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A Review of the Effects of Visual-Spatial Representations and Heuristics on Word Problem Solving in Middle School Mathematics

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Abstract

Mathematics word-problems continue to be an insurmountable challenge for many middle school students. Educators have used pictorial and schematic illustrations within the classroom to help students visualize these problems. However, the data shows that pictorial representations can be more harmful than helpful in that they only display objects or persons while neglecting the spatial relationships between those components (Hegarty & Kozhevnikov, p. 686). Research supports schematic diagrams that highlight spatial relationships and focus on related information within the problem (Hegarty & Kozhevnikov, 1999; van Garderen & Montague, 2003). In addition to these strategies, educators have used heuristics (systematic scripted procedures to solve word problems) as a way to provide structure and routine to a variety of word problems. This review of the literature found varying results for the use of some heuristics being implemented in present-day classrooms. On the other hand, both visual-spatial diagrams and heuristics have demonstrated value in middle school special education classrooms. These findings support the further use and research of visual-spatial representations and problem-solving heuristics in order to solve mathematics word problems.

Key words: Mathematics education; Word problems; Heuristics; Middle school; Special education

Introduction

Mathematics is an integral part of school curricula and student education. Educators consistently strive to increase student comprehension of mathematical terms and concepts, specifically in the area of word problems. Word problems become a primary focus because they help students analyze mathematical processes as well as different uses of arithmetic operations (Jitendra et al., 2007, p. 115). Jitendra et al. (2007) suggest that successful word problem solving can lead to increased quality of computation skills in middle school students. The researchers also reference a breakdown of the skills students use when solving word problems (Jitendra et al., 2007, p. 115). This itemization includes language-based comprehension, translation, mental representation, solution construction, and solution execution (Jitendra et al., 2007, p. 115; Desote, Royers, De Clercq, 2003, p. 188). These metacognitive skills, when combined in a strategic manner, allow students to tackle even the most complex of word problems. As mathematics curricula shift toward more literacy-based instruction, educators are in an even greater need for a large repertoire of effective word problem-solving strategies to help students accomplish each phase of the word problem-solving procedure (van Garderen, 2008, p. 142). According to Barak (2012), educators believe that current Common Core middle school mathematic objectives should be designed to promote problem solving within the classroom in order to facilitate these skills in the real world. These objectives can be accomplished through the implementation of empirically-based strategies and methods.

One emerging strategy is Schema-Based Instruction (SBI). The research of Jitendra, Star, Dupuis, and Rodriguez (2013) suggest positive results when SBI was tested in connection with students' word problem-solving skills (p. 131). This four-step instructional mathematics strategy is designed to help students solve word problems by (1) priming the mathematical structure of problems, (2) utilizing visual-spatial representations as mathematical tools, (3) explicitly instructing through problem-solving heuristics, and (4) employing procedural flexibility (Jitendra, Star, Dupuis, & Rodriguez, 2013, p. 115-117). Jitendra et al. (2013) define priming the mathematical structure as schema training that separates relevant structural features from irrelevant and surface information (p. 115-116). Visual-spatial representations encompass both pictorial and schematic illustrations which include some kind of written or drawn construction of an individual's ideas (Jitendra et al., 2013, p. 116; van Garderen, Scheuermann, & Jackson, 2012, p. 145). After the students draw their representation, they utilize heuristics, or systematic methodologies to represent, analyze, and construct solutions to the problems (Jitendra et al., 2013, p. 116; Koichu, Berman, & Moore, 2007, p. 1). Lastly, procedural flexibility requires an educator to

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modify and adjust details throughout their step-by-step instruction (Jitendra et al., 2013, p. 117; Jitendra, DiPipi, & Perron-Jones, 2002, p. 24). An example word problem is, *"On a geography map test that was worth 25 points, Janie got a grade of 20%. How many points did she earn on the geography test?"* (Jitendra et al., 2013, p. 121). A student using SBI entirely might approach this problem initially by focusing on the relationship between the 25 points and 20 percent. Next they might construct a diagram that shows the part to whole ratio equation:

$$\frac{\substack{X \\ (Janie's\ total\ points\ earned)}}{\substack{25 \\ (Total\ Points)}} = \frac{20}{100}$$

Utilizing this diagram, or something similar, to identify the appropriate heuristic procedure, a student would then seek to execute a solution while appropriately applying any necessary flexibility.

