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Students' Understanding of the Definite Integral Concept

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Abstract

This study investigated students' procedural and conceptual knowledge of the definite integral.Twenty five students enrolled in one section of an undergraduate Calculus II class participated in this study. Data were collected from a test that was conducted during the fourth week of the semester. The test aimed at collecting information about the students' procedural and conceptual knowledge of the definite integral.. The results indicated that the students' most dominant knowledge was the procedural one, and that they had limited understanding of the definite integral. Their abilities to represent the concept in different ways were limited; they could represent the concept using only two different representations.

Key words: Calculus; Definite Integral; Mathematics Education; Understanding

Introduction

University students encounter many Calculus concepts during their college education. These concepts are used not only in Mathematics classes, but also in Chemistry, Engineering, Physics, as well as many others. The definite integral concept is one of the main concepts introduced in Calculus and important for students to master. Many research studies indicated that students have difficulties with the concept of the definite integral as well as the concepts of function, limit, and derivative (Grundmeier, Hansen, & Sousa, 2006; Mahir, 2009; Orton, 1984; Serhan, 2009; Tall & Vinner, 1981; Tall, 1987).

A major tenet of understanding is the capacity to make connections between conceptual and procedural knowledge. Hiebert & Lefevre (1986) defined conceptual knowledge as "knowledge that is rich in relationships. It can be thought of as a connected web of knowledge" (p. 3). Students' conceptual knowledge is developed by the construction of relationships between pieces of information. On the other hand, procedural knowledge, is divided into two distinct parts "One part is composed of the formal language, or symbol representation system, of mathematics. The other part consists of the algorithms, or rules, for completing mathematical tasks"(p. 6).

According to Hiebert & Lefevre (1986) understanding takes place when new information is connected through appropriate relationships to existing knowledge. It is very important to link conceptual and procedural knowledge; students who are able to link the two together are able to develop a strong mathematical knowledge. While students who are deficient in either kind of knowledge, or who developed both of them as separate entities are not fully competent in dealing with mathematics concepts.

According to Tall & Vinner (1981) the *concept image* consists of all the mental pictures that are associated with a given concept in the individual's mind. The student's image, that is developed based on his/her own experiences with the concept, consists of everything a student associates with the concept; symbols, words, pictures, etc. It takes years of all kinds of accumulated experiences to build that image which changes as the individual matures and encounters new stimuli. Whenever the student processes new information, change may occur on existing concept images. New difficulties, conceptions, and misconceptions may be encountered and may cause uncertainty or conflicts with some parts of the concept image. However, the *concept definition* "is a form of words used to specify the concept" (Tall & Vinner, 1981, p. 152). This may be a definition the student has learned, or it may be a personal reconstruction of a definition by the student. The concept definition is then the form of words the student uses for his/her own explanations of his/her (evoked) concept images (Tall & Vinner, 1981).

One of the key concepts in Calculus is the definite integral of a function. While learning this concept, students encounter Riemann sums, limits, derivatives, area, and many other concepts. To have a good understanding of the definite integral, students should be able to make connections between all of these concepts as indicated by

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Hiebert & Lefevre (1986). Research in student understanding of these concepts informs us of common misconceptions students hold about integration as well as common computational errors that students encounter. Research also sheds information about the ways that students tend to view integration and points towards suggested instructional methods. The present study adds to the research in the field of investigating students' procedural and conceptual understanding and concept images of the definite integral. The research investigating students' understanding of the definite integral indicated that students associate the definite integral with finding the area under the curve (Mahir, 2009; Rasslan & Tall, 2002; Sealey, 2006). In other studies, researchers found that students were good at computing the integral (Abdul-Rahman, 2005; Ferrini-Mundy & Graham, 1994; Orton, 1983; Pettersson & Scheja, 2008)

In a study that explored students' understanding of symbolic and verbal definitions of the definite integral, Grundmeier et al. (2006) found that a large number of students were able to evaluate the integrals correctly but had difficulty with both the symbolic and verbal definitions of a definite integral. Mahir (2009) administered a questionnaire that aimed at investigating students' understandings of the concepts associated with the definite integral as well as their computation skills. Mahir found that students did not have a good conceptual understanding of integration. Also Rasslan and Tall (2002) used a survey that included a question involving a definite integral in which the function crossed the horizontal axis (changing the sign). They found out that many students did not know how to calculate the area when the function changed its sign.

