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Abstract

The aim of this research was to examine mathematics teachers' performances in defining special types of quadrilaterals, identifying their family and hierarchically classifying them. In this vein, 33 of 58 primary school mathematics teachers working in the province of Yozgat, Turkey were voluntarily recruited for this survey, and they were asked to complete a data collection form consisting of three open-ended questions. In the first question, participants were asked to define the special types of quadrilaterals. In the second question, teachers were asked to recognise kites, trapezoids, parallelograms, rectangles and rhombuses among 15 quadrilaterals. Finally, participants were prompted to make a hierarchical classification of the special types of quadrilaterals. The results of this study illustrated that while 20 mathematics teachers could define a kite hierarchically, only 1 of the sample could hierarchically define a trapezoid. In addition, participants encountered difficulties not only in determining the kite and trapezoid family, but also in demonstrating the relations of kite-rhombus, trapezoid-parallelogram and trapezoid-rectangle quadrilaterals. Hence, in-service training programs for primary school mathematics teachers should be organized to improve content knowledge of geometry in general and quadrilaterals in particular.

Introduction

The National Council of Teachers of Mathematics (2000) reported that teachers should have deep knowledge and understanding of the subjects which they teach. Shulman (1986) claimed that this type of knowledge referred to content knowledge and argued that teachers' pedagogical knowledge is also crucial for effective teaching. Pedagogic knowledge spans the principles and strategies of classroom management and organization that are cross-curricular. Effective teachers possess subject-specific pedagogic knowledge, know their students, and use appropriate teaching materials and processes of teaching (Marks, 1990). On the other hand, Ball et al. (2005) asserted that content knowledge and pedagogic knowledge are distinctive, yet content knowledge is related to mathematical knowledge for teaching. Additionally, numerous studies have shown that students' mathematics competencies are closely associated with their teachers' knowledge of mathematics (Ball et al., 2005; Darling-Hammond, 2000; Goldhaber & Brewer, 2000; Hill et al., 2008; Morris, Hiebert, & Spitzer, 2009).

In Turkey, the elementary school mathematics curriculum includes geometry but students' performance is seen as middling. In the Trends in International Mathematics and Science Study (TIMSS) 2015 report, Mullis et al. (2016) ranked the geometry achievement of students in 39 countries. Turkey was placed 22nd in this index, with both male and female students performing slightly below world average. This finding suggests that local studies are necessary to monitor mathematics teachers' knowledge of Geometry and to identify where gaps can be addressed.

The topic of quadrilaterals is fundamental to the study of geometry and is therefore included in grades 6, 7 and 8 of the mathematics curriculum in Turkey (T.C. Millî Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı, 2013). The topic is mostly taught in Turkey by providing information to students about the definitions and properties of the special types of quadrilaterals as well as relations between these quadrilaterals (Akkaş & Türnüklü, 2015). Usiskin et al. (2008) indicated that the special types of quadrilaterals can be defined exclusively and inclusively. While an exclusive definition is true for a specific quadrilateral, an inclusive definition would be valid for a family of quadrilaterals. For instance, the exclusive definition of a trapezoid is a quadrilateral with only one pair of parallel sides whilst the inclusive definition of a trapezoid is a quadrilateral with at least one pair of parallel sides. The exclusive and inclusive definitions of a trapezoid also result in two different hierarchical classifications of quadrilaterals as seen in Figure 1.a and Figure 1.b, respectively (Popovic, 2012). Josefsson (2013) expressed that both definitions have limitations and benefits. The exclusive definition is useful when students are first shown the special types of quadrilaterals including a trapezoid or rhombus and taught their basic properties. On the other hand, the inclusive definition is beneficial when students proceed to higher levels of study. For example, when the inclusive definition of a trapezoid is learned, students realize that the properties of a trapezoid are also true for a parallelogram, rectangle, rhombus, and square. Thus, knowledge of the inclusive definition can be an advantage over the exclusive definition.

Given the importance of the inclusive definition of quadrilaterals, the following section highlighted studies on exploring pre-service teachers' and mathematics teachers' knowledge of quadrilaterals.

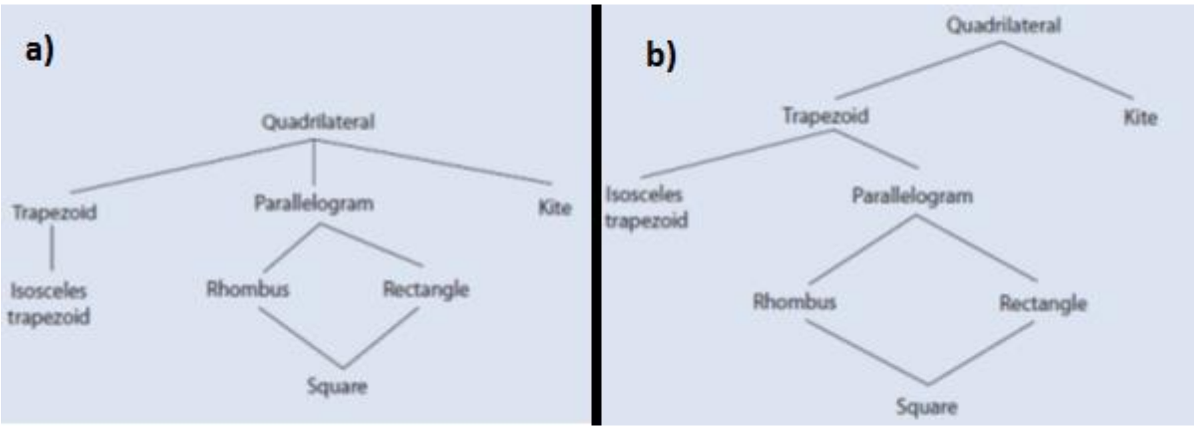

Figure 1. Hierarchical classifications of quadrilaterals based on the exclusive definition (a) and inclusive definition (b) of quadrilaterals (Popovic, 2012)

Literature Review of Studies on Quadrilaterals

Fujita and Jones (2007) performed two studies to investigate not only how pre-service teachers define a trapezium, parallelogram, rectangle and square, but also how well they demonstrate relations among them. 158 pre-service teachers participated in the first study and were asked to determine whether a square is a trapezium, a square is a rectangle, and a parallelogram is a trapezium as well as define the so-called types of quadrilaterals. It was found that 14 pre-service teachers (8.9%) indicated that a square is a trapezium; 29 (18.4%) that a parallelogram is a trapezium; and 20 (12.7%) that a square is a rectangle. Additionally, 19 (12%), 93 (58.9%), 34 (21.5%) and 60 (38%) pre-service teachers correctly defined a trapezium, parallelogram, rectangle and square, respectively. In the second study, 105 pre-service teachers were asked to define a parallelogram and recognise parallelograms among a collection of 15 quadrilaterals. While 86 pre-service teachers (82%) almost accurately defined a parallelogram, only 21 (20%) could correctly identify parallelograms among 15 quadrilaterals.

