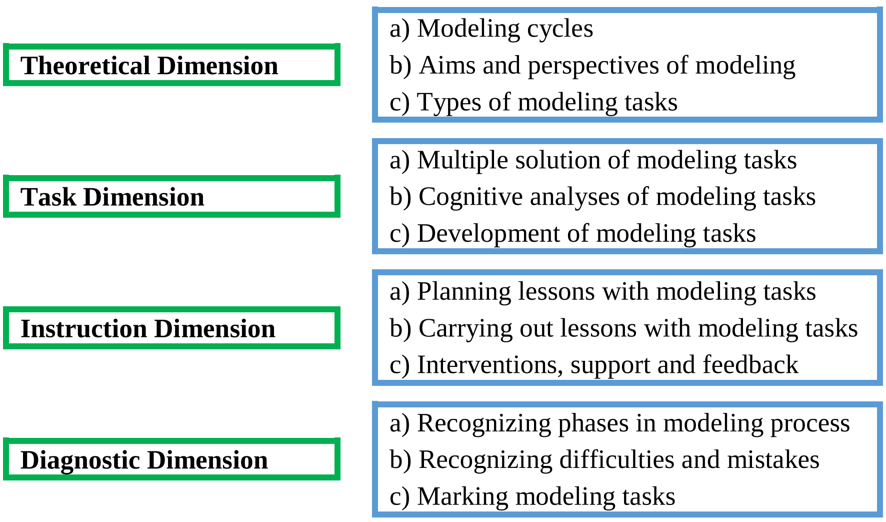

Figure 1. Model for Competencies Needed in Teaching Mathematical Modeling (Borromeo Ferri and Blum, 2009; Borromeo Ferri, 2014, p.29)

Theoretical Dimension

The question of *"what is meant by mathematical modeling?"* is the basis of the theoretical dimension. The teacher should be aware of how mathematical modeling is defined, for what its purpose it is, and why it is important. Teachers' perspective of mathematical modeling is a factor that will directly affect their learning and teaching activities. It is teachers' responsibility to determine their own perspective on mathematical modeling and to teach in accordance with this understanding of modeling. Therefore, prospective teachers need to know the modeling cycles. What is expected from the prospective teachers is not to know how many steps a modeling cycle consists of and what the details of each step are, but to be aware of the purpose of each cycle (Borromeo Ferri, 2018). In teacher education, introducing the theoretical knowledge of mathematical modeling at an initial stage may be difficult and even meaningless for prospective teachers (Borromeo Ferri, 2018). However, with the implementation of the activities, they can understand that it is necessary for being able to see the whole picture (Borromeo Ferri, 2018). For this reason, confronting prospective teachers firstly with modeling activities and ensuring that they experience the modeling process themselves can be effective in understanding what mathematical modeling is, its purpose and the perspectives of modeling.

Task Dimension

The task dimension is the second competence that teachers should have, which is to solve mathematical modeling activities and to determine the criteria for modeling activities. Essentially, question of *"what are the criteria that a good mathematical modeling activity should have?"* is to be answered. It is expected that teachers will be able to organize mathematical modeling activities in accordance with the modeling process, to determine the characteristics that distinguish these activities from traditional problems, and to prepare mathematical modeling activities.

Instruction Dimension

Instruction dimension, the third dimension of teacher competencies, includes the competencies of preparing and implementing lesson plans appropriate for mathematical modeling problems. This competence is very important to ensure the balance between theory and practice. The teaching dimension covers all practices such as how teachers will carry out modeling activities in the classroom and how they give feedback to students during implementation (Borromeo Ferri, 2018).

Diagnostic Dimension

The fourth and last competence that teachers should have for teaching mathematical modeling is diagnostic competence. The diagnostic dimension includes the ability of teachers to see the challenges that occur in the mathematical modeling process and to assess mathematical modeling activities. Diagnosing the challenges in order to provide appropriate support and feedback to the students during teaching is the first step. Diagnosis allows teachers to understand the solutions that students produce, and the teacher can decide to give individual support, feedback, or intervention to the student only after the difficulty is identified (Borromeo Ferri and Blum, 2009; Borromeo Ferri, 2018).

The teacher competencies mentioned above cannot be considered independent of each other; each competence is complementary to others. Currently, there is no specific procedure for teaching these qualifications to prospective teachers. However, it is the most important step of quality education that teachers fully acquire these competencies in order to implement mathematical modeling in the classroom. The quality of the teacher directly affects the quality of education; and the quality of education directly affects student learning (Borromeo Ferri, 2018).