Two of the steps involved in SBI, utilizing visual-spatial representations and explicitly teaching through problem-solving heuristics, are of particular interest because they are unique to SBI when compared with similarly complex methods. Jitendra, DiPipi, and Perron-Jones (2002) claim that the component that separates SBI from other empirical strategies is the use of diagrams (p. 24). According to Koichu, Berman, and Moore (2007), universal heuristics have been attracting mathematics educators for years (p. 1). Additional sources have also claimed that there is an interest in visual-spatial strategies and heuristics within the special education community (Jitendra et al., 2002, p. 35; van Garderen, 2008, p. 139).

The purpose of this review is to determine the independent effectiveness of visual-spatial and heuristic strategies, for students with and without disabilities, when used in conjunction with word problem solving at the middle level (grades 4-8). A comprehensive analysis will be presented through the use of three guiding questions. First, what is the effect of using visual-spatial representations as a word problem-solving strategy for middle school students? Second, what is the effect of using problem-solving heuristics as a word problem-solving strategy for middle school students? Finally, what are the associated benefits of visual-spatial representations and problem-solving heuristics for students with learning disabilities?

Visual Spatial Representations as Word Problem-Solving Strategy for Middle School Students

The Difference between Pictorial and Schematic Representations

Visual-spatial representations are part of the mathematical problem-solving process. There are several visual-representation categories that can be used in the classroom (Presmeg, 1986, p. 302; van Garderen & Montague, 2003, p. 246) to help enhance individual students' problem-solving skills. Van Garderen and Montague (2003) found a significant positive correlation between the total number of visual-spatial representations and students' overall mathematical problem-solving ability. A significant portion of research on visual-spatial representations focuses on pictorial and schematic diagrams (Hegarty & Kozhevnikov, 1999; Jitendra et al., 2002; Montague & Applegate; 2000; van Garderen & Montague, 2003; van Garderen et al., 2012). Hegarty and Kozhevnikov (1999) conducted seminal research on the effects of these visual and spatial images; in order to do so they specified what a diagram within each of these categories might contain (p. 685-686). A pictorial representation is one in which a student simply identifies objects or persons that were referenced in the problem, whereas a schematic representation is categorized as one in which the student references spatial relations between objects and persons from the problem (Hegarty & Kozhevnikov, 1999, p. 686). One example word problem is, *Remi usually visits Henry's house by walking two blocks east and three blocks north so that he can bypass the woods in between their houses. Today he wants to take a shortcut through the woods to get there as quickly as possible. What is the length of the shortest distance between Remi's and Henry's houses?* A student responding pictorially to this problem might draw a picture of two boys and some evergreen trees (Figure 2a). Subsequently, a schematic response to Remi's word problem might show some of the same characteristics as the pictorial response, but it would also show the path through the woods as well as the distance east and north to represent the original route (Figure 2b).

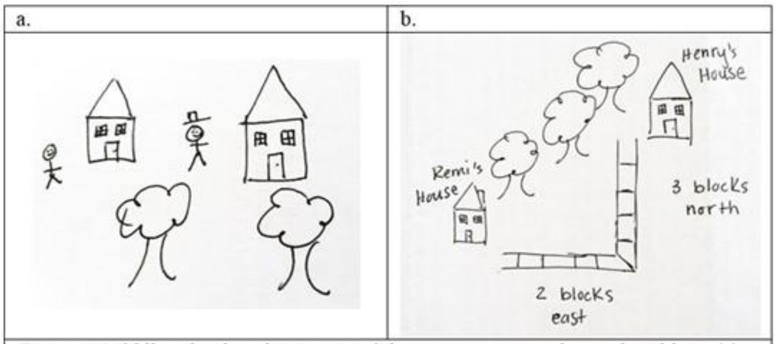

Figure 2. Middle school student's potential responses to sample word problem: (a) pictorial response and (b) schematic response.

This schematic image could also lead a student to use the Pythagorean Theorem to find the length of the shortest route to Henry's house. To arrive at this point in the schematic procedure, students need to have achieved language-based comprehension, translation, mental representation, and part of solution construction (Jitendra et al., 2007, p. 115; Desote et al., 2003, p. 188). If these steps have not been completed properly, accurate solution execution, in this case, is unlikely no matter the visual-spatial illustration strategy used.

