



ISSN: 2148-9955

## International Journal of Research in Education and Science (IJRES)

[www.ijres.net](http://www.ijres.net)

### University-School Collaboration as a Tool for Promoting Pre-Service Mathematics Teachers' Professional Skills

Hulya Kilic<sup>1</sup>, Zelha Tunc Pekkan<sup>2</sup>

<sup>1</sup>Yeditepe University

<sup>2</sup>MEF University

#### To cite this article:

Kilic, H. & Tunc Pekkan, Z. (2017). University-school collaboration as a tool for promoting pre-service mathematics teachers' professional skills. *International Journal of Research in Education and Science (IJRES)*, 3(2), 383- 394. DOI: 10.21890/ijres.327897

This article may be used for research, teaching, and private study purposes.

Any substantial or systematic reproduction, redistribution, reselling, loan, sub-licensing, systematic supply, or distribution in any form to anyone is expressly forbidden.

Authors alone are responsible for the contents of their articles. The journal owns the copyright of the articles.

The publisher shall not be liable for any loss, actions, claims, proceedings, demand, or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of the research material.

## University-School Collaboration as a Tool for Promoting Pre-Service Mathematics Teachers' Professional Skills

Hulya Kilic, Zelha Tunc Pekkan

<b>Article Info</b>	<b>Abstract</b>
<p><i>Article History</i></p> <p>Received: 02 November 2016</p> <p>Accepted: 13 January 2017</p> <hr style="border: 0.5px solid black;"/> <p><i>Keywords</i></p> <p>Modeling Noticing Mathematical thinking</p>	<p>In this paper, we discuss pre-service mathematics teachers' professional gains from a university-school collaboration where they were given opportunity to observe two teacher educators' instructional practices in a 6th grade classroom, interact with students in one-to-one fashion and reflect on the teacher educators' and their own practices. Three pre-service teachers out of nine volunteers attended all modelling and practice sessions for 20 weeks. The data collected through interviews, field notes, videos of both class activities and group reflections and written reflections. All nine pre-service teachers appreciated the value of working closely with students in a school setting rather than just observing the teachers. Furthermore, they gained experience about noticing students' mathematical thinking and critically analyzing their own practices.</p>

### Introduction

In many countries, one of the major concerns is to increase the quality of the education (The Organization for Economic Co-operation and Development [OECD], 2014). Teachers are at critical positions because they have responsibility of providing appropriate teaching-learning environment for students to achieve the educational goals identified in the curriculum (Sowder, 2007). Therefore, teachers should be equipped with various knowledge and skills to attain those educational goals (Shulman, 1987). In theory, teacher education programs are designed to offer such professional knowledge and skills for pre-service teachers (PSTs). In practice, teacher educators are responsible for providing opportunities for PSTs to gain those skills (Aleccia, 2011; Anthony, Hunter & Hunter, 2015; Bates, Ramirez, & Drits, 2009; McGinnis, Watanabe, & McDuffie, 2005). Because teachers have a tendency to teach in a manner they were taught (McGinnis et al., 2005) teacher educators not only teach for being a teacher but also should try to be good models for PSTs.

Many scholars agreed that student-centered teaching practices are better in terms of satisfying permanent learning and supporting higher order thinking (Franke, Kazemi, & Battey, 2007; Sowder, 2007). Although many teacher educators emphasize the importance of such practices in their courses, they may not be embraced by the PSTs unless they do not instruct in that manner (Aleccia, 2011). Definitely, teacher educators, themselves, should enable PSTs to be active learners and experience the steps of lesson planning, effective use of instructional techniques, preparing instructional tasks, etc. (Grossman, Hammerness, & McDonald, 2009; Loughran & Berry, 2005). The studies in teaching mathematics indicated that providing opportunities for teaching in a school setting and analyzing the videos of own or others' instructional practices help PSTs understand the essence of student-centered practices as well as improve their pedagogical content knowledge (PCK), specifically, their knowledge about students and their learning (Friel & Carboni, 2000; Lannin et al., 2013; Roller, 2015; Stockero, 2008). Teacher educators' major roles in such experiences would be the organizer of such teaching environment and guide for teaching, noticing and scaffolding practices (Anthony et al., 2015; McGinnis et al., 2005). Besides, some scholars (e.g., Ball, 1988) attempted to be a model for PSTs by being the teacher of a classroom in a school setting such that PSTs are able to observe how the teacher educator plans the flow of the lesson, interacts with students, engages them into lesson, handles with unexpected situations and assesses their understanding.

In this study, we provided a teaching-learning setting for PSTs to investigate the contributions of such setting on PSTs' professional skills. We afforded PSTs with opportunities that they involved in preparation and implementation processes of mathematical tasks for students and then they designed their own tasks and implemented them. We essentially looked for how teacher educators' modelling practices and PSTs' own experiences with students contributed to their professional skills, specifically their noticing of students' mathematical thinking.