In this study, the task dimension of the above teacher competencies is analyzed with respect to 'the cognitive analysis of modeling problems'. Analyzing modeling problems can be defined as the ability to know the characteristics that a mathematical modeling problem should have and distinguish mathematical modeling problems from traditional problems (Borromeo Ferri, 2018). It is important to know how teachers address modeling-specific features that distinguish modeling from traditional problem solving when designing, implementing, and evaluating modeling activities to be used in the classroom (Gould, 2016). However, it is observed that there is a lack of research in the literature that investigate how teachers or prospective teachers define mathematical modeling and that how mathematical modeling criteria are applied in a practical way. Consequently, the research question of this study is “What is the level of mathematics teacher candidates' theoretical knowledge about mathematical modeling and what is the level of their competencies to determine whether a problem is a mathematical modeling problem taking into account the criteria that mathematical modeling problems should have?”

Method

The Case Study

This case study investigated prospective teachers’ ability to analyze and distinguish mathematical modeling problems. A case study is defined as examination and description of an event, case or situation (Yin, 2009). The aim was to examine the situation in its real context and in all its aspects. In this study, the knowledge and opinions of prospective teachers about mathematical modeling were examined in detail and qualitative methods were adopted.

Participants

The study was conducted in 2016-2017 academic year with 27 second-year mathematics teacher candidates. Participants took the elective course on ‘Modeling in Mathematics Teaching’ where mathematical modeling was examined theoretically and mathematical modeling problems at middle/high school level(s) were practiced.

Data Collection Tools and Processes

The data were collected in two stages. The first stage of the data collection was conducted at the beginning of the course. Students were informed that they would need to prepare a mathematical modeling problem without subject and class level constraints and the problems they had prepared were collected at the end of the course period. Twenty-seven problems were created by the students, 4 problems were selected by the researchers and the participants were asked to evaluate these problems. When determining these 4 problems which constituted the data collection tools of the first stage, some criteria were taken into account. In determining these problems, it was paid attention that they were appropriate to the level of the students and expressed in an understandable language. In addition, to identify evulating criteria of students it has been considered that the problems selected were not clear to understand whether they were mathematical modeling problems at the first glance. In order to set an example for the problems identified, ‘Galata Tower Problem’ (Figure 2), which is a mathematical modeling problem, and ‘Polybius Encryption Problem’ (Figure 3), which is among three non-mathematical modeling problems, are presented below.

GALATA TOWER PROBLEM

Ceyda, a local tourist, is visiting the historical sites of Istanbul. Finally, Ceyda visits Galata Tower which is one of the oldest towers in the world and which was built by the Byzantine Emperor Anastasius in 528 as a lighthouse tower. While visiting the Galata Tower, Ceyda also learns the following information in addition to the history of the tower: The height of the Galata Tower from the floor to the end of the roof is 60,9 meters, the wall thickness is 3,75 meters, the inner diameter 8,95 meters and the outside diameter 16,45 meter. After getting information about the Galata Tower, Ceyda wants to have a photo taken while she stands 34,275 meters away from the tower and in a way that the top of the roof of the tower is visible. Since the length of Ceyda is 1.70 meters, how far should the person who is taking the picture should stay away from the tower?

Figure 2. Galata Tower problem

This problem, which has the characteristics explained in theoretical framework, is a mathematical modeling problem. The students were expected to solve the problem by considering the height of the person who took the photograph and deciding how he/she took the photograph (standing, sitting, reaching out, etc.). Therefore, there was a structure in which students could form their own assumptions and each student could create his/her own model. In addition, the fact that the problem reflects a real life situation and associates with different disciplines by giving the architectural features of Galata Tower are the strengths of the problem.

Ali realizes that his students have difficulty in learning exponential numbers. He begins to search for a fun way to make them easier to learn, and decides to use cryptography. Cryptography is a set of mathematical methods that works to provide information security concepts such as confidentiality, authentication, and integrity. Thus, Ali decides to use Polybius (Checkers) encryption. Polybius encryption is a two-dimensional wood-based encryption system found by the person named Polybius in Ancient Greek. The feature of this system is to place the letters in the alphabet in a two-dimensional table and then read them according to the row and column numbers in this table.

The version of the table adapted to Turkish alphabet is as follows:

	1	2	3	4	5	6
1	A	B	C	Ç	D	E
2	F	G	Ğ	H	I	İ
3	J	K	L	M	N	O
4	Ö	P	R	S	Ş	T
5	U	Ü	V	Y	Z	

Encryption system: During encryption, the system simply finds a result of two numbers for each letter. The first number is a row and the second is a column element. For example, the word Ali is encrypted in the form of 113326.