In summary, the research on definite integrals found that student knowledge was limited to procedural knowledge since they were good at computing the integral but had difficulty explaining the negative area as well as connecting the different representations of the definite integral. The present study was designed to investigate students' procedural and conceptual knowledge of the definite integral by examining their understanding through the concepts that they form in association with the definite integral. Based on literature research, no similar study was conducted in the UAE; therefore, the undertaking here is important and makes significant contribution to the field. It is important for instructors to be aware of how and what students learn in their introductory calculus courses so they may optimize student understanding. This study serves as a stepping stone for further investigation of student learning in calculus.

Research Questions

The main aim of this study was to examine students' procedural and conceptual knowledge of the definite integral. The study aimed at addressing the following research questions:

1. Which is the most dominant knowledge of the definite integral for students is it procedural knowledge or conceptual knowledge?
2. Are students capable of dealing with negative areas and explaining their answers?
3. What concept images do Calculus II students associate with the definite integral concept?

Method

Participants

The participants in this study were 25 undergraduate students enrolled in a Calculus II course at a major university in the United Arab Emirates. Students in this class had already been introduced to anti-derivatives, definite integrals, Fundamental Theorem of Calculus, integration by substitution, area between curves, and improper integrals in a Calculus I course that they finished the previous semester. The concepts in this class were introduced to students verbally, algebraically, graphically and numerically.

Procedure

At the beginning of the semester, the researcher explained to the instructor the purpose of the study. Data were collected from a test that was conducted during the fourth week of the semester. The test aimed at collecting as much information as possible about the students' procedural and conceptual knowledge of the definite integral. The test consisted of six questions that focused on students' procedural and conceptual knowledge. The questions were distributed as follows: the first two questions were computational in nature, asking the students to evaluate given definite integrals; the third question focused on graphical representation especially of the negative area; the fourth question was about connecting derivatives with definite integrals given the graph of the derivative of a function; the fifth question was about improper integrals, and the last question was the following:

"What does $\int_a^b f(x)dx$ mean to you? Give as much details as you can."

The first four questions were designed to examine students' procedural knowledge and whether or not students were capable of recognizing and connecting the different representations of integrals. The third question, in particular, examined students' understanding of negative area when the function changes its sign. The fifth question was designed to examine if students would be able to recognize improper integrals when the integral is undefined over the given interval. The aim of the last question was to evoke students' concept images of the definite integral concept and to check if students were able to make associations between the different representations of the definite integral.

Analysis and Results

The study followed an in-depth analysis that focused on the students' responses to each question. The objective of the data analysis was to investigate students' procedural and conceptual knowledge of the definite integral. Students' responses to the first five questions were given a numerical grade and were analyzed to get a good understanding of students' conceptual knowledge. Students' responses to the sixth question were organized in categories.

Most of the participants in this study were able to compute the given definite integrals in the first two questions,; 21 out of 25 participants (84%) were able to give a correct answer for both questions, while the other four students were able to give a correct answer to only one of the two questions.

For the third question, the area above the x-axis between *a* and *b* and the area below the x-axis between *b* and *c* were given. Based on these given areas students were asked to find $\int_a^c f(x)dx$ and explain their answers. Out of the 25 participants, 18 (72%) were able to come up with the correct answer. Only five of these 18 students were able to explain their answers correctly. Following are some of the students' explanations:

a. Because it is under the x-axis, the area is negative.
b. First area – second area
c. $\int_a^b Adx + \int_b^c Bdx$
d. $\int_a^b Adx + \int_b^c - Bdx$
e. Area = $\int_a^c f(x)dx$
f. $\int_a^b f(x)dx - \int_a^c f(x)dx$

Based on the graph of the derivative of a function that was given in the fourth question, students were asked to find the critical points of the function. Only five out of the 25 students (20%) were able to come up with the correct critical points.

The fifth question focused on one type of improper integral in which the integral was undefined over the given interval. None of the students were able to figure out the answer to this question, most of them, 20 out of 25 stated the given answer was wrong and evaluated the integral and came up with different inaccurate values.

Students' responses to the last question; "What does $\int_a^b f(x)dx$ mean to you? Give as much details as you can", were categorized into the following four categories :'Area", 'Integral', "Antiderivative" and "Unclear". These categories were establised based on students' responses only. Table 1, provides a summary of students' reponses within these four categories.

Table 1. Categories mentioned by students

Category	Number of Students
Area: The area between the curve and he x-axis from *a* to *b*	12
Integral: The integral of the function *f*(*x*) from *a* to *b*	7
Antiderivative	4
Unclear or no answer	7

Table 2 gives details of the number of students who mentioned multiple categories.