## Theoretical Framework

### Modeling

Bandura (1977) claimed that people learn by observing others and modelling their behaviors. Teaching through modelling-intentionally or not, is used by many school teachers and teacher educators (Anthony et al., 2015; Parker & Hurry, 2007; Santagata, 2009; White, 2011). Teacher educators could be model for effective use of instructional strategies, how to elicit and assess students' thinking, how to design instructional tasks and how to use manipulatives and technology for teaching (Anthony et al., 2015; Kazemi & Waege, 2015; Loughran & Berry, 2005). Teacher educators' such modelling might be rehearsed by PSTs in an artificial classroom setting, in the form of microteaching, or in an actual classroom setting, in the form of practicum (Anthony et al., 2015). Although PSTs could get benefit from such modelling practices, it is not much known whether they would internalize those practices and convey in their future teaching. However, recently, teacher educators put more emphasize on how to enable PSTs to notice the classroom events that have potential to elicit or understand students' mathematical thinking (Anthony et al., 2015; Sherin, Jacobs, & Philipp, 2011). They use videos of classroom instructions (PSTs' own or others) or students' written work to model noticing and they assess PSTs' noticing practices by analyzing their oral or written reflections about the instructions that they delivered, watched or observed (Sherin et al., 2011). In this study, as teacher educators, we tried to be model for PSTs in terms of preparing and implementing mathematical tasks as well as eliciting and assessing students' mathematical understanding and thinking. Then, we gave opportunities to the PSTs to make practice of understanding students' mathematical thinking and scaffolding their learning.

### Noticing Students' Thinking

Noticing is usually described as "attending to particular events in an instructional setting and making sense of those events" (Sherin et al., 2011, p. 9). However, scholars' definitions differ in terms of what to attend and what making sense involves in. Van Es and Sherin (2002) identified three aspects of noticing as "(a) identifying what is important and noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about classroom interactions" (p. 573). Van Es and Sherin (2002, 2008) also categorized what teachers paid attention to in a lesson as mathematical thinking, pedagogy, climate and management. Some scholars, however, specifically focused on noticing of students' mathematical thinking rather than the other things could be observed in a lesson as van Es and Sherin found out. For instance, Jacobs and her colleagues (Jacobs, Lamb, & Philipp, 2010) identified noticing as involving three phases: attending students' thinking, making sense of it by interpreting students' understanding and deciding how to respond to them. They stated that noticing is initiated by paying attention to students' written work and oral explanations to catch any evidence about students' mathematical understanding and thinking. Then, teachers use those evidences to interpret students' understanding. And finally, they decide how to react to the students either to fill the gaps in their knowledge or scaffold their understanding. In our study, because we worked with a group of volunteer 6<sup>th</sup> grade students rather than a definite 6<sup>th</sup> grade class, we decided to focus on noticing students' mathematical thinking and tried to understand how the PSTs attended to students' thinking, made sense of it and reacted to it as Jacobs et al. (2010) proposed for noticing.

It is evident that attending any instances occurs in a classroom setting or making decision about what is noteworthy to attend is not easy, even controversial for teachers. Leatham and his colleagues (Leatham, Peterson, Stockero, & van Zoest, 2015) attempted to identify what is noteworthy to attend in order to understand students' mathematical thinking. They defined instances as *Mathematically Significant Pedagogical Opportunity to Build on Student Thinking* (MOST) such that the instance needs to be student mathematical thinking where the student's mathematics can be inferred from that particular instance and there is a mathematical point related to it. Furthermore, that particular mathematical thinking should be significant in terms of being central learning goal for the students and appropriate for their level. And finally, pedagogical opportunity occurs when the student's expressions lead an opening to build on his thinking as well as it is the right time to initiate that opening (see Leatham et al., 2015 for details). Leatham et al.'s (2015) description of MOST provides some criteria for deciding what is noteworthy to attend and how to interpret that instance as proposed in Jacobs' et al. (2010) first two steps of noticing. Therefore, in this study we attempted to address to the "mathematical opportunities" occurred in the classroom setting to be model for noticing of students' mathematical thinking and then we evaluated how the PSTs attended such opportunities while they were interacting with students.

Furthermore, because teachers' PCK involves in recognizing students' mathematical thinking and providing appropriate instruction and scaffolding to enhance students' understanding (Hill, Ball, & Schilling, 2008), teachers' noticing skills inform about their PCK and vice versa (Sherin, Russ, & Colestock, 2011; Star & Strickland, 2008). Thus, any instance about teachers' noticing could be counted for their knowledge and professional skills. Therefore, in this study we accepted that PSTs' noticing skills and their PCK nourished each other and we interpreted the findings accordingly.

## Method

In Turkey, mathematics teacher education programs mostly consist of theoretical courses with little space for PSTs to make practice in teaching. There are field experience and teaching practice courses in the last year of the 4-year program where the PSTs mostly do observations and just teach once or twice in a year. However, involvement in teaching-learning process actively and reflecting on their observations and practices contribute more to PSTs' PCK (Lannin et al., 2013). Therefore, we decided to set up a collaboration between a school and the faculty to provide first hand experiences to the PSTs to learn about how students learn mathematics as well as how to teach mathematics.

The data discussed in this paper comes from a university-school collaboration between the Mathematics Education Department of a large university in İstanbul in Turkey and a local middle school. The data was collected in 2012-2013 academic year from November to May, for 20 weeks. This study consisted of two phases: modelling and practice. In the first phase, we, as teacher educators, attempted to be model for PSTs by developing plan for the mathematical tasks for 6<sup>th</sup> grade students and implementing them in the school setting. Each implementation took approximately 45 minutes. We followed the order of the topics taught in the 6<sup>th</sup> grade math lessons while preparing our tasks because we wanted to put on what students already learned in their regular math lessons rather than to teach something new for them. We asked PSTs to share their observations about students' practices after each implementation and make verbal comments about how the following activity should be designed. In the second phase, we allowed PSTs to prepare and implement tasks in the classroom and work closely with students to understand their mathematical thinking. We wanted them to videotape their interactions with students. Immediately after each session, we asked them to discuss how it went and then write a reflection about the session by critically analyzing their own videos.