Note: Leaving a space between numbers leads to a weak password because it contains information about the number length of the words. Therefore, it is not recommended.

In accordance with the information given, answer the following questions:

1) (7^2+2), (6^2-2), (6^2+15), (6^2+10), (7^2+2^2), (3^2+2), (6^2+2^2+3) solve these exponentials to find the secret word encrypted by binary numbers.

2) Encrypt your name and write down as exponential numbers.

Figure 3. Polybius Encryption problem

The students were asked to encrypt their own names in the second stage of Polybius encryption problem where they were first given a special encryption system and were asked to find the word coded by various exponential number operations. Although there is a real life story in the question, it can be said that the problem does not bear the characteristics of mathematical modeling problem because it has only one answer and does not require any assumptions to be made. The students were asked to evaluate whether the problems identified were mathematical modeling activities together with their explanations and their answers were collected in writing.

In the second stage of the data collection, individual semi-structured interviews were conducted with 12 prospective teachers, who were randomly selected among the participants, in order to further examine their views on mathematical modeling. The questions asked to understand the prospective teachers' knowledge and ideas about mathematical modeling in the interviews are as follows: What is a mathematical model? What is mathematical modeling? What are the three most important characteristics of a mathematical modeling activity? In addition to these questions, the participants were presented with two problems, one of which was mathematical modeling problem and the other one was an application problem, and they were asked to evaluate these problems.

The researchers took into account the data collected in the first phase of the study when determining the problems in the interview form. As a result of the preliminary analysis of the data obtained in the first stage, it was determined that most of prospective teachers have a perception that modeling problems should have numerical data. For this reason, in the interviews, the researchers presented a mathematical modeling problem that does not contain any numerical data and a non-modeling problem that contains all the numerical data needed to solve the problem. Thus, it is aimed to evaluate the participants' understanding of mathematical modeling more accurately. The first problem, School Party, was a mathematical modeling problem developed by Henning and Keune (2007) and adapted to Turkish by Doruk (2010). The application problem was 'Giapetto's Woodcarving' developed by Winston (2004) and adapted by the authors (see Appendix). The data collection process was completed with the interviews.

Data Analysis

Grounded Theory method was used in the analysis of the data (Strauss and Corbin, 1998). In Grounded Theory, where the environment is observed, depicted and the reasons are put into effect without any interference, the present situation is examined without using any previously prepared analysis of framework. The aim here is to evaluate the current problem as a whole without comparing it to another situation. In this study, it is aimed to examine how pre-service teachers define mathematical modeling problem and what criteria they consider when evaluating a mathematical modeling problem together with their reasons. Grounded theory uses open, axial and selective coding as data analysis techniques. Open coding is the first stage of analysis. At this stage, the data are analyzed by continuously comparing and the codes reflecting the situation are formed (Vollstedt, 2015). Axial coding is the second stage in which the codes that emerge in open coding are associated with each other under certain categories. Finally, selective coding is the third stage where the identified categories are associated with other categories and verified.

In the study, the written documents obtained in the first stage of the data collection and the transcripts of the interviews obtained in the second stage were examined. Subsequently, in order to ensure the coding reliability, a part of the written documents and four interview transcripts were individually coded by the researchers and compatibility of the codes were examined by comparing the codes. After achieving a common code list, the entire document was coded and categories were created by associating the codes. The categories and codes that obtained by open and axial coding and that represent the characteristics that students take into account when evaluating mathematical modeling activities are presented in Table 1. The data collected in the study were analyzed and interpreted according to the categories and codes given below.

Table 1. Categories and codes obtained from open and axial coding

Categories	Codes	Explanation
Content Features	Real life connection	Being related to real/daily life
	Being interesting	Being interesting/remarkable
	Being thought-provoking (high level thinking)	Being thought provoking
	Being complicated or simple	The number of variables in the problem
	Complete data	All numeric data being included in the problem
	Being clear and accurate	Clearly indicating the problem and what is asked
	Interdisciplinary transition	Including different disciplines in the problem
Formal features	Length of the problem	The problem text being long or short
	Having a title	The problem having a title
	Having a visual	Having an image representing the problem
Features related to the solution	Having different solutions	Having more than one solution
	Model emergence	The emergence of a model in solving the problem
	Being generalizable	Reaching a generalizable model at the end of the problem
	Being non-operational	Solving the problem not only through mathematical operations
	Being interpretable	The problem can be solved from different perspectives

Results

Mathematical Modeling Problems Should be related to Real Life

When the problem evaluations of all the participants were examined, it was determined that 10 of the 11 participants who thought that the Paint Problem was a mathematical modeling problem; 11 of the 17 participants who think that the Encryption Problem was a modeling problem; 15 of the 17 participants who thought that the Diet Problem was a mathematical modeling problem and 16 of the 21 participants who thought that the Galata Tower problem was a modeling problem, considered the real life situation as a criterion when evaluating the problems (Table 2). The findings showed that the most important criterion considered by prospective teachers was being related to real life.