Table 2. Multiple categories mentioned by students

Number of Categories Mentioned by Students	Number of Students
0	7
1	11
2	6

Discussion and Conclusion

The main aim of this study was to examine students' procedural and conceptual knowledge of the definite integral. Based on the previous results, all students worked on the first two questions and most of them were able to evaluate the given definite integrals. In addition to that, students were able to evaluate the integral based on the given negative area. However, they had difficulty explaining their answers. It was difficult for them to connect the derivative and the integral in addition to dealing with improper integrals. The results of this study agree with with Orton's findings that students have the ability to use procedural knowledge and solve integration problems but have a limited understanidng of the basic concepts of integration.

Based on the categories of the last question, area was the most popular image of the definite integral that was used by students. This agrees with the findings of previous research studies (Grundmeier, Hansen, & Sousa, 2006; Mahir, 2009; Rasslan & Tall, 2002; Sealey, 2006). The findings of the study also show that students had difficulty explaining the negative area, which is also similar to findings of other research studies (Orton, 1983; Rasslan & Tall, 2002). The second popular image that was given by students of $\int_a^b f(x)dx$ indicated that this was the integral of the function *f*(*x*) from *a* to *b*. This points to a mere language understanding of $\int_a^b f(x)dx$.

The results of this study indicated that students had limited understanding of the definite integral. Only six students were able to come up with more than one representation of the concept. And none of them was able to come up with more than two representations. The most dominant knowledge of the integral for students was procedural knowledge. It is very important for students to develop the ability to make connections between different representations. Every opportunity in the classroom should be used to help students accomplish that.

In addition to that, the results of this study revealed that none of the students mentioned the Riemann sum in their image of the definite integral concept and that only a few of them mentioned the antiderivative in their images. Therefore, it is important for instructors to review the way that this concept is presented and taught in class. It is clear from the results of this study that there needs to be more emphasis on the multiple representations and their connections with the concept definition. In addition to that, there should also be more emphasis on the Riemann sum and how students may use it to enhance their structural understanding of the definite integral.

This study was not designed to study a specific concept image of the definite integral; rather its aim was to investigate students' procedural and conceptual understanding of the definite integral concept in general. The findings of this study shed light on students' thinking and understanding. Further studies are needed to investigate students' thinking of other related concepts such as the area, Riemann sum and the Fundamental Theorem of Calculus.

References

Abdul-Rahman, S. (2005). Learning with examples and students' understanding of integration. In A. Rogerson (Ed.), *Proceedings of the Eighth International Conference of Mathematics Education into the 21st Century Project on Reform, Revolution and Paradigm Shifts in Mathematics Education*. Johor Bahru: UTM.

Ferrini-Mundy, J. & Graham, K. (1994). Research in calculus learning: Understanding of limits, derivatives, and integrals. In J. Kaput & E. Dubinsky (Eds.), *Research Issues in Undergraduate Mathematics Learning,* MAA Notes #33. Washington D.C.: Mathematical Association of America.

Grundmeier, T. A., Hansen, J., & Sousa, E. (2006). An exploration of definition and procedural fluency in integral calculus. *PRIMUS*, *16*(2), 178-191.

Hiebert, J., & Lefevre, P. (1986). *Conceptual and procedural knowledge in mathematics: An introductory analysis*. In J. Hiebert (Ed.), Conceptual and procedural knowledge: The case of mathematics (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Mahir, N. (2009). Conceptual and procedural performance of undergraduate students integration. *International Journal of Mathematical Education in Science and Technology*, *40*(2), 201-211.

Orton, A. (1983). Students' understanding of integration. *Educational Studies in Mathematics. 14,* 1-18.

Orton, A. (1984). Understanding rate of change. *Math. School*, *13*(5) 23-26.

Pettersson, K., & Scheja, M. (2008). Algorithmic contexts and learning potentiality: A case study of students understanding of calculus. *International Journal of Mathematical Education in Science and Technology*, *39*(6), 767-784.

Rasslan, S. & Tall, D. (2002). Definitions and Images for the Definite Integral Concept, in: A. D. Cockburn & E. Nardi (eds.) *Proceedings of the 26th Conference PME*, *Norwich*, *4*, 89-96.

Sealey, V. (2006). Definite integrals, Riemann sums, and area under a curve: What is necessary and sufficient? *Proceedings of the 28th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, *Merida, Mexico: Universidad Pedagogica Nacional*, *2-46.*

Serhan, D. (2009). Using concept maps to assess the effect of graphing calculators use on students' concept images of the derivative at a point. *International Journal for Mathematics Teaching and Learning.* Retrieved from http://www.cimt.plymouth.ac.uk/journal/on 09/20/2014.

Tall, D.O. (1987). Whither Calculus. *Mathematics Teaching*, 11, 50-54.

Tall, D.O. & Vinner, S. (1981). Concept image and concept definition in mathematics, with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*, 151 - 169.