Çontay and Paksu (2012) examined pre-service mathematics teachers' understandings of the relations between a kite and square. They asked five pre-service mathematics teachers (PT1, PT2, PT3, PT4 and PT5) to classify a set of quadrilaterals (two non-specific quadrilaterals, a kite, trapezoid, parallelogram, rectangle and square) as a kite or none-kite. Afterwards, individual interviews were conducted to explore teachers' reasoning for their classifications. Teachers PT1, PT2, PT3 and PT5 accurately categorized the given quadrilaterals, while PT4 indicated that a square is not a kite, yet a trapezoid and one of the non-specific quadrilaterals are a kite. During the individual interview stage, PT1 elected to place a square in the non-kite category as they were unable to explain why it was necessary to include a square in the kite category. Consequently, only three of the five pre-service mathematics teachers could demonstrate an understanding of the relations between kite and square.

Türnüklü, Akkaş, et al. (2013) determined how 9 primary school mathematics teachers hierarchically organize the special types of quadrilaterals: trapezium, parallelogram, rectangle, rhombus and square. They found that 3 teachers (T1, T2 and T5) could not hierarchically classify the given quadrilaterals, 2 teachers (T4 and T7) chose to tabulate these quadrilaterals based on their different and common properties, and 2 teachers (T3 and T8) hierarchically organized these quadrilaterals according to their length of sides. Therefore, they revealed that a square is the special case of a rhombus, and a rectangle is the special case of a parallelogram. On the other hand, they did not express that a square is the special case of a rectangle, and a rhombus is the special case of a parallelogram. Teacher (T9) hierarchically classified the given quadrilaterals based on their angles and reported that a square is the special case of a rectangle, and a rhombus is the special case of a parallelogram. However, T9 did not mention that a square is the special case of a rhombus, and a rectangle is the special case of a parallelogram. Teacher (T6) stated that a parallelogram, rectangle, rhombus and square are the special cases of a trapezoid, and a rectangle, rhombus and square are the special cases of a parallelogram. However, T6 did not indicate that a square is the special case of a rhombus.

Erdogan and Dur (2014) investigated 57 pre-service high school mathematics teachers' ability to define a trapezoid, parallelogram, rectangle, rhombus and square, identify the parallelogram, rhombus, rectangle and square families among a set of 15 quadrilaterals, and to make a hierarchical classification of a kite, trapezoid, parallelogram, rectangle, rhombus and square. They reported that 26 pre-service high school mathematics teachers (46%) correctly stated the definition of a trapezoid. Most of the sample 55 (96%) provided the correct definition of a parallelogram but only half (51%) detected the parallelogram family. Similarly, 55 (96%) accurately defined a rectangle while only 26 (46%) recognized the rectangle family. Only 19 of the pre-service high school mathematics teachers (33%) could correctly indicate the definition of a rhombus. Only 28 (49%) determined the rhombus family. Most (95%) could provide the correct definition of a square and 48 (84%) detected the square family. However in terms of the hierarchical classification of quadrilaterals, only 23 (40%) of the sample correctly demonstrated the hierarchical relations among a parallelogram, rectangle, rhombus and square, and only 4 of those accurately included a trapezoid in their diagrams.

Pickreign (2007) asked 40 pre-service teachers to define a rectangle and rhombus. It was found that while 39 pre-service teachers (98%) try defining a rectangle, only 9 of those correctly provide the hierarchical definition of a rectangle which includes a square, yet excludes a parallelogram. While 29 pre-service teachers (73%) attempted to define a rhombus only one of them accurately gave the hierarchical definition of a rhombus which contains a square, but excludes a parallelogram.

Shir ve Zaslavsky (2001) requested 20 elementary school mathematics teachers to detect the accurateness of eight different definitions of a square. Teachers first reviewed the given definitions and chose the correct definition from among those provided. Afterwards, groups of 3-5 mathematics teachers compared and discussed their responses. They found that only five teachers could determine that all of the definitions they were presented with were correct.

The Aim of This Study

The previous literature review section illustrated that:

- Individuals' knowledge of quadrilaterals can be detected by asking them to define quadrilaterals, determine their family and make a hierarchical classification of them.
- Turkish pre-service mathematics teachers encounter problems in defining quadrilaterals, determining their family and hierarchically organizing them.
- A limited number of studies have been conducted to determine Turkish mathematics teachers' content knowledge of quadrilaterals.

In this regard, this study aimed to further investigate and determine mathematics teachers' content knowledge of quadrilaterals.

Methodology

In this study, mathematics teachers' ability to define quadrilaterals, recognizing their family and hierarchically classifying them were examined via a survey method which allows researchers to explore the phenomena which have already taken place (Çepni, 2014; DePoy & Gitlin, 2011). After creating a data collection tool based on previous studies (Erdogan & Dur, 2014; Fujita & Jones, 2007), all required permissions to perform this research were obtained from Yozgat governorship, Turkey. Thereafter, all elementary school principals in Yozgat were informed about this study and meetings arranged with School principals and 58 elementary school mathematics teachers to introduce the research.

Participants

To recruit participants, convenience sampling method was used in this study. In this method, researchers determine a set of inclusion and exclusion criteria. Then, those meeting these criteria are asked to voluntarily participate in the research (DePoy & Gitlin, 2011). In this vein, this research was presented to 58 elementary school mathematics teachers in Yozgat, and 33 of those consented to enrol in this study.

As illustrated in Figure 2.a, there were 19 female and 14 male participants and their mathematics teaching experience varied from 1 to 17 years with a mean of 10 years. As seen in Figure 2.b, 7 teachers had worked for 5 or fewer years, 9 had taught mathematics for 6 to 10 years, 12 for 11 to 15 years, and 5 possessed 16 or more year experience.

Generalizations from convenience sampling method is arguable due to the representativeness issue (Beins & McCarthy, 2011; L. Cohen, Manion, & Morrison, 2013). On the other hand, this study is considered representative of elementary school mathematics teachers in Yozgat province where this research was conducted, as 58% of local teachers elected to take part and participants' maths teaching experience ranged between one and seventeen years (Can, 2014; L. Cohen et al., 2013).