Table 2. Evaluating the problems as mathematical modeling problems for being suitable for real life

Problem	**Number of participants using real-life eligibility criteria**	**Number of participants considering mathematical modeling**
Paint Problem	10	11
Encryption Problem	11	17
Diet Problem	15	17
Galata Tower Problem*	16	21

*Mathematical modeling problem

When the answers given to the interview question "*What do you think mathematical modeling means?*" were examined, it was seen that the participants mostly emphasized the real life feature of mathematical modeling. However, the first of these two important findings was that mathematical modeling was not defined as solving the real life problems with the help of mathematics, but defined as associating a problem encountered in mathematics with real life. For example, one of the paticapants, Kubilay, stated that "*Mathematical modeling is constructing a general mathematical formula or creating a general model by associating a question encountered in mathematics with real life*". Here, the prospective teacher probably thought that real life examples should be presented while creating a mathematical formula to better understand mathematics. It can be said that Kubilay had difficulty in seeing the transition from real world to the world of mathematics. It is also possible to talk about the transition from the world of mathematics to the real world in Gamze's definition of mathematical modeling where she stated "*I think that if any of the rules of derivation is one of our models for that question, then the stages of adapting it to real life will be the modeling part*". Another participant, Mithat, described mathematical modeling as *"connecting one thing in mathematics with everyday life, I mean, intertwining these two*".

The second important finding encountered in interviews and written evaluations was the meaning attributed to *real life*. According to the research findings, almost all participants stated that the first condition they sought in a problem in order to be a mathematical modeling problem was that the problem included a real life situation. However, it was determined that the real life situation was expressed by the participants in different ways such as "real life" and "daily life". While real life situations have a broad meaning, daily life has a more limited meaning refering to the environment in which a person lives, and this difference of meaning has an important place in understanding mathematical modeling. Thus, one of the participants, Burak, defined mathematical modeling as the adaptation of daily life situations to mathematics. During the interviews, it was determined that the reason for continuously using "daily life" instead of "real life" was caused by understanding mathematical modeling as relating one's own living space. According to Burak, whether or not a problem is a mathematical modeling problem may vary depending on the individual. If there is no possibility of encountering the problem situation in daily life for a person, this problem is not a mathematical modeling problem for him/her even if it is suitable for creating a mathematical model. While explaining why School Party Problem wass a mathematical modeling problem, stating "*It's been reconciled with daily life, normally, when we think of it somewhere, how many people would fit in a bus, I mean I would try to find the maximum number, they would think while doing it*", showed that Burak evaluateed the problem by adapting it to his daily life.

One of the other findings wass Zehra's statement that the notion that mathematical modeling was related to real life was a powerful feature, but that it was not always necessary. She thought that it was sufficient to have a story belonging to the problem, but it was important that this story reflected the real life in terms of being interesting. In spite of this, it was seen that Zehra considered treating the problem in a real life context as a criterion in problem evaluation stage. Although it was accepted that real life was an indispensable feature of mathematical modeling by the great majority, it was found that some participants did not consider this at the problem evaluation stage.

According to the findings of the study, it was seen that there were participants who said that a problem was a mathematical modeling problem only because it was related to real life. For example, in evaluating the Paint problem, Tuğçe stated "*It is a modeling problem in terms of adapting a real life problem to mathematics*", and similarly, when evaluating the Diet problem Kubilay stated "*This problem is suitable for modeling. The model was chosen from daily life*". When evaluating the problems, Sevgi's statements such as "*This is a mathematical* *modeling problem because, I mean, it is a situation we can face in daily life*" and "*Yes (it is a mathematical modeling problem). This is a question in which daily life is applied to mathematics*" and Tuğçe's statement of "*This is a mathematical expression of a problem in everyday life. In the process of finding the most profitable, we are asked for help. So, it is a mathematical modeling problem*" showed that they considered being in daily life sufficient for being a modeling problem. Clearly, these participants had the idea that "if the problem situation reflects real life, it is a mathematical modeling problem". However, although this is a requirement, it is not enough for a problem to be a mathematical modeling problem.