The Value of Pictorial and Schematic Representations

These subtle variances between pictorial and schematic strategies have intrigued many researchers (Hegarty & Kozhevnikov, 1999; Jitendra et al., 2002; Montague & Applegate; 2000; van Garderen & Montague, 2003; van Garderen et al., 2012). In 2008, van Garderen administered a survey to 99 New York state special education teachers to determine their instructional practices when solving word problems (p. 141). Participants indicated that they put more instructional emphasis on concrete-visualizing strategies (e.g. pictorial diagrams) rather than more analytical strategies (e.g. schematic diagrams) when instructing students (van Garderen, 2008, p. 139). This focus is misplaced and problematic for students because van Garderen and Montague (2003) found that concrete imagery was the least effective visual representation strategy choice (p. 247; van Garderen et al., 2012, p. 157). With these instructional decisions, educators would expect that solutions to Remi's word problem be more similar to the pictorial example rather than the schematic one (Figure 1). These educators might be more engrossed in the language-based comprehension of the word problem rather than the relationships between the mathematical information embedded within the language.

Additional studies have not gone so far as to determine a strategic order to the effectiveness of visual representation strategies, but they have studied the value of schematic and pictorial representations (Hegarty & Kozhevnikov, 1999; Jitendra et al., 2007; van Garderen & Montague, 2003; van Garderen et al., 2012). A general consensus has emerged from this research that would agree with van Garderen's (2008) findings. That is, pictorial representations are significantly negatively correlated with problem-solving performance (Hegarty & Kozhevnikov, 1999, p. 687; van Garderen & Montague, 2003, p. 250-251). This correlation means that students are less likely to be successful if they only draw a pictorial image. This finding alone indicates that students creating pictures for the sake of pictures will most likely not improve problem-solving skills. These diagrams need to be developed deliberately in order to help students improve their mathematical word problem-solving aptitude. Van Garderen and Montague (2003) evaluated student mathematical problem solving through an assessment called The Mathematical Processing Instrument test (MPI; p. 247) which was developed by Hegarty and Kozhevnikov (1999, p. 686). The results from this 15-item assessment found that 70 percent of the total number of incorrect solutions on the MPI were connected to a pictorial representation (p. 250). This means that students who use pictorial representations have a 30 percent chance of getting a correct solution. Educators cannot rely on this kind of empirical result in the classroom. In conjunction with their research of pictorial methods, these scholars looked at schematic strategies. They determined that schematic representations are significantly positively correlated with problem-solving skills (Hegarty & Kozhevnikov, 1999, p. 688; van Garderen & Montague, 2003, p. 250). In the same van Garderen and Montague (2003) study that found a 30 percent success rate when using pictorial representations, the researchers also determined that 76 percent of the total number of correct solutions on the MPI were connected to a schematic representation (p. 250). These findings support the use of schematic representations in order to increase word problem-solving ability.

Pictorial and schematic representations are at the forefront of visual-spatial representation research. Pictorial representations display objects or persons from a word problem whereas schematic representations identify the spatial relationships between those problems (Hegarty & Kozhevnikov, p. 686). Some studies confirm the negative correlations between pictorial representations and student performance as well as the positive correlations between schematic representations and student performance (Hegarty & Kozhevnikov, 1999; van Garderen & Montague, 2003). This evidence indicates that educators should focus on the "essential information… and omit superfluous details" (Hegarty & Kozhevnikov, p. 688). Due to these pragmatic statistics many studies are calling for strategic diagram instruction in the classroom (Hegarty & Kozhevnikov, 1999; van Garderen & Montague, 2003; van Garderen et al., 2012). This instruction in schematic visual-spatial representations would provide some much needed structure, focused instruction, and support to the mathematics classroom.

Problem-Solving Heuristics as Word Problem-Solving Strategy for Middle School Students

Mathematical problem-solving heuristics are another common practice in the classroom. Jitendra et al. (2003) define heuristics as "systematic approaches to represent, analyze, and solve problems" (p. 116). This is such a broad definition that even some visual-spatial representation strategies could be considered a comprehensive heuristic process. For the purposes of this analysis, the definition has been adjusted to exclude all but the systematic scripted procedures used to solve word problems. Several studies focused on acronym-related step-by-step methods such as *DISC*, *SOLVED*, and *Solve it!*. These studies concentrated on and the effectiveness of these methods within the mathematics classroom (Barak, 2013; Hohn & Frey, 2002; Jitendra et al., 2013; Schaefer Whitby, 2012). These studies are of particular interest because this routine procedure is essential to the implementation of SBI.