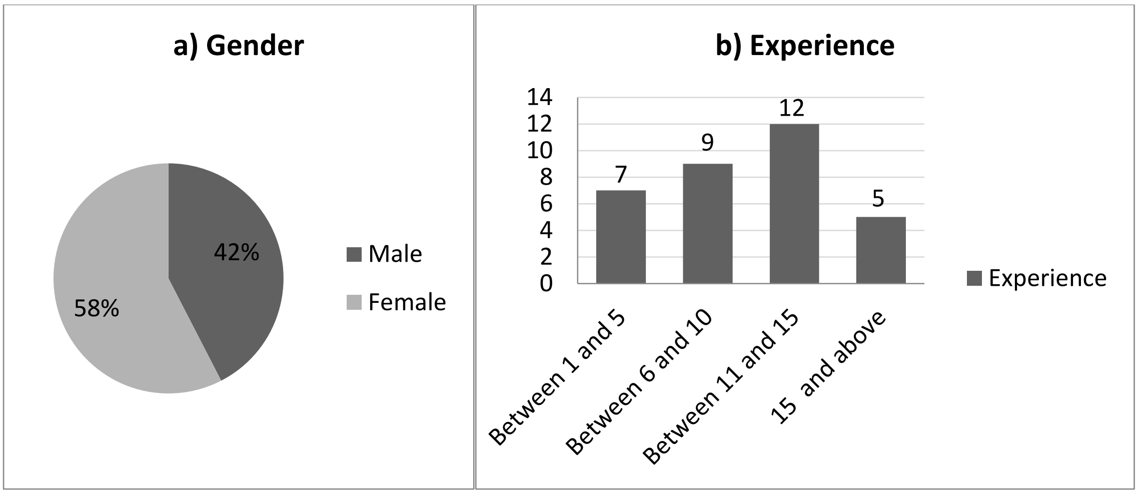

Figure 1. Demographics of participants by gender (a) and experience (b)

Data Collection Tool and Data Analysis

The question form of this research consisted of three open-ended questions which were adapted from previous studies (Erdogan & Dur, 2014; Fujita & Jones, 2007). In the first question, mathematics teachers were given a set of special types of quadrilaterals (kite, trapezoid, parallelogram, rectangle, rhombus and square), and they were asked to define these quadrilaterals. Teachers' answers to the first question were classified as *erroneous*, *prototype* or *hierarchical* definitions. These categories and their descriptions were inspired by Cansız-Aktaş (2016), Fujita (2012), Fujita and Jones (2007) and Pickreign (2007).

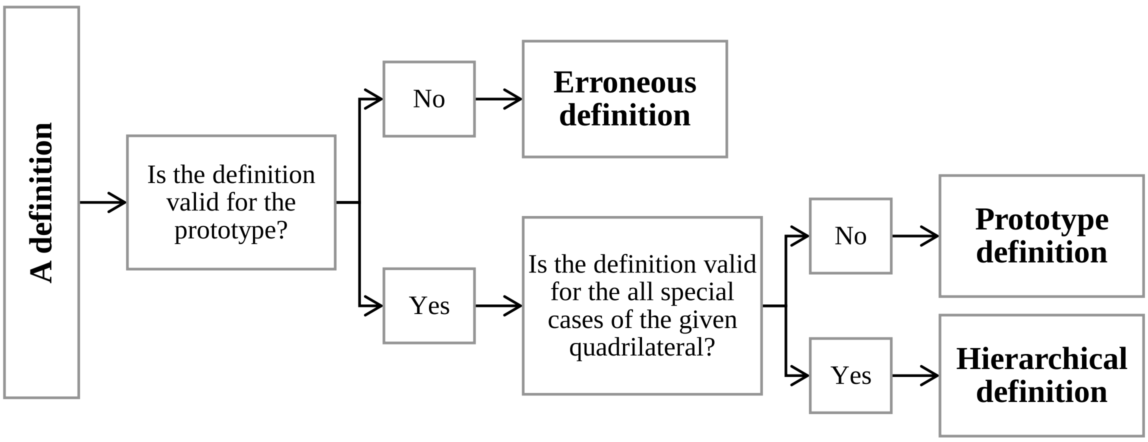

Figure 2. A flowchart for analysing mathematics teachers' definitions of quadrilaterals

As demonstrated in Figure 3, any definition is first evaluated as to whether it is true for the prototype of the given quadrilateral. If it is not, this definition is categorized as an *erroneous* definition. Otherwise, the definition might be either a *prototype* or *hierarchical* definition. Then, the validity of the definition is assessed for the special cases of the given quadrilateral (see Table 1). Here an invalid definition is categorized as a *prototype* definition, and a valid definition is called a *hierarchical* definition. To illustrate, "a kite has two pairs of adjacent equal sides and the sum of the size of opposite angles is 180" is an example of an *erroneous* definition, for the sum of the size of opposite angles in kites is not always 180. Another definition might be "a kite is made up of two different isosceles triangles joined base to base", and this definition is categorized as a *prototype* definition, since this definition is not true for a rhombus and square which are formed by joining two identical isosceles triangles base to base. However, a *hierarchical* definition of a kite can be "a kite contains two isosceles triangles that share a common base", because this definition is true not only for a kite but for also a rhombus and square.

Table 1. Special types of quadrilaterals with their special cases (adapted from Usiskin et al., 2008)

Special types of quadrilaterals	Their special cases
Kite	Rhombus, square
Trapezoid	Parallelogram, rectangle, rhombus, square
Parallelogram	Rectangle, rhombus, square
Rectangle	Square
Rhombus	Square
Square	Not Available

In the second question, mathematics teachers were shown Figure 4 and requested to identify all kites, trapezoids, parallelograms, rectangles and rhombuses. For analysis, one point and two points were first given for each prototype and special case, respectively as shown in Table 2. Then, every teacher's total point score for each quadrilateral was computed. Next, the ratio of their total points score for each quadrilateral to the total possible points they could score plus the number of their erroneous answers was calculated. For instance, "2, 4, 7, 11, 13, 15" might be a teacher's answer to recognizing rectangles among quadrilaterals in the second question. For this answer, the teacher's total point score is computed as 5, with 3 of the points awarded for identifying the prototypes of a rectangle ("2, 7, 13"), and a further 2 points for noting the special case of a rectangle (11). As 15 is neither the prototype or special case of a rectangle, it is scored as an erroneous answer. Thus, the ratio of the teacher's total points (5 points) to the total possible points (7 points) plus the number of their erroneous answers (1 point) is equal to 5/8 for recognizing rectangles.