Mathematical Modeling Problems Should be Open Ended

When examining the problem evaluation process of teacher candidates, it was observed that they emphasized that mathematical modeling problems had a complex structure based on interpretation. In the interviews, most of the participants stated that leading students to think was an important feature of mathematical modeling. For example; Berat underlined the interpretable, open-ended structure of mathematical modeling by saying that "*Not to be considered as simple, to be thought a little higher, to be based on interpretation, so I see a good interpretation before the modeling questions, the capture of a starting point*". Moreover, Tuğçe refered that mathematical modeling was based on assumptions by stating that "*In mathematical modeling, the aim is not always to find one result. Therefore, we can solve the question for ourselves. There may be questions where my results and your results are both correct but different*". However, the findings of the study showed that the opinions and practices of many prospective teachers did not overlap. It was found that the participants had a serious mental complexity especially in School Party and Giapetto's Woodcarving problems related to this issue. For example, Berat expressed that School Party problem was not a nathematical modeling problem by saying "*So there is no information given at this stage. Since there is no information, we cannot create anything, for example, what is the size or shape of the garden, what will happen in that music concert area, is it just a student and, for instance, a place of music? Since there is no information on them, it would be totally fictitious to create this*". From these statements, it was understood that Berat defined making an assumption as a "*totally fictitious*" thing. For mathematical modeling, Zehra said "*I also think that being challenging makes it a qualified problem. When the solution is single, it doesn't seem like much modeling. With just one solution… without thinking*". She said that Giapetto's Woodcarving problem was a mathematical modeling problem and never changed her mind. While explaining her reason, she stated "*There is a delimitation in the problem, you may consider selling 40 toys at most. So, you can sell less or more. The student can calculate what he/she can earn the most by thinking that*". Obviously, although she thought that when the problem had a single solution, it was a traditional problem. She accepted that Giapetto's Woodcarving problem, which had only one solution, was a mathematical modeling problem. It can be said that the answers of Zehra reflected the ideas of the participant group in the evaluation of these two problems. Almost all participants remained in the dilemma in School Party problem similar to Zehra. It is noteworthy that the participants emphasized that all data were given when evaluating Giapetto's Woodcarving problem.

Sinan, who thought that Paint problem was not a mathematical modeling problem, said "*It does not require any mathematical interpretation. A different approach cannot be applied to the problem. No comments varying from person to person can be expected. It is not a complex mathematical problem*." for justification. For Sinan, the problem must be open to assumptions and interpretations. When the participant is talking about the complexity of the problem, he might mean that the problem carries these characteristics. For the Encryption problem, Ceren put forward her ideas by stating "*There is nothing to think about. Every person who knows mathematics can solve this problem. It is a simple mathematics question*". While explaining why Galata Tower problem was a mathematical modeling problem Aydan said "*It is not a simple question. It allows different and realistic thinking*". Here, the participant emphasized that the problem could not be solved by using only mathematical operations and the problem would lead the students to think.

Mathematical Modeling Problems Should be Complex or Thought-Provoking

The research findings actually showed that prospective teachers separated mathematical modeling from traditional problems and that mathematical modeling was a kind of problem solving beyond arithmetic operations. For example, Burak stated: "*Mathematical operation is, you know, writing what you know without thinking, but mathematical modeling is with thinking, I mean, we solved this in class, when you add them up, thinking if it's true in real life is modeling, but it's not the case for mathematical operations. It's true or not, we just do the math. In modeling though, we associate it with real life, so we think if it's true or not*". In addition, as Zehra compared traditional problem solving with mathematical modeling "*Those problems (traditional* *problems) do not lead to thinking, I mean, direct solution. The kid is not aware of what he is doing, just puts the numbers in place. Students are kind of like robots*", it was observed that traditional problems could be solved by following the steps based on mathematical information provided in the problem, without the need of students to think. When evaluating School Party problem, Zehra expressed that it could be a mathematical modeling problem only after some modifications and stated her ideas as "*It has some missing parts. It's missing. For example, if he gave me some more information about the garden. Because in this way each student will draw a different shape. It's very open-ended. ... So there will not be a solution for this solution will not be a general solution. He just says we have a garden like this. He can't do the rest, the student… But here he wants a solution from us*". It should be noted that theoretically, for Zehra, mathematical modeling should be thought-provoking and open-ended, but in practice, when she encountered a problem with these features she thought that there was not enough information to provide a solution. This participant experienced ambiguity about whether the School Party problem involved mathematical modeling and finally decided that it was a mathematical modeling problem.

In Paint problem, Diet problem and Galata Tower problem, 8 participants (for each) considered complete mathematical information as an evaluation criterion. In Encryption problem, it was determined that no participants reported their opinions on mathematical information. For the Encryption problem, no participant has commented whether mathematical information is complete or incomplete.