## Participants

A total of nine PSTs, one male and eight female, took part in the collaboration program. Because it was a voluntary program, just three of them attended both modeling and practice phases. Other PSTs did not attend to the program regularly because of the changes in their course schedule or their course load. A similar pattern was observed with the 6<sup>th</sup> grade students. They were voluntarily attending to the after-school program. During the second semester, some students who were the members of school's sports club or music club were not able to attend because of the schedule of their workouts.

The PSTs were senior students and they all had some tutoring experiences. During data collection process, all PSTs were taking the methods courses and the field experience courses. The methods courses were given by the authors. In those courses, the authors attempted to provide necessary background for the PSTs to catch and to reflect on the significant issues occurred during the implementations at collaboration school. In this paper, we will mainly discuss the data obtained from whom attended both phases (namely, PST1, PST2 and PST3).

## Setting

The tasks done during the first 12 weeks, so called *the modelling phase*, were planned and implemented by the first author. Initially, the PSTs were helping to set up the learning environment, observing the teaching and helping the students whenever needed. In addition to those, time to time they were asked to lead a part of the tasks such that the first author instructed them what to do before the implementations. Immediately after each implementations, the authors and the PSTs discussed their thoughts about how it went, how the students performed and what should be done for the following week. Those oral reflections and in-class implementations were videotaped.

The following eight weeks, so called *the practice phase*, were coordinated by the second author. Because of the decrease in the number of 6<sup>th</sup> grade students and the PSTs who attended the sessions, we decided to let the PSTs work with a pair of students rather than leading the whole class. We thought that such setting might be more beneficial for promoting their noticing skills because they had to attend just two students. As in the modelling phase, the second author managed the planning of the sessions and tried to include PSTs in the planning process. After four weeks, we asked PSTs to prepare two set of tasks one of which specifically for their students and the other one for whole class implementation. Before the implementations, they shared what they planned to do in the class with us and other PSTs.

### Data Collection

The data was collected through interviews, videos of implementation sessions and group reflections, PSTs' written reflections, authors' field notes and 6<sup>th</sup> grade students' worksheets. All PSTs were interviewed about their views of teaching and learning mathematics and their informal teaching experiences. Furthermore, during the pre-interview, we asked for their expectations from the collaboration program and in the post-interview we asked them to reflect on the program.

We conducted pre-interviews with the PSTs at the beginning of the modelling phase. During this phase, we videotaped in-class activities and group reflections after each in-class sessions. In the practice phase, in addition to videos of in-class activities and group reflections, we had PSTs' own recorded videos of their interactions with students and their written reflections. At the end of the practice phase, we conducted post-interviews with the PSTs. We used students' worksheets and field notes to validate PSTs' comments about the students' performances. During the modelling phase, we tried to focus on the instances that students had difficulty to understand or had misconceptions as the mathematical opportunities that should be attended (Jacobs, et al., 2010; Leatham et al., 2015). During the group reflections, we asked for PSTs' views about those opportunities as well as we explained our reasoning to attend them.

### Data Analysis

The videos of PSTs' interactions with students, group reflections and interviews were transcribed. We read the field notes, reviewed students' worksheets and analyzed the PSTs' written reflections. We coded interactions, group reflections and written reflections according to scheme developed in the line of Jacobs et al.'s (2010) definition for noticing of students' mathematical thinking and Leatham et al.'s (2015) description of MOSTs.

Based on Jacobs et al.'s (2010) definition, we focused on three phases of noticing: what to attend in particular moment, how to interpret students' understanding and how to respond to them. To specify what to attend, we used Leatham et al.'s (2015) description of MOSTs such that we specifically looked for the cases that students had difficulties to understand and it should be addressed at that moment to fill in the gaps in students' thinking. After watching the in-class and PSTs' own-recorded videos separately and looking at the students' worksheets, we identified a total of 27 (12 of them in the modeling phase, 15 of them in the practice phase) different mathematical opportunities for all nine PSTs as they were both interacting with students individually and also leading whole-class discussions for the tasks. For instance, one of the opportunities was about ordering unit fractions on a number line. The students placed  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{5}$  and  $\frac{1}{6}$  on the number line in this given order as if they are natural numbers like 2, 3, 4, 5, and 6 (we explained this opportunity more in the following section). When the students were asked about their reasoning for such order, they noted that on a number line the numbers get bigger as they go from left to right. Such an answer of students revealed that they did not yet conceptualize the fact that those fractions are the elements of another type of number set and they should think of it as a quantity and relevant to unit of "1" rather than just paying attention to only the denominator part. That is, this instance was an evidence of students' mathematical thinking about a significant mathematical point which should be addressed by the PST at that moment immediately. However, not all PSTs experienced all of those 27 different opportunities during the implementations because the students they worked with or they observed did not experience the same difficulties but the different ones.

After we agreed on the mathematical opportunities, we developed 4-level scales to describe the PSTs' noticing skills in terms of whether or not they attended to those opportunities and if so, how they attended to them. We decided to give more credit to PSTs when they attempted to elicit students' thinking through probing questions rather than just simply telling the solution or leading students to the correct answer through prompting questions because the former contributes to students' conceptual understanding more than the latter. Furthermore, we

thought that PSTs' justifications in terms of providing mathematical and pedagogical evidences for their comments about students' mathematical thinking informed about the level of their PCK. Therefore, we placed such evaluations of PSTs at the highest level of noticing in our coding scheme. We used interactions with students scale (SI Scale) to evaluate how the PSTs communicated with students. We watched the videos of group reflections and evaluate their reflections and comments, if available, according to group reflections scale (GR Scale). We used the same criteria in GR scale to evaluate the PSTs' written reflections (WR Scale). The levels of each scale are as shown in Table 1.