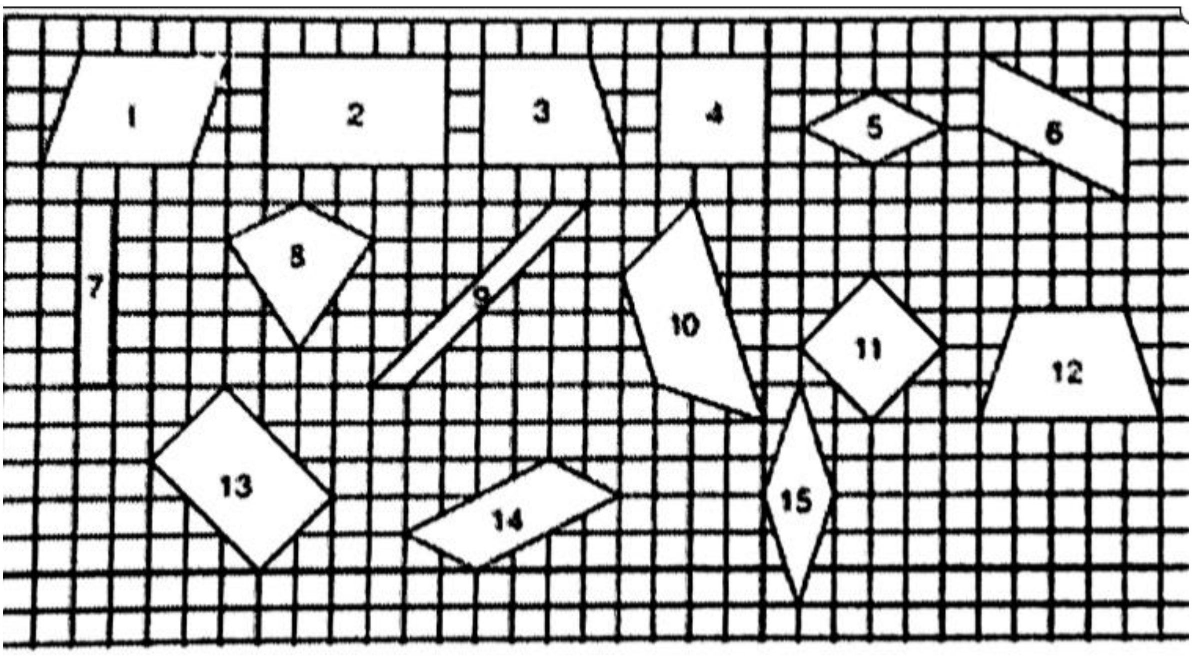

Figure 3. Given quadrilaterals in the second question

Table 2. Special types of quadrilaterals with their special cases in the second question

Quadrilaterals	1 point for each prototype	2 points for each special case	Total possible points
Kites	8	4, 5, 11, 15	9 (=1x1+2x4)
Trapezoids	3, 10, 12	1, 2, 4, 5, 6, 7, 9, 11, 13, 14, 15	25 (=1x3+2x11)
Parallelograms	1, 6, 9, 14	2, 4, 5, 7, 11, 13, 15	18 (=1x4+2x7)
Rectangles	2, 7, 13	4, 11	7 (=1x3+2x2)
Rhombuses	5, 15	4, 11	6 (=1x2+2x2)

In the last question, the same set of special types of quadrilaterals as in the first question was given to mathematics teachers, yet they were asked to classify these quadrilaterals hierarchically and show these hierarchical relationships in a knowledge map. Knowledge maps in this study were evaluated by using the similarity index which is equal to the ratio of the number of common links in two knowledge maps to the number of links in both knowledge maps (Goldsmith, Johnson, & Acton, 1991). According to Kudikyala and Vaughn (2004), the level of similarity between two knowledge maps can be little, moderate and strong where the cut-off values are .4 and.7. On the other hand, Sarwar and Trumpower (2015) argued that these cut-off values are reasonable for novice learners, but not experts. Hence, the cut-off values in this study were assumed to be .50 and .75 for moderate and strong similarity, respectively. As an example, the referent knowledge map, which was adapted from Usiskin et al. (2008), and an example of a teacher's knowledge map are illustrated in Figure 5.a and Figure 5.b, respectively. The number of common links between two knowledge maps (*relevant links*) is 6. Additionally, while the number of unique links in the referent map (*missing links*) is 1, the number of unique links in the teacher's knowledge map (*extraneous links*) is 2. Thus, the similarity index is equal to the ratio of 6 to 9 which corresponds to a moderate similarity.

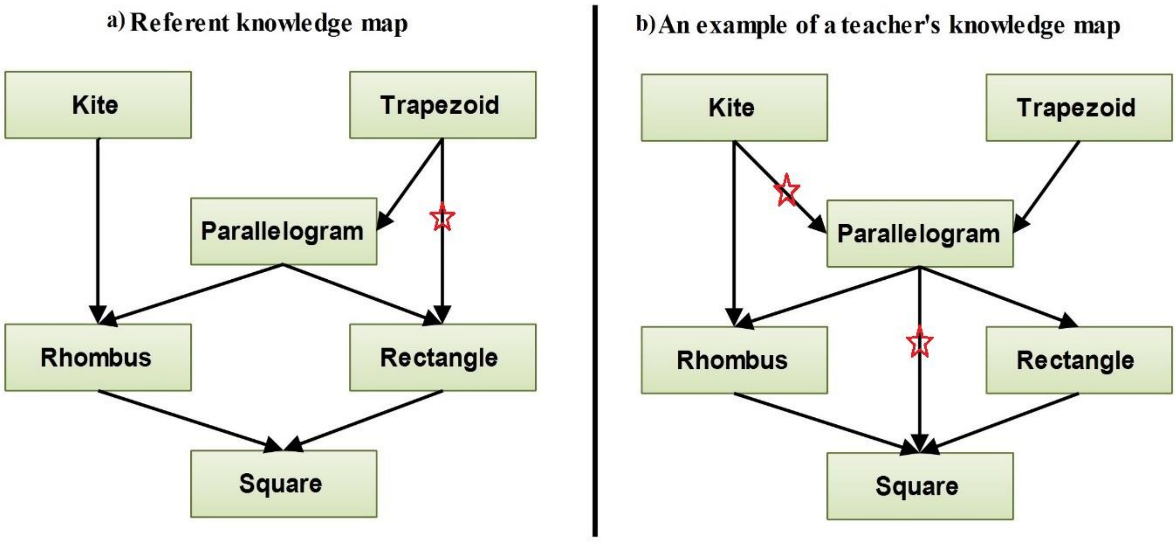

Figure 4. Referent knowledge map (a) and an example of a teacher's knowledge map (b)

Before identifying the aforementioned evaluation criteria for each question, two researchers conducted the literature review on how these questions had been analyzed in previous studies. Afterwards, they had a few online meetings in which different approaches for evaluating each question item were compared and identified as the most suitable evaluation criterion for each question. Thereafter, two researchers independently categorized teachers' answers to each question based on the corresponding evaluation criteria. Next, their coding were compared, and Cohen's kappa coefficient was computed as 0.90 indicating that the inter-rater agreement was almost perfect because it is between 0.81 and 1.00 (J. Cohen, 1960; Gwet, 2014; Landis & Koch, 1977). Discrepancies were reviewed and reconciled through consensus. Finally, the frequencies of the resultant codes were illustrated in the tables.

