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How Could Mathematics Teachers Know Complexity Inquiry?

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Abstract

In order to provide a transforming way of knowing for teachers' professional learning, this paper uses narrative to present the knowledge emerging from teachers as participants in a concept study derived from complexity science. There are three components in the narrative: what knowledge emerged from concept study; what has been changed for the emerged knowledge; and where we are going with complexity. The narrative provides not only a lived example but a framework for the action towards the transforming way of knowing for teachers' professional learning.

Key words: Concept study; Equivalent fractions; Complexity

Introduction

As a mathematics teacher educator, I have been working on teacher professional learning for many years. However, I realize that it is only too often to take teachers as students to accept "planned enculturation or training" (Osberg & Biesta, 2008, p. 315) as Zeichner (2012) reported that teachers were believed to be trained as technicians to implement teacher-proof teaching scripts provided by authorized sources. Therefore, I always deliberate on how teachers could not learn in transferring and transmitting way as a "closed system approach" rather than in a transforming way as an "open systems approach" (Doll, 2012, p. 19).

A month ago, it was a coincidence that I had a chance to try a different way of knowing a concept through a concept study advocated by Davis and Renert (2014). Concept study aimed to organize activities of the cohort around "the collective as a cognizing agent" (Davis and Renert, 2014, p. 53) based on complexity science. Thus, I was very excited to expect that the concept study might provide a sense of transforming learning for teachers because the collective learning was to "recognize events of emergence" (Davis and Simmt, 2003, p. 144).

My concept study was titled as equivalent fractions, referring to that it would work on the concept of equivalent fractions. In my preparation for the concept study, I knew that it was not going to present or deliver the knowledge as "a mastery" (Davis & Renert, 2014, p. 47) of equivalent fractions in "a pre-determined direction" (Osberg & Biesta, 2008, p. 319) but as "an open disposition" (Davis & Renert, 2014, p.47). Thus, I had to turn away from my familiar predetermined presentation towards the open discussion of concept study. However, during my preparation, I was always haunted by the question "what should I prepare for the concept study". In case the open discussion might go off my topic, I had prepared the guiding questions to prevent it from going off the track. Another question "what could I get from concept study" also perplexed me even though I knew there were no placed objects for the concept study. With this regard, I decided to try reading through all the literatures related to equivalent fractions as possible as I could find in the library. At that time, I thought to myself that if I would not obtain any fresh viewpoint from the concept study, I could at least write out a literature review on it with the accumulation of related rich information. Obviously, the "frustration of intentions" was the logic of conventional education (Osberg & Biesta, 2008, p. 325).

As known to us, concept study has a structure made up of four emphases such as realizations, landscapes, entailments, and blends by Davis and Renert (2014). They further presented the interpretations for the four emphases: realizations were to collect learners' all manners of associations in making sense of a mathematical construct; landscapes were to organize realizations into a macro-level map; entailments were to study how different realizations shape the understanding of related mathematics concepts; and blends were to seek the meta-level coherences by exploring the deep connections among realizations. From the above-said structure of concept study, it can be evidently seen that the realizations are the basic one of its emphases. Naturally, my

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focus would be highly on how to stimulate realizations to initiate my concept study, and the four emphases were also employed as a framework of examining what emerged from concept study.

What Emerged from Concept Study?

Multiple Realizations

The initial activity for the collective study of equivalent fractions was designed to stimulate deep and vast realizations. Specifically, participants were asked to interpret such equivalent fractions as $\frac{1}{2}=\frac{2}{4}$ and produce as many interpretations to them as possible by using "enactive, iconic or symbolic" (Bruner, 1966, as cited in Davis & Renert, 2014, p. 58) methods. This way was proved very effective in our later discussion. Firstly, each participant had produced more than three kinds of interpretations as "potential randomness" (Davis & Simmt, 2003, p. 147). Secondly, all the participants were very engaged with the discussion about the posted presentations, which highlighted "neighbor interaction" (Davis & Simmt, 2003, p. 147) for the purpose of reblending different interpretations as "internal diversity" (Davis & Simmt, 2003, p. 147). And lastly, there were 12 kinds of realizations "emerged bottom-up" (Newell, 2008, p. 14) from the participants' contributions such as area, length, ratio, multiplicative thinking, rotation, number line, set model, set symbolic expression, degree of angle, symbolic rule one and two, realistic context, and enacting objects. Surprisingly, such realizations were completely beyond the interpretations from textbooks and researches, and certainly went beyond the "considerations of any member" (Newell, 2008, p. 13). I was totally "surprised" (Newell, 2008, p. 12) at the realizations like all the participants at that moment. Particularly, we were very astonished about Symbolic Rule One: multiplication by one e.g. $\frac{1}{2}\times\frac{2}{2}=\frac{2}{4}$.