Table 1. Coding scheme to analyze data

Levels	Interaction with students (SI Scale)	Group reflections and Written reflections (GR / WR Scale)
Level 0	Missed the opportunity	Missed the opportunity
Level 1	Attended to the opportunity and the conversation with the students is mostly in question-answer manner	Mentioned and/or only described the mathematical opportunity
Level 2	Attended to the opportunity and attempted to elicit students' thinking through probing and prompting questions but it is mostly teacher dominated conversation with his/her mathematical goals in mind	Described the mathematical opportunity and the interactions, evaluated students' thinking, and their own actions
Level 3	Attended to the opportunity and used questioning to guide towards a legitimate mathematical idea. Students have some ownership of the ideas developed during conversation. PST addressed misconception (if there were any) by providing mathematically valid examples	Made legitimate claims based on evidence and provided reasoning related to students' possible reasoning underlying the mathematical opportunities

For the modelling phase, we only analyzed group reflections according to GR scale. For the practice phase, we analyzed 1) group reflections according to GR scale, 2) written reflections according to WR scale and 3) videos of both PSTs' one-to-one interactions with students and their whole class instructions according to SI scale. We separately coded each of three PST's interactions with students, their oral reflections and written reflections. For each of three PSTs we obtained 0.88 or more consistency for coding. We mostly differed in making decision between Level 1 and Level 2 in SI scale because in some cases the PSTs' conversations were mostly in question-answer manner but they also used some prompting questions. However, we decided to code the conversation in terms of what pattern was dominated mostly. Thus, we discussed the differences in coding and arrived at consensus finally.

## Findings

### The First Phase: Modelling to Notice

We identified a total of 12 different opportunities in the first phase. They were about recognizing similar polygons, constructing and measuring angles, constructing a square, identifying intersection of circles, operations with sets ("at most" and "at least" concepts), recognizing prime numbers, least common multiple (LCM) and greatest common factor (GCF) of given numbers, solving word problems and solving equations. For a few weeks, at the beginning of the group reflection the teacher educator (first author) mentioned about the opportunities occurred in the implementation and asked the PSTs whether they noticed those opportunities or not. The teacher educator also mentioned what she inferred from those opportunities and what actions she took in the class or plan to take for the next implementation. For instance in the first week, about recognizing similar polygons task she said the followings:

The aim of our task was to check whether students were able to distinguish between similar and congruent shapes and whether they know basic transformations like translation, rotation and symmetry. First, I asked them to define congruent and similar polygons and Emre [pseudonym, the 6<sup>th</sup> grade student] stated them correctly. Then I let them work on the questions. As I was walking around I noticed that majority of students failed to construct similar polygons even though they drew

congruent figures correctly. I asked Emre, whether two shapes were similar to each other [see Figure 1] and he said yes. Then I asked him to give me the definition of similar figures again. He said that each side should be a multiple of [corresponding] side of the other. I asked him whether his figure is appropriate, that is, he multiplied all sides by the same amount. He looked at the shapes and then corrected his answer by doubling each side of the original figure.'

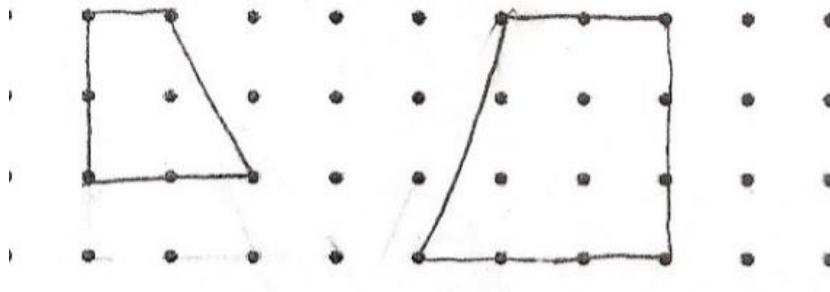


Figure 1. A Typical student answer for similar polygons

Then the teacher educator asked the PSTs whether they noticed that difficulty with other students and how they guided them when it was the case. The PSTs told that they corrected students' answers by telling the procedure of getting a similar figure for the given one. That is, the PSTs caught that opportunity however their way of attending that opportunity was different from the teacher educator's. Then, the teacher educator asked them what would be the reasoning behind this difficulty rather than offering her suggestions firstly. One of the PSTs said that they might think that similar figures mean to the figures that look like each other. The teacher educator agreed that it might be one of the reasoning and asked for others' opinions. Another PST told that they likely misunderstood "the amount of increase or decrease." She said that they might think that they would add two units to each side or decrease one unit from each side rather than thinking about multiplying each side with the same amount. The teacher educator also noted that it might be another reason. At the end of the group reflection, she shared her planning for the next session but she told that first she would look over the students' worksheets in detail and then finalize the task for the next implementation. She also asked for PSTs' opinions about what to do next. They agreed that there should be a review of similar polygons. The similar pattern was followed in the group reflection sessions for a few weeks. Then, the teacher educator initially let PSTs share their experiences and thoughts about students' mathematical thinking and then she shared her own observations.