Findings

In the first question, mathematics teachers were asked to define a few special types of quadrilaterals including a kite, trapezoid, parallelogram, rectangle, rhombus and square. As demonstrated in Table 3, 20 teachers (60%) hierarchically defined a kite whilst 11 teachers (33%) provided the prototype definition of a kite. In terms of trapezoid definition, only 3 teachers (9%) gave a hierarchical definition while 30 teachers (91%) gave a prototype one. For parallelogram and rectangle, 31 teachers (94%) offered a hierarchical definition, while 1 teacher (3%) and 2 teachers (6%) stated a prototype definition of a parallelogram and rectangle, respectively. For rhombus, 26 teachers (79%) hierarchically defined it and 5 (15%) expressed a prototype definition. Teachers were not asked to classify squares as they appear at the base of the hierarchy of quadrilaterals. Instead, researchers were concerned with the correctness of each definition of a square provided. All teachers (33; 100%) could accurately define a square but they chose to emphasize different characteristics, for instance, a few teachers mentioned that a square is a special case of a rectangle.

Table 3. Mathematics teachers' definitions of special types of quadrilaterals

The name of quadrilaterals	The classification of definitions	Frequencies	Examples
Kite	Hierarchical	20	is a quadrilateral formed with two isosceles triangles sharing a common base.
	Prototype	11	is a quadrilateral formed with two different isosceles triangles joined base to base.
	Erroneous or partial	2	is a quadrilateral which has two pairs of equal sides, and the sum of its opposite angles is 180.
Trapezoid	Hierarchical	3	is a quadrilateral having at least a pair of parallel sides.
	Prototype	30	is a quadrilateral having only one pair of parallel sides.
	Erroneous or partial	0	
Parallelogram	Hierarchical	31	is a quadrilateral whose both opposite sides are parallel, and opposite angles are equal.
	Prototype	1	is a quadrilateral whose both opposite sides are parallel, and its diagonals bisect each other, yet the angle between these diagonals is not 90.
	Erroneous or partial	1	is a quadrilateral whose opposite angles are equal, and the size of its opposite sides are same.
Rectangle	Hierarchical	31	is a quadrilateral whose opposite sides are equal, and the size of its angles is 90.
	Prototype	2	is a quadrilateral having a pair of equal short parallel sides and a pair of equal long parallel sides as well as the size of its angles is 90.
	Erroneous or partial	0	
Rhombus	Hierarchical	26	is a parallelogram whose all sides are equal.
	Prototype	5	is a quadrilateral whose all sides are equal, and the sizes of its opposite angles are same, but none of them is equal to 90.
	Erroneous or partial	2	is a quadrilateral whose all sides are equal.
Square	Accurate	33	is a rectangle whose sides are equal, and the size of its angles is 90. is a quadrilateral whose sides are equal, its opposite sides are parallel, and the size of its angles is 90.
	Erroneous or partial	0	

In the second question, mathematics teachers were given 15 quadrilaterals and asked to recognise kites, trapezoids, parallelograms, rectangles and rhombuses among them. As seen in Table 4, while 4 teachers (12%) could identify the kite family, 21 (64%) could only recognise prototypes of a kite. Similarly, in the trapezoid family, 4 teachers (12%) observed the relations, while 22 teachers (67%) could only find the prototypes. For parallelograms, 15 teachers (45%) identified the family, and 11 (33%) only prototypes. For rectangles, 14 teachers (42%) determined the family, and 12 teachers (36%) recognized only prototypes. In the rhombus case 16 teachers (48%) indicated the rhombus family, while 9 (27%) discovered only the prototypes. Consequently, mathematics teachers' average scores for this question were low. As illustrated in Figure 6, the mean scores obtained by 15 teachers (45%) were found lower than .50, 10 (30%) were computed between .50 and .74, 8 (24%) were calculated as .75 or above, and only three of the participants (9%) scored .95 or above.

Table 4. Discrepancies between mathematics teachers' answers and the evaluation criteria

	Kite		Trapezoid		Parallelogram		Rectangle		Rhombus		Mean
	Dis.	Sco.	Dis.	Sco.	Dis.	Sco.	Dis.	Sco.	Dis.	Sco.	
T1	4, 5	5/9	-	25/25	7	16/18	"7", "13"	5/7	4	4/6	0.77
T2	4, 5	5/9	-	25/25	4, 7	14/18	"7", "13"	5/7	4	4/6	0.74
T3	NSC	1/9	"12"; NSC	2/25	-	18/18	-	7/7	-	6/6	0.64
T4	4, 5, 11	3/9	"3"; 1, 2, 4, 5, 6, 7, 11, 13, 14, 15	4/25	"6", "9"; 15	14/18	NSC	3/7	4	4/6	0.47
T5	- ; (1)	9/10	9, 11, 13, 14, 15	15/25	-	18/18	"13"; 11	4/7	"15"	5/6	0.78
T6	NSC	1/9	NSC	3/25	-	18/18	-	7/7	-	6/6	0.65
T7	NSC	1/9	NSC	3/25	-	18/18	-	7/7	-	6/6	0.65
T8	NSC	1/9	NSC	3/25	-	18/18	4	5/7	-	6/6	0.59
T9	-	9/9	NSC	3/25	-	18/18	-	7/7	-	6/6	0.82
T10	-	9/9	-	25/25	-	18/18	-	7/7	-	6/6	1.00
T11	NSC	1/9	NSC	3/25	NSC	4/18	NSC	3/7	4	4/6	0.31
T12	NSC	1/9	NSC	3/25	NSC	4/18	-	7/7	NSC	2/6	0.36
T13	NSC	1/9	NSC	3/25	-	18/18	-	7/7	-	6/6	0.65
T14	NSC	1/9	NSC	3/25	NSC	4/18	NSC	3/7	-	6/6	0.38
T15	NSC	1/9	NSC	3/25	2, 5, 7, 13, 15	8/18	NSC	3/7	NSC	2/6	0.29
T16	NSC	1/9	NSC	3/25	NSC	4/18	"7"; NSC	2/7	"5", "15"; NSC; (1),(6),(13),(14)	0/10	0.18
T17	NSC	1/9	NSC	3/25	NSC	4/18	-	7/7	NSC	2/6	0.36
T18	NSC	1/9	NSC	3/25	NSC	4/18	NSC	3/7	-	6/6	0.38
T19	NSC	1/9	NSC	3/25	2, 4, 7, 11, 13	8/18	-	7/7	-	6/6	0.54
T20	NSC	1/9	NSC	3/25	NSC	4/18	NSC	3/7	-	6/6	0.38
T21	NSC	1/9	NSC	3/25	-	18/18	-	7/7	-	6/6	0.65
T22	NSC	1/9	NSC	3/25	-	18/18	-	7/7	NSC	2/6	0.51
T23	NSC; (10)	1/10	"10"; NSC	2/25	NSC	4/18	NSC	3/7	NSC	2/6	0.23
T24	-	9/9	NSC	3/25	-	18/18	-	7/7	-	6/6	0.82
T25	NSC	1/9	1, 2, 5, 6, 7, 9, 13, 14, 15	7/25	NSC	4/18	NSC	3/7	NSC	2/6	0.28
T26	4	7/9	- ; (8)	25/26	-	18/18	-	7/7	-	6/6	0.95
T27	NSC	1/9	NSC	3/25	NSC	4/18	NSC	3/7	NSC	2/6	0.24
T28	NSC	1/9	NSC	2/25	13	16/18	NSC	3/7	NSC	2/6	0.37
T29	NSC	1/9	NSC	3/25	NSC	4/18	NSC	3/7	4	4/6	0.31
T30	NSC	1/9	NSC	3/25	-	18/18	-	7/7	NSC	2/6	0.51
T31	4	7/9	NSC	3/25	-	18/18	"13"	6/7	-	6/6	0.75
T32	4, 11	5/9	NSC	3/25	5, 11, 13	12/18	NSC	3/7	"5", "15"; 4	2/6	0.42
T33	-	9/9	-	25/25	-	18/18	"7"	6/7	-	6/6	0.97