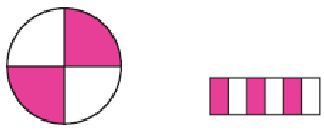
Nonlinear Landscapes

The symbolic rule one was referred to by a participant from a college as "tacit knowledge" (Davis & Renert, 2014, p. 27). To her surprise, she had never realized according to her words that operating such symbolic rule was more reasonably than doing the classical one: multiplication by sameness on both numerator and denominator e.g. $\frac{1\times2}{2\times2}=\frac{2}{4}$.
Just at the moment when the discussion was centering on the symbolic rule one and her surprise, I was wondering where the discussion would lead to and whether or not I would need to pose a guiding question for the further discussion in case that the discussion might go off the topic. In fact, it was my hesitation that made it possible to leave "the space of emergence" (Osberg & Biesta, 2008, p. 325) and maintain the "system dynamics" (Smitherman, 2004, p. 9). After a while, the participant who spoke of the symbolic rule one suddenly stood up, went to the front of class, and was eager to present the following rational function such as $f(x)=\frac{x^2}{x^2+x}=\frac{x}{x+1}\times\frac{x}{x}=\frac{x}{x+1}\times1=\frac{x}{x+1}$ to us for it occurred to her that she had ever applied it to rational function before. The application of equivalent fraction in the rational function was emergent in a nonlinear way but created a pathway of presenting landscapes of equivalent fractions, which really gave me an inspirational flash. At this very moment, I came to realize that it was absolutely unnecessary for me to "control the system or predict system behavior" (Smitherman, 2004, p.9) by throwing out guiding questions.

Excavating Entailments

In addition to rational function, the symbolic rule one-multiplication by one-also touched off another heated discussion about the concept of fractions. Frankly speaking, at that time, notwithstanding I did not know how the discussion of fractions concept would benefit my concept study-equivalent fractions as an "unsettling doing" (Osberg & Biesta, 2008, p. 325), I would like to "allow the conversations to reach whatever conclusions they might veer towards" (Newell, 2008, p. 14) due to the fact that I have not only experienced the surprise of the occasioned emergence in realizations and landscapes but also known that any pre-determined direction would "close down other possibilities for the subject's emergence" (Osberg & Biesta, 2008, p. 319). And then the participants started to enquiry about what fractions were. On the discussion, such interpretations of fractions were emergent as 'part-whole, percentages, operators, decimals, probabilities, ratios, and quotients'. However, the form of fraction $\frac{a}{b}$ was indicated to be quite problematic. For example, it was easy to confirm that $\frac{2}{4}$ was equivalent to $\frac{3}{6}$ while, if the two fractions were set in the figure A and B (see figure1), respectively, it was not proper to say that both of them were equivalent because fractions expressed not only the relationships between parts and whole but operators as well.

Such problem, which might make students, even teachers confused, is rarely mentioned in practical mathematics teaching. In fact, it really helps students to build the real concept of fraction and equivalent fractions. Until now, other participants and I could be feeling that “what would be produced, opened up and generated was unpredictable” (Smitherman, 2004, p. 25) and that the discussions had reached “meaningful connections that had a magnification to a greater connection” (Smitherman, 2004, p. 22) in terms of fractions concept.
Near the end of my concept study, I demonstrated a novel way of finding equivalent fraction of concept study from multiplication chart (see figure 2), which I had learnt from Coker and Cook (1992).



A $\frac{2}{4}$ B $\frac{3}{6}$

Figure 1. Fractions

	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	3	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	37	36	45	54	63	72	81

Figure 2. An open multiplication chart

For example, it was easy to find equivalent fractions of $\frac{2}{3}$ by identifying a multiple of 2 such as 10 and then finding the multiple of 3, 15. Thus, $\frac{10}{15}$ was an equivalent fraction of $\frac{2}{3}$. In fact, I would like only to present the fresh idea without any intention of triggering further discussion or thinking, and I did not know the implication of such idea, either.

Unexpectedly, after my introduction of multiplication chart, all the participants revealed that this method was a really practical and fresh one. Interestingly, one of the participants, who is a primary teacher, noted that she would try using the method in her teaching practice as soon as possible. So far, I thought my concept study could be characterized as an “open disposition” (Davis & Renert, 2014, p. 47) and “collective cognition” (Davis & Renert, 2014, p. 53).