We found that as the PSTs were interacting with the students they noticed all opportunities that occurred. However, in Table 2, we only presented the number of opportunities that PST1, PST2 and PST3 had experienced and the analysis of those opportunities in terms of levels. Because, not each opportunity was available in each PST-student interaction or whole class discussion, the number of opportunities attended by the PSTs was different. On the other hand, because some of the opportunities were common in the PST-student interactions, the total number of opportunities does not add up 12 (the total number of the opportunities) in the table.

Table 2. Analysis of PSTs' noticing of students' mathematical thinking in the modelling phase

PSTs	Number of opportunities	Group reflections (GR)
PST1	9	Level 2: 9
PST2	10	Level 1: 4 Level 2: 5 Level 3: 1
PST3	6	Level 1: 5 Level 3: 1
<b>Total</b>		<b>Level 1: 9</b> <b>Level 2: 14</b> <b>Level 3: 2</b>

As given in Table 2, PST2 experienced the most of the opportunities and she mostly attended them at Level 2. That is, during the group reflections, she described the opportunity and made comments about the student's thinking and her scaffolding practices. For instance, during the implementation of *operations with sets* task, she realized that the students could not interpret given Venn diagram to figure out number of students who were taking "at most" one course. She stated that the students did not recognize that "at most" meant to upper boundary while "at least" meant to lower boundary. She told that she gave an example from the concrete objects on their desks to help their understanding of what "at most one" or "at least two" meant as well as she asked them to tell about what the given regions of the Venn diagram represented for. On the other hand, PST3 experienced the least number of opportunities and she only mentioned about those opportunities without giving further details about the opportunity and how she attended to it. Overall, the PSTs recognized all 12 different opportunities occurred in the modelling phase and during group reflections the PSTs usually mentioned about the opportunities, evaluated students' thinking and their own practices.

### The Second Phase: Practicing to Notice

In the second phase of the study, the PSTs were matched with a pair of students and they worked with them for eight weeks. The second author designed the tasks for four weeks and all PSTs applied those tasks in their groups. In the fifth week, each PST prepared their own tasks for their own groups. For the last three weeks the PSTs prepared set of tasks for whole class and led the implementation session. We analyzed PSTs' interactions with students (one-to-one and whole class), their group reflections and written reflections. We identified a total of 15 different opportunities occurred in this phase. Those opportunities were about representing a fraction on a given shape, ordering fractions on a number line, finding a fraction between two fractions, division and multiplication of fractions and solving fraction problems. In Table 3, the number of opportunities that each PST had experienced and their level of attendance of those opportunities are given. As similar to Table 2, the total number of opportunities does not add up to 15 because some opportunities were common for the PSTs.

Table 3. Analysis of PSTs' noticing of students' mathematical thinking in the practice phase

PSTs	Number of opportunities	Student interactions (SI)	Group reflections (GR)	Written reflections (WR)
PST1	10	Level 0: 1 Level 1: 2 Level 2: 6 Level 3: 1	Level 0: 1 Level 1: 3 Level 2: 6	Level 1: 2 Level 2: 6 Level 3: 2
PST2	9	Level 0: 1 Level 2: 8	Level 0: 1 Level 1: 2 Level 2: 6	Level 2: 8 Level 3: 1
PST3	7	Level 0: 1 Level 1: 5 Level 2: 1	Level 0: 1 Level 1: 5 Level 2: 1	Level 1: 5 Level 2: 2
<b>Total</b>		<b>Level 0: 3</b> <b>Level 1: 7</b> <b>Level 2: 15</b> <b>Level 3: 1</b>	<b>Level 0: 3</b> <b>Level 1: 10</b> <b>Level 2: 13</b>	<b>Level 1: 7</b> <b>Level 2: 16</b> <b>Level 3: 3</b>

As shown in Table 3, each of PSTs missed an opportunity during their interactions with students, however, they noticed those opportunities while watching their videos because they wrote about those opportunities in their reflection reports. The PSTs' interactions with students were usually at Level 2 as shown in the table (Level 2=15, out of 26 interactions). That is, the PSTs attended to the opportunities and tried to understand students' thinking by asking for clarification or explanation of their reasoning. However, they were mostly teacher-dominated conversations such that the PSTs attempted to guide students towards the mathematical goals in their minds. For instance, one of PST2's student put the given fractions in order as follows:  $1/8 < 9/40 < 3/7 < 11/12 = 4/5$ . She asked him to explain his reasoning:

Student:  $1/8$  is closer to 0,  $9/40$  is closer to a quarter,  $3/7$  is closer to half, and  $11/12$  and  $4/5$  are closer to a whole.

PST2: Do you mean that if two fractions are close to a whole or a quarter then they are equal to each other?

- Student: I thought that both  $11/12$  and  $4/5$  has one piece to be a whole so they are equal to each other.  
 PST 2: What are those pieces?  
 Student:  $1/12$  and  $1/5$ .  
 PST 2: Are they equal to each other?  
 Student: No.  $11/12$  and  $4/5$  equal to each other.  
 PST 2: Now, construct two identical lines or boxes and show me  $11/12$  and  $4/5$  separately on them.

Then the student drew two lines and plotted the fractions. He realized that  $11/12$  was closer to 1 than  $4/5$ . PST2 convinced that the student understood his mistake and she did not discuss more why the student's reasoning was wrong. Therefore, we coded her interaction with the student as Level 2. During the group reflections the PSTs mostly mentioned the opportunities but they sometimes talked about what might be reasoning behind students' difficulties or confusions. On the other hand, in their written reflections the PSTs shared their thoughts about students' performance more freely, sometimes with evidence from the students' worksheets or their conversations with students.