An example heuristic, one which has been used in tandem with SBI, is the *DISC* strategy (Jitendra et al., 2013, p. 120). *DISC* stands for D – Discover the problem type, I – Identify information in the problem to represent in a diagram, S – Solve the problem, and C – Check the solution (Jitendra et al., 2013, p. 120). While this method displays the structure of a heuristic and demonstrates student expectations, it will not be applicable because it has not been separated from the SBI context. It was, however, a model for the analysis involving other heuristics.

Hohn and Frey (2002) completed a study which sought to determine the difference between standard textbook-based instruction and the *SOLVED* heuristic (p. 375). In this study, *SOLVED* stood for S – State the problem, O – Options to use, L – Links to the past, V – Visual aid, E – Execute your answer, and D – Do check back (Hohn & Frey, 2002, p. 374). This method is different from visual-spatial representation strategies because it uses general visual representations to support the process as a whole rather than specifically applying strategic diagrams independently. In order to execute a solution with the *SOLVED* method, students would need to first master language-based comprehension in order to state the problem and evaluate necessary operations for mathematics translation (Hohn & Frey, 2002, p. 374; Jitendra et al., 2007, p. 115; Desote et al., 2003, p. 188). This foundation allows students to successfully progress with the rest of this method.

Students using the *SOLVED* heuristic would approach Remi's word problem differently than those using a pictorial or schematic visual-spatial representation. Recall Remi's problem: *Remi usually walks to Henry's house by walking two blocks east and three blocks north so that he can bypass the woods in between their houses. Today he wants to take a shortcut through the woods to get there as quickly as possible. What is the length of the shortest distance between Remi's and Henry's houses?* Students using the *SOLVED* system would first need to state the problem. Remi's problem is that he needs to know the shortest distance between the two houses. Following the acronym, a student would determine the possible options available. These options could include approaching the problem in geometric, algebraic, or even trigonometric manner and choosing appropriate operations accordingly.

After stating the problem and choosing an option, students are ready to comtinue with the remaining steps of the SOLVED heuristic. The next step is making links to similar problems or situations they have completed in the past. This particular step can be difficult for students who interpret this step literally. Some may say that they have never tried to determine the shortest distance between two homes, so they have never seen anything like the mathematics in Remi's problem. Following this reflection, students would draw a visual aid of some kind. This could be pictorial, schematic, or some other kind of illustration. No matter which they choose, the representation needs to support a student's mental image rather than provide an answer. Once all of these steps have been completed, a student can begin to execute an answer. This can be another difficult step for students as they take all of the information gathered and endeavor to put it together in a logical manner. The *SOLVED* process, at this point, involves the entire breakdown referenced by Jitendra et al. (2007) in that students have demonstrated their comprehension, translated the problem from written language into an equation, made a mental representation with connections to previous representations, as well as constructed and executed their solution (p. 115; Desote et al., 2003, p. 188). After this intensive procedure, students still have to complete the final step of the *SOLVED* method by completing a check back. This is another instance in which this heuristic encourages reflection. After reaching this point, many students move on to the next problem and assume that their work is complete when, in fact, this reflection is extremely important and should not be neglected. This check back allows students to consider the problem they have completed for future reference and possibly identify errors in the process. This is in no way a minor achievement. Many consider mathematics to be simple due to its logical nature but when broken into its component parts it is clear that students need to master multiple skills in order to be successful in a mathematics classroom or profession.

Visual-spatial representations and their methodical procedure are not as demanding as those of a heuristic. The *SOLVED* method is longer than the *DISC*, and *Solve it!* because it supplements the breakdown of each mathematical process with two reflection components. Hohn and Frey (2002) used *SOLVED* to gather evidence on its effectiveness for third-, fourth-, and fifth-grade students (p. 375). It is important to note that the researchers did maintain consistency with instruction of the heuristic, not just its use, so that all treatment group students received the same heuristic training during the intervention. They implemented this intervention to treatment and control groups in each grade (Hohn & Frey, 2002, p. 375). When the results from all grade levels were combined, the statistically significant findings indicated a level of improvement that was greater than the improvement for those in the control group ($p < .01$; Hohn & Frey, 2002, p. 377). These results demonstrate that the *SOLVED* program increased student success (Hohn & Frey, 2002, p. 379). In addition to these outcomes, the researchers found that initial mathematics ability did not affect student success in that the *SOLVED* program helped all students equally (Hohn & Frey, 2002, p. 379).