* Dis.: discrepancies; Sco.: score; NSC: none of special cases were mentioned; missing prototypes were enclosed in quotation marks; missing special cases were underlined; erroneous answers were presented in round brackets; dashes demonstrated that a teacher's answer includes the prototypes of the corresponding quadrilateral and all its special cases.

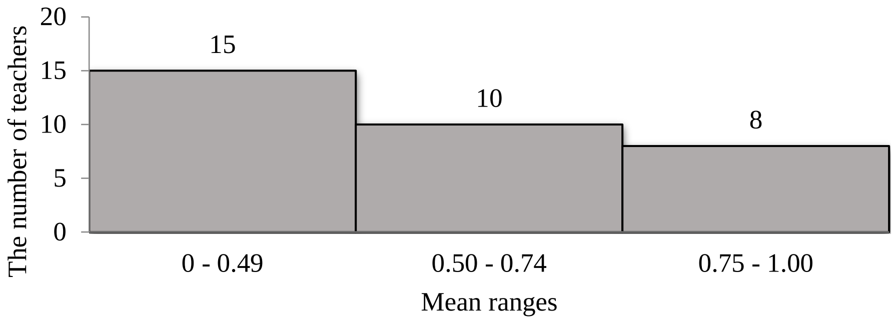

Figure 5. The distributions of mathematics teachers' mean scores in the second question

In the third question, mathematics teachers were asked to illustrate the relationship among a kite, trapezoid, parallelogram, rectangle, rhombus and square, but 2 of them did not answer this question. As demonstrated in Figure 7, there was a link between a kite and a rhombus in the referent knowledge map, but this relationship was included in only 10 teachers' knowledge maps (32%). Additionally, a trapezoid was linked with both a rectangle and parallelogram in the referent knowledge map, yet these relationships were highlighted by only 1 teacher (3%) and 7 teachers (23%), respectively. Elsewhere, while there was a link between a parallelogram, and rectangle in the referent knowledge map, 26 teachers (84%) linked these quadrilaterals in their knowledge maps. While a parallelogram and rhombus were linked in the referent knowledge map, 23 teachers (74%) included this relationship in their maps. Similarly, a link between a rectangle and square was shown in the referent knowledge map, and 23 teachers' (74%) produced this link in their maps. The relationship between a rhombus and a square was also shown in the referent knowledge map, yet only 17 (55%) teachers included this in their knowledge map. Finally, even though no link was shown between a parallelogram and square in the referent knowledge map, 10 teachers (32%) linked these quadrilaterals. Consequently, as seen in Table 5, while the highest similarity index between the referent knowledge map and teachers' knowledge maps was found as .86, the average was calculated as .446. Table 5 illustrates that little similarity exists between the referent knowledge map and the knowledge maps created by 17 teachers (55%), whereas moderate similarity was seen in 13 teachers' knowledge maps (42%). Only one teacher's knowledge map (3%) showed a strong similarity with the referent map.

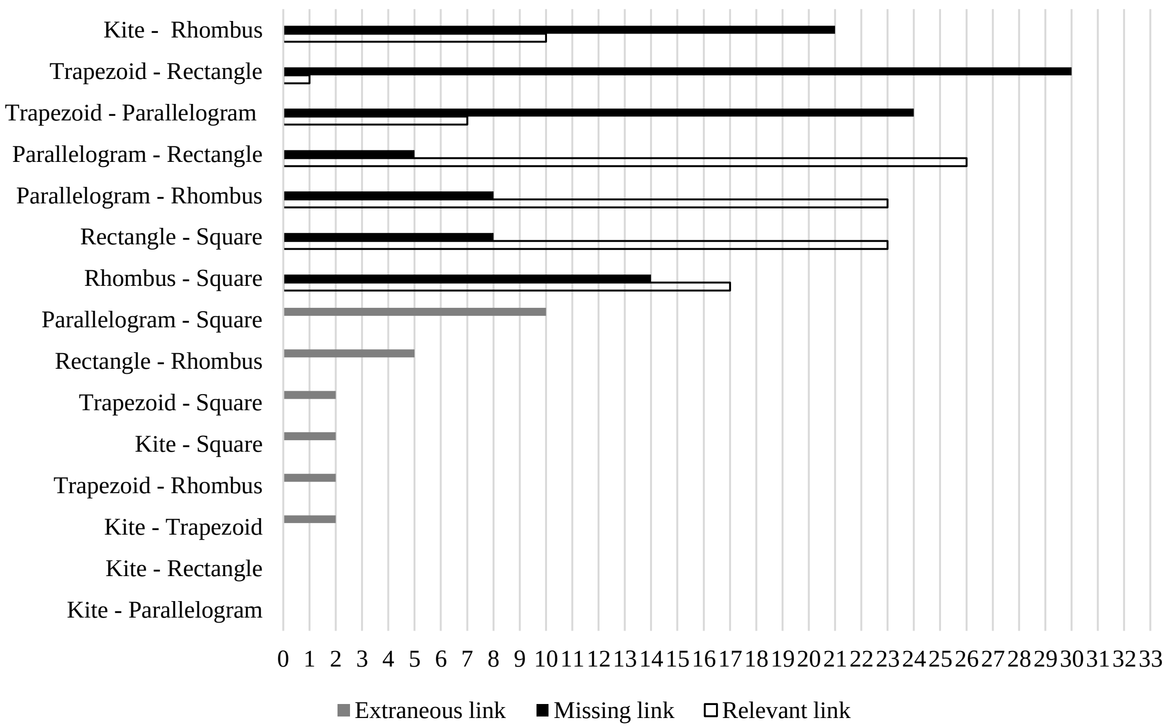

Figure 6. The number of relevant links, missing links and extraneous links in knowledge maps