The performances of PST1, PST2 and PST3 during the second phase were similar to the first phase. PST1's and PST2's interactions with students, their GRs and WRs were mostly at Level 2 while PST3's were at Level 1. We found that writing reflections helped PST1 and PST2 to elaborate on students' thinking and their own interactions. For instance, the students were asked to place unit fractions  $1/2$ ,  $1/3$ ,  $1/4$ ,  $1/5$ ,  $1/6$  on a number line and PST1's group misplaced them such that they placed them equidistantly as given order. During the interactions, PST1 noticed this misconception but was not able to finalize the discussion with students because time was up (Level 1). In the GR, she only mentioned the opportunity (Level 1) but in her WR she discussed possible reasoning behind this difficulty by providing examples from students' work and group discussions. She wrote the followings in her WR:

The students knew that as going from left to right on a number line the numbers are getting bigger. However, they were not aware of that we are ordering numbers with respect to their distance from 0. When I asked them "which one is larger  $1/2$  or  $1/3$ ?" they were able to say that  $1/2$  is larger. But when we asked them fold a paper strip and label the fractions as  $1/2$  and  $1/4$ , they labeled each piece. That is, they did not think of the fractions as representing the "distance from the initial point". Perhaps, folding paper was confusing for them. I guess teachers are using folding paper method to represent fractions rather than a number line. Because students were able to compare given fractions, I think they could put the fractions on a number line correctly if we tell them to pay attention to "0", the initial point.

We coded her WR as Level 3 because she recognized that students were not able to take "0" as benchmark to order the fractions. Although placing fractions on a number line entails more abstract thinking of fractions as "rational numbers" rather than "representing equal parts of a concrete object", in Turkish mathematics curriculum, the students are expected to find equivalent fractions of given fractions and then place them on a number line by dividing the line into equal pieces. Therefore, we did not expect the PSTs to initiate discussion about the difference between perceiving a fraction as an operator or as a number. That is, we did not evaluate PSTs' reflections in terms of whether they discussed how placing fractions on a number line differs from more concrete problems of fractions but we only focused on their thoughts about possible reasoning behind students' difficulties in placing fractions on a number line.

Similarly, we coded one of PST2's opportunities as Level 2 in SI and GR but as Level 3 in WR. In that opportunity, the students were given a problem such that three people were sharing an unknown amount; the first person is getting one third of it, the second person is getting one third of the rest and the last person is getting what is left. The students initially thought that the first person would get the most and the last one would get the least. PST2 asked them to use manipulatives or draw sketches to check their answer. The students used fraction tiles to model the problem. Because there is no one-ninth pieces in the fraction strips set they hardly found out one third of  $2/3$ . The conversation between PST2 and students were mostly at Level 2 because she aimed to lead students to think of how to enlarge  $2/3$  by 3 in order to share the pieces equally. In her WR she wrote the following:

They understood that they would find  $1/3$  of  $2/3$  but their choice of manipulative to model this problem was inappropriate. They had two pieces of one thirds and they knew that they had to divide it into three. They also knew that they would enlarge  $2/3$ . They replaced one-sixths and one-twelfths with two thirds but they were not helpful. They needed one-ninth pieces but in the fraction tiles set

there was any. Therefore, they spent too much time to figure out the solution. Although there were unit cubes available on the table they couldn't think of how to use it. If they had used them, they would have solved it earlier. However, they eventually drew a sketch of it and found out the answer.

PST3 had the least number of opportunities occurred during the interactions with students. Although she missed an opportunity during the interactions, she realized it while writing her reflection paper. As different from PST1 and PST2, in her reflection reports she preferred to describe the opportunity without attempting to evaluate students' thinking or her own scaffolding. For instance, her students also misplaced the given fractions on a number line as in PST1's case. PST3 only wrote that the students did not know about how to put fractions in order. She neither discuss possible reasoning behind that answer nor provide suggestions to eliminate such misunderstanding.

### **Activities Led by Participants PST1, PST2 and PST3**

In the second phase, PST1, PST2 and PST3 prepared an in-class activity and implemented in the class. PST1 prepared an activity about division with fractions. She used a commercial software to simulate division operation and gave some exercises. PST2 prepared an activity about fraction problems. PST3 prepared a worksheet consist of ordering fractions, placing fractions on number line and division with fractions.

The software PST1 used was operating fraction division in a different way. She tried to explain the method and showed a couple of examples. Then she asked students to solve the division problems by using that method. However, the students failed to apply the procedure correctly. Initially, PST1 did not conceptualize what the students got confused about but then she made some clarifications to facilitate students' understanding. We coded her interaction with students as Level 1. In GR and WR she noted that she did not expect that students would get confused. She criticized herself that she should explain the division method more clearly and provided suggestions about what she should do. Therefore, we coded her GR and WR as Level 2 and Level 3, respectively.

PST2 asked problems about fraction division and multiplication. One of the students volunteered to share his answer, however his solution was wrong. PST2 attended that opportunity by asking the meaning of division operation but she did not attempt to understand the student's reasoning behind that incorrect solution. Therefore, we coded that interaction as Level 1. In her GR and WR she said that it would be better to start with simpler problems and she gave a few examples. We coded each of GR and WR as Level 2.