Such findings imply that the SOLVED program would be effective among pre-teens; however, a deeper analysis of the data shows that not all grade levels were as successful as the average. Hohn and Frey (2002) found a statistically significant interaction between time and grade that prompted them to examine each grade level's results separately (p. 378). When the researchers analyzed the fifth grade results, they found that there appeared to be steady improvement over the initial trials but students were not able to maintain that achievement or steady progress on a two-week delayed posttest (Hohn & Frey, 2002, p. 378). On a superficial level, the fourth grade results appeared to improve over time but this trend never approached significance (Hohn & Frey, 2002, p. 378). When examining the results of the third grade assessments, Hohn and Frey (2002) found that not only did the mathematical abilities of third graders' steadily improve throughout the initial trials, but this trend continued on the delayed posttest (p. 378). These results persuaded the authors to conclude that third grade might be the ideal age for heuristic introduction and instruction (Hohn & Frey, 2002, p. 379). They went so far as to say that students at this point in school are more open to new methods because they are just beginning to solve mathematical word problems (Hohn & Frey, 2002, p. 379). These conclusions cause one to question the effectiveness of a heuristic strategy at a middle grade level. If in fact, students are open to heuristics as early as eight- or nine-years old because mathematics word problems are new, a thirteen-year old may be less likely to employ these methods. Future longitudinal studies would be able to determine heuristic effectiveness when taught in third grade and continuously re-taught and practiced into the middle grades.

Results have varied between studies depending on the heuristic used and the population studied. Schaefer Whitby (2012) conducted a study to determine the effectiveness of *Solve It!* on the percent of correct mathematical word problems for students with Autism Spectrum Disorder (ASD; p. 79). The *Solve It!* routine has many of the same strategic steps as the *SOLVED* method but does not have the mnemonic device to support student recall. *Solve It!* involves seven procedural stages: (1) Read – for understanding, (2) Paraphrase – your own words, (3) Visualize – a picture of a diagram, (4) Hypothesize – a plan to solve the problem, (5) Estimate – predict the answer, (6) Compute – do the arithmetic, (7) Check – make sure everything is right (Schaefer Whitby, 2012, p. 80). Schaefer Whitby hypothesized that this method would support the remediation of the executive functioning deficits that many students with ASD possess (p.78). This study reported the results as interrelated case studies. The three students involved had large gains and minor losses in various areas throughout the seven procedural stages (Schaefer Whitby, 2012, p. 85). The overall results demonstrate that *Solve It!* is an effective word problem-solving routine for students with ASD (Schaefer Whitby, 2012, p. 86).

There are additional historical heuristic methods such as, action-sequence-structure model, wanted-given model, and universal heuristics (Clark & Eads, 1954, p. 8; Koichu et al., 2007, p. 13; Parmar, Cawley, & Frazita, 1996, p. 416; Pólya, 2014). The action-sequence-structure model and wanted-given model are similar to Pólya's seminal universal heuristics in that they are stated in such general terms they could be used in middle level word problems or to solve a financial dispute at a major corporation. This review does not evaluate the effectiveness of these methods; however, it was these methods that set the foundation for modern heuristics and heuristic research. Koichu et al. (2007) used Pólya's methods in order to conduct research on the importance of heuristic literacy and found that on average, the treatment group had significantly higher posttest scores than the contrast group (p. 10). Similar to the research for *SOLVED* and *Solve It!*, Pólya's universal heuristic has been criticized for not being specific enough or providing necessary computational detail in order to solve the problem (Jitendra et al., 2013, p. 117).

A number of heuristics have been used in studies with varying populations. Hohn and Frey (2002) speculated that students older than eight or nine might be opposed to utilizing new methods and therefore against heuristics (p. 379). This was challenged when Schaefer Whitby (2012) found positive correlations between students with ASD using the *Solve It!* heuristic and successful mathematical word problem solving (p. 86). Universal heuristics, such as Pólya's, are just as controversial. While Koichu et al. (2007) employs Pólya's method, Jitendra et al. (2013) criticizes its use in tandem with SBI because it is too broad (p. 10; p. 117). Future research is necessary to determine whether or not problem-solving heuristics should be implemented. If problem-solving heuristics are effective, then a clearer definition is imperative for implementation. Broad conclusions cannot be drawn at this point due to a lack of consistency with the evidence. Educators should conduct empirical action research or rely on alternative strategies while this data are gathered.