The uncertainty resulting from the implicity of mathematics in the problem text caused preservice teachers to remain in dilemma. It was understood from Aydan's statement "*I think it is a mathematical modeling problem, although it has shortcomings. Dimensions of the garden are not given, there is no information about the garden, and no information on where the concert will take place in the garden*" that she had been experiencing an instability because both the problem was based on assumptions and there was a lack of numerical information. Later in the interview Aydan was asked how she would solve the problem if she was asked and she solved the problem by creating a mathematical model. In the modeling process, she decided that it was a mathematical modeling problem and commented "*It was a mathematical problem, because I determined the dimensions of the school myself, visualized it in my mind and associated it with daily life*". For the same problem, Sevgi expressed her thoughts as "*But in fact there is no such math. Yeah, there's no math. Then there will be no modeling, I mean, I'm not doing any operations*", meaning she thought that it was not a mathematical modeling problem because of the lack of mathematical information.

When the written exam evaluations of the participants were examined, it was observed that their comments were under the code "the problem is thought-provoking" (requiring high-level thinking). This code was emerged 11 times in Paint problem, 5 times in Encryption problem, 9 times in Diet problem and 9 times in Galata Tower problem, and the participants who considered this criterion thought that not being thought-provoking was a weak side for a problem.

Other Characteristics of Mathematical Modeling Problems

It was observed that prospective teachers generally shaped their mathematical modeling problem evaluation criteria according to the characteristics of mathematical modeling activities. During the interviews, when participants were asked to rank the characteristics of mathematical modeling activities, it was seen that the real life property was in the first place, followed by being thought-provoking (including assumptions) and including more than one solution. However, as mentioned before, the results of the research showed that the candidate teachers' criteria for evaluating the problems depended on the problem itself. The most common codes and repetition frequency are given in the table below.

As seen in Table 3, the most commonly used criterion to determine whether a problem was a mathematical modeling problem has been "real life situation". The criteria of "being thought-provoking" were mostly used in Paint and Galata Tower problems. The "model emergence" code had been one of the most frequently used codes. Although it has a very high frequency, it can be said that pre-service teachers evaluated visual mathematical structures such as geometric shapes, tables and schemas that were present in traditional problems and modeling problems as a "model". The use of this code in Paint, Encryption and Galata Tower problems and not considering the emergence of a model as a necessary characteristic in the Diet problem that requireed only mathematical operations on numerical basis supported this idea. One of the striking results was that the code of "being interesting" was one of the codes frequently used in the Encryption and Galata Tower problems. Indeed, these two problems had a different structure that is interesting compared to other problems. During the interviews, it was determined that the participants considered "being interesting" as an important feature of mathematical modeling. Although giving information from different disciplines together or requiring to use information from different disciplines in the modeling process was one of its strengths of mathematical modeling, it was not imperative. It was determined that pre-service teachers were aware of this, but still they considered it as a criterion when evaluating the Galata Tower problem because they contained information from different disciplines. Apart from the codes in the table above, it was also seen that the participants evaluated the problems under the criteria such as “being understandable and accurate”, “having a title”, “having a visual” and “having a different or single solution”. In addition, the participants namely used mathematical modeling principles based on their theoretical knowledge and they could not explain those principles.

Table 3. Frequencies of categories and codes

Problem	**Codes**	**Frequency**
Paint Problem	Real life situation	20
	Being thought provoking	11
	Model emergence	10
Encryption Problem	Real life situation	18
	Model emergence	14
	Being interesting	11
Diet Problem	Real life situation	20
	Being complicated or simple	12
	Length of the problem text	12
Galata Tower Problem*	Real life situation	20
	Model emergence	14
	Being interesting	9
	Being thought provoking	9
	Interdisciplinary transition	9