Table 5. Similarity indices of mathematics teachers' knowledge maps

Teacher	Score	Teacher	Score	Teacher	Score	Teacher	Score	Teacher	Score
T1	0.5	**T8**	0.44	**T14**	0.71	**T20**	0	**T26**	0.38
T2	0.5	**T9**	0.44	**T15**	0.25	**T21**	0.4	**T27**	0.44
T3	0.57	**T10**	0.86	**T16**	0.71	**T22**	0.43	**T28**	0.57
T4	0.57	**T11**	0	**T17**	0.29	**T23**	0.29	**T29**	0.57
T5	0.43	**T12**	0.5	**T18**	0.5	**T24**	0.29	**T30**	0.43
T6	0.29	**T13**	0.5	**T19**	0.71	**T25**	0.5	**T31**	0.43
T7	0.33				**Average: 0.446**				

Discussions and Conclusions

This study aimed to investigate how well mathematics teachers in Turkey define special types of quadrilaterals, recognize their family and make a hierarchical classification of them. We found that teachers encounter problems in defining special types of quadrilaterals, determining their family and hierarchically organizing them. To our knowledge, the number of studies exploring mathematics teachers' content knowledge of quadrilaterals is very limited, so the findings of this study are discussed in relation to a number of studies that also examined pre-service teachers' content knowledge of quadrilaterals.

The first problem was investigated concerning the definition of a kite, recognition of the kite family and demonstration of the link between a kite and a rhombus. The results of this study showed that 31 teachers (94%) gave the correct definition of a kite while 20 (60%) hierarchically defined it. Additionally, 4 teachers (12%) determined the kite family whereas the kite-rhombus relation was revealed by only 10 teachers (32%). Likewise, Güner and Gülten (2016) found that 36 of 50 pre-service teachers (72%) could accurately define a kite. However, Ndlovu (2014) found that only 4 of 16 pre-service teachers gave the correct definition. In addition, Çontay and Paksu (2012) reported that 3 of 5 pre-service teachers identify the kite family.

Similarly, primary school mathematics teachers faced challenges in defining a trapezoid, determining the trapezoid family and highlighting trapezoid-parallelogram and trapezoid-rectangle relations. In this research, all teachers could produce an accurate definition of a trapezoid, yet only 3 (9%) of those were hierarchical definitions. Additionally, 4 teachers (12%) investigated the trapezoid family. Furthermore, while one teacher (3%) included the link between a trapezoid and rectangle in their knowledge map, the trapezoid-parallelogram relation was demonstrated by only 7 teachers (23%). Likewise, Duatepe-Paksu et al. (2012) found that 10 (22%) of 45 pre-service teachers erroneously define a trapezoid whereas Ndlovu (2014) observed that 12 (75%) of 16 pre-service teachers could give the correct definition. Moreover, Güner and Gülten (2016) reported that 32 (64%) of 50 pre-service teachers accurately define a trapezoid. On the other hand, Erdogan and Dur (2014) found that 26 (46%) of 57 pre-service teachers could provide an accurate definition of a trapezoid. In addition, Türnüklü, Gündoğdu Alaylı et al. (2013) found that 12 (33%) of 36 pre-service teachers could correctly define a trapezoid whilst Fujita and Jones (2006b, 2007) revealed that only 19 (12%) of 158 pre-service teachers could do this. Regarding the hierarchical definition of a trapezoid, Karakuş and Erşen (2016) determined that 7 (12%) of 58 pre-service teachers hierarchically define a trapezoid. Additionally, Türnüklü, Akkaş, et al. (2013) found that only 1 in 9 mathematics teachers in a study conducted in Turkey could link a trapezoid with a rectangle. While Fujita and Jones (2006b) reported that 29 (23%) of 124 pre-service teachers could demonstrate the link between a trapezoid and rectangle, Erdogan and Dur (2014) observed in Turkey that only 4 (7%) of 57 pre-service teachers include this relation in their knowledge maps. In another study conducted in Turkey, Karakuş and Erşen (2016) determined that 16 (25%) of 58 pre-service teachers highlight the relation of trapezoid-rectangle.

However, primary school mathematics teachers performed relatively better when providing the definition of a parallelogram, recognizing the parallelogram family and presenting the links of a parallelogram-rectangle and parallelogram-rhombus. In this study, 32 teachers (97%) accurately defined a parallelogram whilst 31 teachers (94%) hierarchically defined a parallelogram. Additionally, 15 teachers (45%) determined the parallelogram family whereas the relations of a parallelogram-rectangle and parallelogram-rhombus were revealed by 26 teachers (84%) and 23 teachers (74%), respectively. Similarly, Erşen and Karakuş (2013) detected that 6 of 6 pre-service teachers could indicate the accurate definition of parallelogram while Duatepe-Paksu et al. (2012)

found that 37 (82%) of 45 pre-service teachers could correctly define it. In addition, Erdogan and Dur (2014) reported that 46 (81%) of 57 pre-service teachers could produce a correct definition of a parallelogram whilst Ndlovu (2014) found that 12 of 16 pre-service teachers could make an accurate definition. Furthermore, Fujita and Jones (2006b, 2007) observed that 93 (59%) of 158 pre-service teachers could correctly define a parallelogram whereas Güner and Gülten (2016) reported that only 20 (40%) of 50 pre-service teachers could do this accurately. Karakuş and Erşen (2016) reported that 31 (53%) of 58 pre-service teachers provide the hierarchical definition of a parallelogram. Additionally, Erdogan and Dur (2014) found that 29 (51%) of 57 pre-service teachers recognize the parallelogram family whereas Fujita and Jones (2006a, 2007) reported that only 21 (20%) of 105 pre-service teachers could identify the parallelogram family. Concerning the relations of a parallelogram-rectangle and parallelogram-rhombus, Erdogan and Dur (2014) determined that 53 of 57 pre-service teachers could highlight these links. On the other hand, Žilková (2015) indicated that 70 of 159 pre-service teachers could describe a parallelogram-rectangle relation, while 115 of the sample could show the relation of a parallelogram-rhombus. In addition, Okazaki and Fujita (2007) reported that 40% of 111 pre-service teachers could illustrate the relation of a parallelogram-rectangle and 41% of the sample could indicate the parallelogram-rhombus link. Moreover, Fujita and Jones (2006b) found that 43 (34%) of 124 pre-service teachers could highlight the relation between a parallelogram and rectangle while only 10 (8%) of the sample could associate a parallelogram with a rhombus. Furthermore, Ndlovu (2014) reported that although 5 of 16 pre-service teachers could reveal the parallelogram-rectangle link, only 4 of 16 pre-service teachers indicate the parallelogram-rhombus link. Türnüklü, Akkaş, et al. (2013) also found that 3 of 9 mathematics teachers could indicate the parallelogram-rectangle relation, whereas only 2 of the sample could accurately illustrate the parallelogram-rhombus link in their knowledge maps.