Extended Discussion

After the concept study, I was very excited to tell one of my colleagues about my emotional transitions from frustrated intentions at the beginning to the surprises at the end of the concept study. Instead, she was desperate to ask me about how teachers could build the connections between the enactive and iconic and the symbolic representations which were introduced arbitrarily very often. Frankly speaking, I was stuck at such a demanding and unexpected question that my mind went blind because it was never touched upon in my own teaching experiences and research literature, which triggered me to find out their connections.

The next day after my concept study, there was a regular meeting for the graduate students in mathematics, and most of them participated in my concept study. Therefore, I attempted to avail myself of this opportunity to pose the question about how to build the connections between different representations of equivalent fractions. However, to my disappointment, the question did not ignite participants’ further discussion. I came to realize that my “desired end” (Osberg & Biesta, 2008, p. 325) could not be reached and that the participants were only interested in the multiplication chart I had introduced near the end of my concept study.

Except that the equivalent fractions could be found in the chart, for the participants, it seemed that the chart could generate more ideas about the fractions because it demonstrated the area model of the multiplication with which students were quite familiar.

Occasioned Blends

Interestingly, it was found that there were new interpretations of equivalent fractions in using the multiplication chart by blending with area model, set model, set symbolic expression, and the rule of multiplication by sameness on both numerator and denominator. For example, it was very reasonable for students to find the common factor 'm' for the symbolic expression $\frac{a\times m}{b\times m}=\frac{am}{bm}$ from the multiplication chart. Specifically, the steps of moving from 2 to 10 were the same as those from 3 to 15. This could be used to interpret the fact that when the numerator 2 was multiplied by 5, the denominator 5 also done by 5. Moving backward could illustrate dividing the denominator and numerator by sameness. In addition, the openness of the multiplication chart could be understood as visualized symbolic sets such as $\left\{\frac{2}{3},\frac{4}{6},\frac{6}{9},\frac{8}{12},\dots\right\}$. Obviously, the multiplication chart became a context of building the connections of between different representations of equivalent fractions, which completely transcended "the sum of the participants' individual knowledge" (Newell, 2008, p. 11) and emerged as an occasion (Newell, 2008).

In addition to the new and tacit knowledge emerging from our discussions, there remained some confusions about the equivalent fractions.

The Same Value?

It appeared to be quite common for us that two fractions were considered as equivalent when they had the same value (Wong & Evans, 2007). The same value was so obviously demonstrated in the idea of "multiplication by one" of equivalent fractions that we seemed to have no trouble with 'the same value'. However, different voices about the value of a ratio came out of the participants when we moved to set model. Someone queried whether "1 to 2 had the same value as 30 to 60". However, I did not rush for a fixed answer rather than intentionally left such inquiry to the future further discussion in order not to close down the open disposition of the concept study. Thus, the chaos would be regarded as "teachable moments, embracing the notion that not everything that occurs in the classroom can be predicted" (Smitherman, 2004, p. 10).

'Equivalent' or 'Equal'?

In addition to the question of the same value, the set model caused another thought about the differences between equivalent and equal. Taken the area model as an interpretation of equivalent fractions, the area sizes of the shaded parts representing equivalent fractions such as $\frac{1}{2}$ and $\frac{2}{4}$ were equal (see figure 3). However, in set model, being equal was much challenged because the amounts of the shaded parts in the two sets were not equal (see figure 4). The challenge activated one teacher, in particular, to continue investigating the differences between equivalent and equal after the concept study. For example, she found that 'the Math is Fun' website defined equivalent as having the same value and equal as exactly the same amount or value. The two definitions were so indiscernible that the teacher was frustrated at that "I am stuck". Feeling frustrated and confused, to a certain extent, implied the "challenges of thinking of a class a learning system" (Newell, 2008, p. 15).

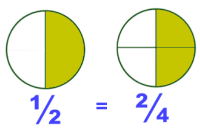


Figure 3. Area Model

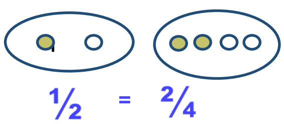


Figure 4. Set Model

What emerged from my concept study including the surprises and concerns definitely was more than what I had described in the above sections. More importantly, I made sure that, except for my reliance on literatures to write a paper on my concept study as what I addressed as before, now I have also had a great number of data resulted from the knowledge and knowing that emerged from my concept study for writing such a paper.

Retrospectively, I would like to see what had been changed for the rich emergences from my concept study including inner consciousness and outer environments. Fortunately, I found that the changes could be expressed by using the four conditions for emergence in a classroom from Davis and Simmt's (2003) work: internal diversity; decentralized control; organized randomness; and neighbour interactions (p. 147) and one comment-risk taking from a professor.