* Mathematical modeling problem

Conclusion and Discussion

Mathematical modeling is one of the basic skills that should be provided to students in the curriculum of many countries. Undoubtedly, teachers have a great responsibility in bringing this skill to students. Therefore, it is very important that prospective teachers have knowledge and experience about mathematical modeling skills and teaching those skills. For this purpose, it is necessary for prospective teachers to gain the qualifications that teachers should have in teaching mathematical modeling. These competencies can be listed as knowing the purpose and importance of mathematical modeling, knowing the properties of mathematical modeling and distinguishing it from traditional problems, preparing mathematical modeling activities, application and evaluation (Borromeo Ferri, 2018). It is possible for teachers to be successful in teaching mathematical modeling by having all of these competences. However, it is important how teachers address modeling-specific features that distinguish modeling from traditional problem solving, while designing, implementing, and evaluating modeling activities (Gould, 2016). Borromeo Ferri and Blum (2009) consider this qualification as “cognitive analysis of modeling problems”. Analyzing modeling problems can be defined as the ability to know the characteristics that a mathematical modeling problem should have and distinguish it from traditional problems (Borromeo Ferri, 2018). Asking them to evaluate a given mathematical problem is an effective method that can be used to determine the level of teacher candidates' such knowledge. What was asked from prospective teachers in this study is to evaluate four problems, one of which is mathematical modeling problem, in writing according to the modeling criteria they have determined. In addition, 12 individuals were randomly selected from the 27 participants and individual interviews were conducted to examine their criteria determination and problem evaluation processes. Thus, mathematical modeling knowledge of the prospective teachers was revealed.

According to the findings of this study, prospective teachers' views on the definition of mathematical modeling and their problem evaluation criteria were grouped under four themes. These themes are “Mathematical Modeling should be related to Real Life”, “Mathematical Modeling Problems should be open-ended”, "Mathematical Modeling Problems should be complex or thought-provoking" and "Other Properties Strengthening Mathematical Modeling Problems".

The results of the study showed that prospective teachers had the knowledge of mathematical modeling as "solving real life situations using mathematics". However, it was also observed that prospective teachers experienced difficulties related to the direct meanings of "real life" and "daily life" concepts. When the related foreign literature is reiewed, it is seen that "real world" or "real life" expressions are used in definitions of mathematical modeling. However, these expressions are translated to Turkish as "gerçek yaşam (real life)" along with "günlük hayat (daily life)". Although they are used interchangeably by mathematics education researchers, these expressions have different meanings. Daily life refers to a restricted living space limited to the environment in which the person lives. However, the problem state in mathematical modeling may not be directly related to the students' real lives/environments. What is important is that the problem is realistic and meaningful.

Mathematical modeling is not only a process in which real life situations are simplified, it is also a process that carries an individual reality that reflects the individual's knowledge, perspective and purpose about the situation (Blum and Niss, 1991). Therefore, the structure of the model to be created depends on the theoretical, experiential and hypothetical knowledge of the person(s) who solve(s) the problem (Blomhoj and Jensen, 2006). As stated in the findings, Tuğçe's statements of "*We can solve the problem by ourselves. Your correct answer and mine may be different*", indicate that she is aware of the fact that modeling activities are open to producing unique solutions.

While the main aim of the modeling activities is to encourage students to solve the real life situation mathematically, another objective is to give students independent thinking and working skills (Meier, 2009). In this study, it was determined that teacher candidates expressed the opinion that mathematical modeling led to different thinking. For example, as mentioned in the findings, while explaining why the Encryption Problem was not a mathematical modeling problem, Ceren's statement of "*There is nothing to trigger the student to think. Every person who knows mathematics can solve this problem*", supported this situation. One of the important features of mathematical modeling problems is to enable the person or people who solve the problem to decide which information that affects the problem state and which ones to be ignored (Borromeo Ferri and Blum, 2009). The fact that mathematical modeling is open-ended and based on assumptions brings about an uncertainty. The results of this study showed that some pre-service teachers had misconceptions that mathematical modeling requires assumptions. In fact, what is often missing in the modeling process is the understanding of the original situation, deciding what to keep and what is to be taken and verifying that the results are meaningful in the real world (Pollak, 2003). Prospective teachers' thinking that mathematical modeling should be thought-provoking and complex comes with this challenge. They found it difficult to make accurate assumptions and predictions, especially in problems that mathematics is implicit. When the teacher candidates' comments about the School Party and Gepetto the Toy Maker problems were examined, it was seen that the participants had a strong belief that mathematical information should be presented in the problem. One of the reasons for the unclearity about whether the School Party problem was mathematical modeling was that it was based on the assumptions, as mentioned above, and the other was that there was no mathematical information in the problem.

As all the information required for the solution of the traditional problems is included in the problem, while the students solve the problem with linear steps computationally, in mathematical modeling, they are expected to formulate the necessary method for solving the problem. When discussing the difference between mathematical modeling and traditional problems, Zehra's comparison as "*Those problems (traditional problems) do not lead to thinking, I mean, direct solution. The kid is not aware of what he is doing, just puts the numbers in place. Students are kind of like robots*", and other participants expressing their views in a similar way showed that prospective teachers were aware that mathematical modeling was a cyclical process based on assumptions. Nevertheless, one of the results obtained was that pre-service teachers were disturbed by the uncertainty of mathematical modeling. Even though they encountered many examples of mathematical modeling during semester, it was seen the expectations of the prospective teachers were that the problem should be sufficiently clear and all the information needed for the solution –mathematical data, in particular- should be given in full. Because it is difficult to solve mathematical modeling problems for students who are used to solve problems by using the formulas and numbers readily provided to them, it was observed that teachers avoided these kinds of problems even though they were in the curriculum (Clement, Lochhead and Monk, 1981).