PST3 focused on comparing fractions and ordering them on a number line. The students were able to compare two unit fractions with each other but some of them had a misconception that  $\frac{3}{4}$  is equivalent to  $\frac{11}{12}$  because the difference between the numerator and the denominator is the same, that is,  $4-3=1$  and  $12-11=1$ . She attempted to eliminate such misunderstanding by using fraction strips during the instruction. We coded her interaction as Level 1. In her GR and WR she mentioned about that opportunity and possible reasoning behind them, so we coded them as Level 2.

Because the PSTs did only one whole class instruction during the second phase, we could not find a major difference in their noticing practices in terms of small group interaction versus whole class instruction. Similarly, there was no big changes in their GR. However, they wrote more about the opportunities, evaluate students' and their own practices in WR.

### **Discussion and Conclusion**

To improve students' mathematical thinking is one of the hot topics of mathematics education research (Sherin et al., 2011). Teachers have a crucial role in planning and providing opportunities for students' thinking as well as noticing significant incidents that reveal such thinking. However, recent studies show that teachers need support to learn how to attend students' thinking (Sherin et al., 2011). Therefore, we set up a university-school collaboration program to enable the PSTs observe and engage in task implementation and evaluation processes. Thus, we attempted to investigate how teacher educators' modelling and PSTs' own practices under this program contributed to PSTs' professional skills, specifically their noticing of students' mathematical thinking.

As we inferred from our field notes and the videos of both in-class discussions and group reflections, we found out that almost all PSTs benefitted from the program such that they were able to recognize students' difficulties

and misconceptions and they attempted to address them as much as they could. Because attendance to this collaboration program was voluntarily, three PSTs attended all sessions regularly. Those three PSTs were able to catch all mathematical opportunities occurred during the modelling phase as shown in Table 1. It seemed that teacher educators' modelling of attending to mathematical opportunities during the implementations raised the PSTs' attention for students' mathematical thinking (Anthony et al., 2015; Kazemi & Waage, 2015). Furthermore, PST1, PST2 and PST3 paid more attention to not give away answers immediately but guide students through questioning or manipulatives as we asked them to do so although many other PSTs had a tendency to tell the procedures or solutions as a mean to address students' difficulties (Tunc Pekkan & Kilic, 2015).

On the other hand, during the practice phase, the PSTs missed some opportunities while they were working with the students in one-to-one fashion or leading an in-class activity. Because they were novice and they do not have strong PCK, it was not surprising that they might miss some opportunities in action (Sherin, Russ, & Colestock, 2011; Star, Lynch, & Perova, 2011). Furthermore, during the practice phase, they felt more stress on their shoulders because they had more responsibility in monitoring students' work and scaffolding their understanding. However, during the modelling phase, they knew that the teacher educators had the responsibility of managing the class and making decisions if anything went wrong, therefore, they comfortably communicated with students.

Although the PSTs missed some opportunities in-action, they were able to catch them while watching their videos. The analysis of PSTs' written reflections revealed that PSTs mentioned about the opportunities that they missed during the implementations or they discussed students' understanding as well as their actions by providing justifications in their papers. Out of 26 discussion-of-opportunity in the written reflections, we coded 16 of them at Level 2 according to our WR scale. This finding is compatible with other studies' findings (e.g., Sherin & van Es, 2005; Stockero, 2008) such that watching own-videos and writing reflection about own practice contributes to PSTs' noticing skills because PSTs had time to re-watch and critically think about the videos. However, the PSTs might get better in noticing students' thinking in action even if they were given opportunity to work with students for a long-term or led a classroom a couple times (Star et al., 2011).

The PSTs mostly used prompting and probing questions to understand students' thinking as shown in Table 3. That is, out of 26 PST-student interactions on the mathematical opportunities, 15 of them were at Level 2. It revealed that although the PSTs were able to notice the mathematical opportunities, their PCK was not strong enough to attend those opportunities in a way to elicit students' mathematical thinking and scaffold their understanding (Sherin, Russ, & Colestock, 2011; Star & Strickland, 2008). However, we realized that during whole class instruction the PSTs recognized the mathematical opportunities but they did not attempt to ask for eliciting students' thinking rather they tried to make students recognize their mistakes. Because it was the only chance given to them to lead a whole class during this collaboration program, they might focus on covering their lesson plan during the class period in sake of taking the advantage of attending those opportunities to promote students' understanding.

Besides our observations and analysis, the post-interviews revealed that PSTs perceived an improvement in their professional skills and practices, namely, in their PCK. They noted that their repertoire of students' misconceptions and difficulties, their knowledge of effective use of manipulatives and their ability to use questioning to elicit students' thinking had improved. The PSTs' such evaluations about themselves were compatible with the results of other studies where the PSTs were given opportunity to practice in teaching or interact with students closely (e.g., Anthony et al., 2015). Although the PSTs had discussed much of those issues in their courses (teaching methods, material development, etc.) they had opportunity to make practice with them as a part of university-school collaboration program. They appreciated the value of working closely with students because during the field experiences they did not have such opportunity to learn about students' mathematical thinking. Star et al. (2011) also pointed to such weakness of the PSTs' field experiences because the mentor teachers may not exemplify how to elicit students' mathematical thinking or how to scaffold learning.

In conclusion, modelling to notice students' mathematical thinking and then providing opportunities for practice contributed the PSTs' noticing skills to some extent. The PSTs were able to recognize the gaps in students' understanding even though they needed to get better in how to respond students after noticing the instances. Although the PSTs attempted to use questioning effectively such that they helped the students to figure out their own mistakes rather than directly telling them, they should be given more opportunities to make practice on the second and the third phases of Jacobs et al.'s (2010) noticing. That is, they should make more practice on how to elicit students' mathematical thinking and scaffold their understanding. Therefore, teacher educators should

provide appropriate learning environments for PSTs to develop such professional skills. Furthermore, teacher education programs should be revised to increase the effectiveness of field experiences and teaching practices such that PSTs should take more active roles in the schools.