What Had Been Changed in Concept Study?

Internal Diversity

There were eight participants in my concept study, including: three were Canadian teacher educator, primary teacher, and college teacher, respectively; three were Tanzanian mathematics professor, teacher educator, and curriculum developer, respectively; one was Caribbean middle school teacher; and one was Chinese teacher educator. Based on the diversity of participants, they brought the possibility of diverse conversations because it could "define the range and contours of possible response" (Davis and Sumara, 2008, p. 39).

Decentralized Control

My primary worries had been about such the discussion digression, if any, out of my concept study that guiding questions were fully prepared to get it back to the track. Fortunately, during the discussion, my directed questions were not brought forward due to my hesitation in taking guiding questions as avoiding the digression or extinguishing the "potential of the collective as a knowledge-producer" (Davis and Sumara, 2008, p. 41). Eventually, the collective of my group became the arbiter of the discussion orientation, "the correctness and appropriateness of the knowledge produced by the system" (Newell, 2008, p. 12).

Organized Randomness

In our discussion, the participants were encouraged to articulate as many ideas about the equivalent fractions as possible. Happily, the multiple realizations of equivalent fractions were well exemplified for "openness to randomness in order to allow for the emergence of unanticipated possibilities" (Newell, 2004, p. 12).

Neighbor Interactions

The landscapes, entailments and blends were the results of neighbor interactions, which emerged occasionally. In fact, enabling neighboring interactions demanded that "one must relinquish any desire to control the structure and outcome of the collective" (Davis & Sumara, 2008, p. 41) as I had disregarded my guiding questions.

Risks Taking

It was risk taking for me to keep the discussion going without predetermined interruption because I was not sure about what would emerge from it and about whether such discussion would benefit my concept study such as the discussion of fractions concept. However, such risk taking also encouraged participants to have a deep exploration of the concept and make bold to express their ideas as a professor had comment on my concept study. Certainly, engaging in emergence was placing my group at risk as Osberg and Biesta (2008) had commented that we "do not know, cannot know, what will happen, only that something will happen" (p. 325).

The changes listed above were necessary for a productive concept study but not sufficient as Newell (2008) indicated that "complexity is not guaranteed" (p. 14). Since occasioning emergence in the learning collective was so difficult and uncertain, where are we going with it?

Where Are We Going with Complexity?

From complexity science, class is "a complex adaptive system and a learning entity" (Newell, 2008, p. 16), which offers a pragmatic advice for teachers that "teaching is not a matter of management, rather a sort of improvising in the jazz music sense of engaging attentively and responsively with others in a collective project" (p. 16). During teaching, knowledge and order emerge from neighbor interactions having just "right amount of tension or difference or imbalance" (Doll, 2012, p. 25) or even "violence" (Osberg & Biesta, 2008, p. 326) among the participants' interactions. Learning now occurs not through "direct transmission from experts to learners" (Doll, 2012, p. 25) in a "sequential form" (Smitherman, 2004, p. 24) but through a "space of emergence" (Osberg & Biesta, 2008, p.326) in a "nonlinear manner" (Doll, 2012, p. 25) just as what the teachers in my concept study had experienced. The nonlinear dynamical notions provide new metaphors with which our purview on curriculum can change.

Now curriculum could be generated by "linking pedagogical goals with the unpredictable behavior of learners" (Simitherman, 2004, p. 10). In Doll's (2012) words, curriculum is now "an emerging one within ongoing process that actually catalyzes itself via interactions within the system or network" (p. 25). As we had done, the curriculum emerged from the concept study of equivalent fractions included the multiple realizations, the rationale of rational function, the deep understanding of fractions concept, the connections of different representations in multiplication chart, and unsettling concerns or confusions such as the differences between equivalent and equal. Such curriculum was unique and we could not find it from any kind of authorized sources or research literatures.

It is worthwhile to try, create or allow learning to occur in "nonlinear patterns, emergent, divergent, and convergent" (Sitherman, 2004, p. 15) even though it is "difficult and provocative, and often uncomfortable" (Osberg & Biesta, 2008, p. 325), and "even a certain violence" (Biesta, 2006, p. 26, as cited in Osberg & Biesta, 2008, p. 325). Therefore, as educators, we are responsible not to deny the difficulty, provocativity, uncomfortability, and violence involved in coming, or as Osberg and Biesta (2008) said "we should be calling, into presence" (p. 325) as "a source of creativity" (Doll, 2012, p. 17).

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