Benefits of Visual-Representation or Heuristics for Middle School Students with Disabilities

Mathematical word problems have proven to be an even more daunting challenge for students with disabilities. Parmar, Cawley, and Frazita (1996) designed a study to quantify the differences between word problem arithmetic performance of students with and without mild disabilities (p. 417). Their results confirm the need for additional effective problem-solving strategies for students with learning disabilities in that, students with disabilities demonstrated minor successes even on the most rudimentary word problems (p. 422). Sharpe, Krawec, and Fults (2014) address a similar concern wherein they explain that students with learning disabilities often have difficulty with simple computation and therefore are particularly challenged by the complex metacognitive skills involved in mathematic word problems (p. 45). For instance, if a student has a learning disability in reading then the language-based comprehension referenced by Jitendra et al. (2007) becomes the first in a series of hurdles. If a student cannot overcome this hurdle, then they do not have an opportunity to demonstrate their mathematics ability. Educators can help students overcome this by reading the problem aloud; however, each subsequent step is comprised of its own challenges. Students with mathematics learning disabilities not only face the same academic challenges as their non-disabled peers, but they are also expected to overcome additional academic deficits in order to achieve similar success. According to Montague and Applegate (2000), students with disabilities often report problems as more difficult than do their average-achieving peers. In addition to this, as that difficulty level increases they have a tendency to take less time than their peers, indicating their belief that they are unable to complete the word problems. This was attributed to the fact that they demonstrated evidence of rushing through the problems and giving up instead of persisting through the computation (Montague & Applegate, 2000).

It is an educator's responsibility to help each student achieve their highest potential, despite these challenges. Word problem-solving strategies are one way in which educators can support mathematic potential. Several studies have proposed visual-spatial and heuristic concepts that would support the word problem-solving success of students with learning disabilities in the classroom (Koichu et al., 2007; Montague & Applegate, 2000; Sharpe, Krawec, & Fults, 2014; van Garderen & Montague, 2003; van Garderen et al, 2012).

Benefits of Visual-Spatial Strategies for Students with Disabilities

Visual-spatial representation strategies are a simple way to ask students to work through a problem. This strategy is used in special education classrooms because it appears to be simple and straightforward (van Garderen & Montague, 2003, p. 138). Research has shown that schematic visual-spatial representations help all students achieve correct solutions more often than pictorial representations. This generalization appears to apply to all minorities but empirical data will show that students with learning disabilities may benefit most from the use of these strategic representations. Van Garderen and Montague (2003) studied the relationship between frequency of the type of visual-spatial representations and successful execution that sixth grade students used while solving word problems (p. 247). In order to measure student achievement, the researchers used a modified 13-item version of the 15-item MPI (van Garderen & Montague, 2003, p. 251). Students encountered word problems on the MPI that were multistep, thought-provoking statements designed to quantify mathematics proficiency (van Garderen & Montague, 2003, p. 251). Assessments from this study showed that all students reported using visual-representation strategies a minimum of eight times; however, in general, the students with learning disabilities drew less visual representations than their average-achieving peers (van Garderen & Montague, 2003, p. 251). The results also exhibited that, on average, students with learning disabilities received the lowest scores among participants and drew more pictorial diagrams rather than schematic ones (van Garderen & Montague, 2003, p. 251). This information permits one to come to the conclusion that if students with disabilities drew more schematic diagrams, like their average-achieving peers, they could be more successful in the mathematics classroom. This speculation is the basis of future research in this area.

Another study, by van Garderen, Scheuermann, and Jackson (2012), studied how students with disabilities used visual-spatial representations to solve mathematics word problems (p. 147). Part of the method of this study was designed to allow the administrator to prompt students in one group and refrain from prompting students in a second group (van Garderen et al., 2012, p. 147). The initial findings of this study paralleled those of the van Garderen and Montague (2003) study in that students with disabilities and their average-achieving peers reported using a similar number of diagrams (p. 157). Initial results also indicated that students in the prompting group generated more diagrams than those in the no-prompt groups (van Garderen et al., 2012, p. 147). As the researchers completed further analyses, they found that the majority of diagrams created by students with learning disabilities were pictorial (van Garderen et al., 2012, p. 157). These results confirm the speculation that a higher quality diagram might support more accurate results (van Garderen et al., 2012, p. 155).