## References

- Aleccia, V. (2011). Walking our talk: The imperative of teacher educator modeling. *The Clearing House*, 84, 87–90.
- Anthony, G., Hunter, J., & Hunter, R. (2015). Supporting prospective teachers to notice students' mathematical thinking through rehearsal activities. *Mathematics Teacher Education and Development*, 17(2), 7–24.
- Ball, D.L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics*, 8(1), 40–48.
- Bandura, A. (1977). *Social learning theory*. New Jersey: Prentice-Hall.
- Bates, A. J., Ramirez, L., & Dritis, D. (2009). Connecting university supervision and critical reflection: mentoring and modeling. *The Teacher Educator*, 44, 90–112.
- Franke, M., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225–256). Charlotte, NC: IAP.
- Friel, S. N. & Carboni, L. W. (2000). Using video-based pedagogy in an elementary mathematics methods course. *School Science & Mathematics*, 100(3), 118–127.
- Grossman, P., Hammerness, K., & McDonald, M. (2009). Redefining teaching, re-imagining teacher education. *Teachers and Teaching: Theory and Practice*, 15(2), 273–289.
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge. *Journal for Research in Mathematics Education*, 39, 372–400.
- Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41(2), 169–202.
- Kazemi, E., & Waege, K. (2015). Learning to teach within practice-based methods courses. *Mathematics Teacher Education and Development*, 17(2), 125–145.
- Lannin, J. K., Webb, M., Chval, K., Arbaugh, F., Hicks, S., Taylor, C., & Bruton, R. (2013). The development of beginning mathematics teacher pedagogical content knowledge. *Journal of Mathematics Teacher Education*, 16, 403–426.
- Leatham, K. R., Peterson, B. E., Stockero, S. L., & van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education*, 46(1), 88–124.
- Loughran J., & Berry, A. (2005). Modeling by teacher educators. *Teaching and Teacher Education*, 21(2), 193–203.
- McGinnis, J. R., Watanabe, T., & McDuffie, A. R. (2005). University mathematics and science faculty modeling their understanding of reform based instruction in a teacher preparation program: Voices of faculty and teacher candidates. *International Journal of Science and Mathematics Education*, 3, 407–428.
- OECD (2014). *Education at a glance 2014: OECD indicators*. OECD Publishing. <http://dx.doi.org/10.1787/eag-2014-en>
- Parker M., & Hurry, J. (2007). Teachers' use of questioning and modelling comprehension skills in primary classrooms. *Educational Review*, 59 (3), 299–314.
- Roller, S. A. (2015). What they notice in video: A study of prospective secondary mathematics teachers learning to teach. *Journal of Mathematics Teacher Education*. DOI 10.1007/s10857-015-9307-x.
- Santagata, R. (2009). Designing video-based professional development for mathematics teachers in low-performing schools. *Journal of Teacher Education*, 60(1), 38–51.
- Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (2011). Situating the study of teacher noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 3–13). New York: Routledge.
- Sherin, M. G., Russ, R. S., & Colestock, A. A. (2011). Assessing mathematics teachers' in-the-moment assessing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 80–94). New York: Routledge.
- Sherin, M. G., & van Es, E. A. (2005). Using video to support teachers' ability to interpret classroom interactions. *Journal of Technology and Teacher Education*, 13, 475–491.
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57, 1–22.

- Sowder, J. T. (2007). The mathematical education and development of teachers. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157–223). Charlotte, NC: IAP.
- Star, J. R., Lynch, K., & Perova, N. (2011). Using video to improve preservice mathematics teachers' abilities to attend to classroom features. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 117–133). New York: Routledge.
- Star, J., & Strickland, S. (2008). Learning to observe: using video to improve preservice mathematics teachers' ability to notice. *Journal of Mathematics Teacher Education*, 11(2), 107-125.
- Stockero, S. L. (2008). Using a video-based curriculum to develop a reflective stance in prospective mathematics teachers. *Journal of Mathematics Teacher Education*, 11, 373–394.
- Tunc-Pekkan, Z. & Kilic, H. (2015). *Mathematical opportunities: Noticing and acting*. In Zehetmeier, S. Bosse, M., Brown, L., Hospesova, A. Malara, N. and Rösken-Winter, B. (Eds.). Proceedings of the 9<sup>th</sup> congress of the European Society for Research in Mathematics Education (pp. 368-374). Prague, Czech Republic: CERME.
- Van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education*, 10(4), 571–596.
- van Es, E. A. & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. *Teaching and Teacher Education*, 24, 244-276.
- White, E. (2011). Working towards explicit modelling: Experiences of a new teacher educator. *Professional Development in Education*, 37(4), 483–497.

---

### Author Information

---

**Hulya Kilic**

Yeditepe University  
 Inonu M. Kayisdagi C. GSF 5. kat Egitim Fak.  
 Atasehir 34755 Istanbul, Turkey  
 Contact e-mail: [hulya.kilic@yeditepe.edu.tr](mailto:hulya.kilic@yeditepe.edu.tr)

**Zelha Tunc Pekkan**

MEF University  
 Ayazaga C. No: 4 Maslak  
 Sariyer 34396 Istanbul, Turkey

---