One of the important results obtained in the study was that the participants could not reflect the theoretical knowledge of mathematical modeling on their applications. It is of course of great importance to have theoretical knowledge on the subject, but it is possible to see the effect of traditional education on students. Lesh and Doerr's (2003) principles of mathematical modeling have been used as a criterion in significantly high rates in written evaluations. However, it was detected that teachers could not explain the principles, only used expressions such as "*This is a mathematical modeling problem, because it is appropriate to the principle of model emergence*" or "*I think that the problem is in accordance with the principle of generalization*". On the other hand, during the interviews, all the participants made statements that revealed their individual views. In addition, the findings of the research showed that the common characteristics of the problems encountered in the course were effective in determining the evaluation criteria for teacher candidates. For example, having a title and a related picture for the problem was adopted as a criterion. In addition, they found mathematical modeling problems interesting, because mathematical modeling problems that they encountered during the course period were structurally different from the problems they solved before. For this reason, it has been observed that "being interesting" has become a criterion for teacher candidates. Among the reasons why pre-service teachers described mathematical modeling problems as interesting was the fact that they were different from traditional problems, they were open-ended, and they usually havd an interdisciplinary structure.

When the results of this study are taken into consideration, it is possible to say that prospective teachers did not have enough knowledge about mathematical modeling. This result showd that the prospective teachers were not sufficient in theoretical dimension from the qualifications determined by Borromeo Ferri and Blum (2009). In addition, it can be said that the lack of theoretical knowledge affected their skills of cognitive analysis of modeling problems. Although it is not surprising that such a result is encountered, it is necessary to be more careful in undergraduate education. While creating the content of the mathematical modeling course, pre-service teachers should be provided with activities related to modeling knowledge and skills, as well as problem preparation, implementation and problem evaluation. Teachers should have these expected qualifications in teacher education. Mathematical Modeling, which was taught as an elective course in the Mathematics Education Departments of universities in Turkey, has been added to the undergraduate program as a compulsory course since the 2018-2019 academic year. Thus, all mathematics teacher candidates will have the opportunity to receive the necessary training.

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Author Information

Seda Sahin
Adiyaman University
Adiyaman/Turkey
Contact e-mail: *ssahin@adiyaman.edu.tr*

Muhammed Fatih Dogan
Adiyaman University
Adiyaman/Turkey

Zeynep Cavus Erdem
Ministry of National Education
Mehmet Akif Ersoy Secondary School
Adiyaman/Turkey

Ramazan Gurbuz
Adiyaman University
Adiyaman/Turkey

Ali Temurtas
Adiyaman University
Adiyaman/Turkey

Appendix. Problems Used in the Study

SCHOOL PARTY

It has been announced that a famous band is going to play in the gym at a school party in our school. Almost all the students from your school and many students from neighboring schools would like to come to the concert. From the organizers of the party you receive the task of calculating the maximium possible number of spectators for the gym.

l) Plan how you will proceed with solving the problem and write out the steps needed for the solution.
2) Complete the task which the organizers gave you. If any details are missing, figure them out by estimating. The organizers would like you to show your work to the heads of the school in a short presentation.

GIAPETTO'S WOODCARVING

Giapetto's Woodcarving, Inc., manufactures two types of wooden toys: soldiers and trains. A soldier sells for $27 and uses $10 worth of raw materials. Each soldier that is manufactured increases Giapetto's variable labor and overhead costs by $14. A train sells for $21 and uses $9 worth of raw materials. Each train built increases Giapetto's variable labor and overhead costs by $10. The manufacture of wooden soldiers and trains requires two types of skilled labor: carpentry and finishing. A soldier requires 2 hours of finishing labor and 1 hour of carpentry labor. A train requires 1 hour of finishing and 1 hour of carpentry labor. Each week, Giapetto can obtain all the needed raw material but only 100 finishing hours and 80 carpentry hours. Demand for trains is unlimited, but at most 40 soldiers are bought each week. Giapetto wants to maximize weekly profit (revenues _ costs). Formulate Giapetto's situation that can be used to maximize Giapetto's weekly profit.