The findings of van Garderen and Montague (2003) as well as van Garderen et al. (2012) are promising. However, there are no guarantees that explicit instruction in schematic diagrams will improve mathematics ability for students with learning disabilities. The results indicate that students with learning disabilities have a tendency to draw more pictorial diagrams yet the data shows that students with more pictorial diagrams generate less correct solutions (van Garderen & Montague, 2003, p. 251; van Garderen et al., 2012, p. 157). Due to the parallel nature of the results for students with learning disabilities when compared to general education students, there is still a call for more research in strategic diagram instruction (Hegarty & Kozhevnikov, 1999; van Garderen & Montague, 2003; van Garderen et al., 2012).

Benefits of Problem-solving Heuristic Strategies for Students with Disabilities

Heuristic strategies are in an ambiguous state of acceptance in the education community. Studies contradict each other regarding heuristic selection and application effectiveness. The one concept that remains consistent throughout heuristic research is its application for special education students. Many of the studies that explored heuristics and their applications for students with learning disabilities acknowledged their benefits for the special education classroom (Hohn & Frey, 2002; Koichu et al., 2007; Schaefer Whitby, 2012).

In Schaefer Whitby’s 2012 study on the effectiveness of *Solve It!* problem-solving routine for three students with ASD, the researcher hypothesized that heuristics would provide the necessary structure and support that students with ASD need in order to counteract possible executive functioning deficits (Schaefer Whitby, 2012, p. 78). It is important to note that a limitation of this heuristic study is the small sample size (3 students). The results for these three students were clear in that, despite minor setbacks, all students had large gains and their mathematics ability improved through the use of this heuristic intervention (Schaefer Whitby, 2012, p. 86).

Koichu et al. (2007) discussed a potential explanation for this success among students with learning disabilities. In a final analysis of this research, Koichu et al. (2007) segregated the students into ‘stronger’ and ‘weaker’ groups based on their pretest scores (p. 12). Students in both the treatment and contrast groups were labeled this way and their results compared. The initial data confirmed that the weaker students were weaker than stronger students; however, the weaker treatment group demonstrated momentous gains (Koichu et al., 2007, p. 13). This group improved their mathematics ability more than all other groups in the study (Koichu et al., 2007, p. 13). Koichu et al. (2007) concluded that the weaker students succeeded because the heuristic activities were structured to allow contributions from all students to whole-class discussions (p. 15). While this specific comment may be attributed to speculation it is based on the success of this heuristic strategy and does not lessen the success of the study.

A third study showed similar success in the use of heuristic strategies when applied to students with learning disabilities. Hohn and Frey (2002) researched the effects of using the *SOLVED* method in third, fourth, and fifth grade students (p. 375). As stated earlier, the results of the *SOLVED* program were positive when the entire control and treatment groups were compared; however, when a grade-dependent analysis was conducted the researchers found that age played a role in student success (Hohn & Frey, 2002, p. 378). During their broad spectrum analysis of the results they discovered that there was no interaction between a student's mathematics skills and their performance on the assessments. This leads to the idea that the *SOLVED* program helped all students equally (Hohn & Frey, 2002, p. 379). It is important to note that the final results indicated that this intervention was most effective for third grade and therefore it might be most effective for students with learning disabilities in the third grade as well (Hohn & Frey, 2002, p. 379).

Much of the current research supports the use of visual-spatial schematic representations. Similar research has been conducted on problem-solving heuristic strategies and the results have been more ambiguous. However, when looking at the benefits for students with learning disabilities the results of these two methods are more parallel. Visual-spatial representation research has led to the conclusion that the field needs more research. There is minimal evidence of potential effectiveness but not enough to make all-encompassing conclusions. Heuristic studies, despite differences in their results, have all agreed on the one fundamental notion that students with learning disabilities can benefit from heuristic strategies (Hohn & Frey, 2002; Koichu et al., 2007; Schaefer Whitby, 2012). This evidence is not enough to determine the most effective heuristic for students with learning disabilities but it is plenty to establish a foundation for more empirically-based instruction.

Conclusion and Recommendations

Given the prevalence of word problems, educators and administrators are searching for effective word problem-solving strategies to help increase students' mathematic success (van Garderen, 2008, p. 142). Many of these professionals are looking for alternate methods which include structured approaches with scripted instruction (Allington, 2006, p. 5; Ede, 2006, p. 30; Parks & Bridges-Rhoads, 2012, p. 309). Throughout this review, SBI has emerged as an effective word problem-solving strategy at the middle level (Jitendra et al., 2013, p. 131). While all four steps of this process are successful when combined, the goal of this study was to determine the individual practical applications of two of the stages (visual-spatial representations and problem-solving heuristics) in both the general education and special education classrooms (Jitendra et al., 2002, p. 24; Koichu et al., 2007, p. 1; van Garderen, 2008, p. 139). The results of this review demonstrate the advantages and disadvantages of these strategies.

The first of these stages, visual-spatial representations, was defined as pictorial or schematic illustrations in which students display objects or persons referenced in the problem (e.g. pictorial images) or where students show spatial relationships between those objects or persons (e.g. spatial images; Hegarty & Kozhevnikov, 1999, p. 686). Studies that have looked into the value of these representations have come to the consensus that pictorial diagrams are significantly negatively correlated with mathematic problem-solving performance (Hegarty & Kozhevnikov, 1999, p. 687; van Garderen & Montague, 2003, p. 250-251). This means that students cannot solely draw superficial components and expect to succeed. Students need to have a deliberate strategy in order to be successful. In search of a more structured strategy, researchers investigated the effectiveness of schematic diagrams. Several studies agreed that schematic diagrams were more effective than pictorial ones (Hegarty & Kozhevnikov, 1999, p. 688; van Garderen & Montague, 2003, p. 250). While these results were clean, those related to heuristics were not.

The second stage, problem-solving heuristics, was defined as systematic procedures used to solve word problems. Research has been conducted on several different methods (e.g. *DISC*, *SOLVED*, *Solve it!*) that meet the parameters of the definition. While the results of these studies did not come to a consensus about the effectiveness of problem-solving heuristics, they did present an interesting controversy related to their use. The routine of the *SOLVED* heuristic produced results that showed steady significant progress for third-grade students, insignificant progress for fourth-grade students, and regression on a delayed posttest for fifth-grade students (Hohn & Frey, 2002, p. 378). It was these findings that led Hohn and Frey (2002) to believe students in older grades would be unwilling to learn new methods, such as heuristics, and therefore would not benefit from their use (p. 379). However, another study found that the *Solve It!* routine allowed middle school students with ASD to effectively improve their word problem-solving skills (Schaefer Whitby, 2012, p. 86). Additional controversy arose related to the use of universal heuristics. One study criticized Pólya's (2014) universal heuristics for being too broad when in relation with middle school mathematics word problems (Jitendra et al., 2013). However, another study used Pólya's (2014) heuristic and found that students using the method produced higher posttest scores than the contrast group (Koichu et al., 2007). Despite these controversies, heuristics, as well as visual-spatial representations, demonstrated effectiveness when used by students with learning disabilities.

Research has shown that students with learning disabilities have particular difficulty in the area of mathematics word problems (Parmar et al., 1996, p. 422) due to the metacognitive steps involved (Sharpe et al., 2014, p. 45). In order to contest these difficulties, researchers determined the benefits of visual-spatial strategies and problem-solving heuristic strategies on mathematic success for students with disabilities. The main finding in regards to visual-spatial representation strategies for special education students is that they are effective because the predictable and purposeful images are helpful to many students with learning disabilities (van Garderen et al., 2012, p. 157; van Garderen & Montague, 2003, p. 251). The main finding in regards to problem-solving heuristic strategies for special education students is that they are effective as well because they have a specific routine and structure. Despite the controversies, all of the studies discussed in this review recognized additional benefits for students with disabilities (Hohn & Frey, 2002; Koichu et al., 2007; Schaefer Whitby, 2012). Effective schematic instruction is beneficial for all students whereas the data about the present benefits of effective heuristic instruction only applies to special education students.

The results for the effectiveness of schematic strategies are positive; however, it is still important for educators to maintain a layer of skepticism. Representations, whether pictorial or schematic, are difficult to categorize and, even if drawn properly, they can lead students astray though the misinterpretation of the diagram. Additional research is needed in the areas of schematic construction and application for mathematics word problems. Problem-solving heuristics need additional empirical findings as they are still shrouded in controversy. Broad conclusions cannot be drawn at this point but additional studies related to specific gaps in the research may help. A future longitudinal study should be created to determine whether or not heuristics are effective when taught in third grade and retaught regularly for several years. This research would help to determine the actual benefits of the consistent use of a strategy.

Throughout this review, a few developments have become clear. Educators need to understand the value of diagrams and the effects that the schematic or pictorial nature of a diagram can have on mathematic success. They should be aware of the heuristic controversies related to word problem solving and their implications for the classroom. Most importantly, educators should recognize that utilizing the components of an effective strategy independently of the designated procedure does not guarantee student success.

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