All teachers in this study could accurately define a rectangle and 31 (94%) could indicate the hierarchical definition. Furthermore, 14 teachers (42%) investigated the rectangle family and 23 (74%) included the link between a rectangle and square in their knowledge maps. Similarly, Ndlovu (2014) reported that 14 of 16 pre-service teachers could provide the correct definition of a rectangle and Erşen and Karakuş (2013) observed that 5 of 6 pre-service teachers could do so. Additionally, Duatepe-Paksu et al. (2012) found that 10 (22%) of 45 pre-service teachers would erroneously define a rectangle while Brunheira and da Ponte (2015) reported that 22 (39%) of 57 pre-service teachers indicate an erroneous definition. In Turkey, Erdogan and Dur (2014) reported that 39% of 57 pre-service teachers could accurately define a rectangle and Güner and Gülten (2016) observed that only 18 (34%) of 50 pre-service teachers could do this. Likewise, Türnüklü, Gündoğdu Alaylı et al. (2013) determined that only 33% of 36 pre-service teachers could give the correct definition of a rectangle. Elsewhere, an earlier study by Fujita and Jones (2006b, 2007) found that only 34 (26%) of 158 pre-service teachers could correctly define a rectangle. Karakuş and Erşen (2016) reported that 42 (72%) of 58 pre-service teachers could provide the hierarchical definition of a rectangle whereas Pickreign (2007) found that only 9 (22%) of 40 pre-service teachers could do so. Erdogan and Dur (2014) found that 24 (46%) of 57 pre-service teachers recognize the rectangle family. Regarding the rectangle-square link, Erdogan and Dur (2014) found that 54 (95%) of 57 pre-service teachers could indicate this relation while Karakuş and Erşen (2016) detected that 49 (85%) of 58 pre-service teachers reveal this link. Ndlovu (2014) also reported that 12 of 16 pre-service teachers highlight this relation. However, Okazaki and Fujita (2007) found that 37% of 111 pre-service teachers illustrate this link whereas Fujita and Jones (2006b) found that 39 (31%) of 124 pre-service teachers could indicate this relation. Additionally, Brunheira and da Ponte (2015) found that only 14 (25%) of 57 pre-service teachers reveal the link between a rectangle and square.

Moreover, 31 teachers (94%) correctly defined a rhombus and 26 (79%) could do so hierarchically. Of this sample, 16 teachers (48%) determined the rhombus family, and the rhombus-square relation was seen by 17 teachers (55%). Likewise, Duatepe-Paksu et al. (2012) reported that 10 (22%) of 45 pre-service teachers provide the erroneous definition of a rhombus whereas Ndlovu (2014) observed that 11 of 16 pre-service teachers accurately define a rhombus. In Turkey, Türnüklü, Gündoğdu Alaylı et al. (2013) found that 14 (40%) of 36 pre-service teachers provide the correct definition of a rhombus and Güner and Gülten (2016) indicated that 21 (42%) of 50 pre-service teachers give an accurate definition. Additionally, Erdogan and Dur (2014) determined that 19 (33.3%) of 57 pre-service teachers reveal the correct definition of a rhombus whilst Pickreign (2007) found that only 1 (2.5%) of 40 pre-service teachers could provide the hierarchical definition of a rhombus. Moreover, Erdogan and Dur (2014) reported that 28 (49%) of 57 pre-service teachers identify the rhombus family. Concerning the relation of a rhombus-square, Erdogan and Dur (2014) reported that 52 (91%) of 57 pre-service teachers include this relation in their knowledge map whilst Ndlovu (2014) determined that only 10 of 16 pre-service teachers could do so. Earlier, Okazaki and Fujita (2007) found that 28 (25%) of 111 pre-service teachers indicate this relation whereas Fujita and Jones (2006b) reported that 24 (19%) of 124 pre-service teachers could do so. In addition, Žilková (2015) indicated that 24 (15%) of 159 pre-service teachers illustrate this relation while Türnüklü, Akkaş, et al. (2013) observed that only 2 of 9 mathematics teachers demonstrate this relation in their knowledge maps.

Finally, all mathematics teachers in this study provided an accurate definition of a square, yet 10 of them (32%) mistakenly demonstrated the parallelogram-square relation. Similarly, Erşen and Karakuş (2013) reported that 6 of 6 pre-service teachers correctly define a square while Ndlovu (2014) found that 15 of 16 pre-service teachers gave an accurate definition. Additionally, Erdogan and Dur (2014) observed that 54 (95%) of 57 pre-service teachers could provide the correct definition of a square whereas Brunheira and da Ponte (2015) found that 49 (86%) of 57 pre-service teachers accurately define a square. Furthermore, Karakuş and Erşen (2016) revealed that 48 (83%) of 58 pre-service teachers could give the accurate definition of a square whilst Türnüklü, Gündoğdu Alaylı et al. (2013) reported that 19 (53%) of 36 pre-service teachers provided the correct definition. However, Fujita and Jones (2006b, 2007) reported that only 60 (38%) of 158 pre-service teachers could correctly define a square while in Turkey Güner and Gülten (2016) determined that 22 (44%) of 50 pre-service teachers could do so. Moreover, Erdogan and Dur (2014) found that 21 (37%) of 57 pre-service teachers erroneously indicate the parallelogram-square relation.

In conclusion, the results of this study show that primary school mathematics teachers encounter challenges when defining special types of quadrilaterals, determining their family and hierarchically classifying them. Consequently, in-service trainings should be planned and implemented to enhance teachers' content knowledge of quadrilaterals. Such training should include dynamics geometry software such as Cabri Geometry and Geometer's Sketchpad. Öztoprakçı (2014) reported that the use of Geometer's Sketchpad was seen to increase Turkish pre-service teachers' performance in identifying the basic properties of quadrilaterals, defining them, exploring the relations between them and organizing them hierarchically. GeoGebra software might also be used for improving mathematics teachers' knowledge in terms of defining a kite and trapezoid and identifying the kite and trapezoid family.

During such training, mathematics teachers can be asked to create two isosceles triangles whose bases are the same size, using a graphics program like Geometer's Sketchpad. They can then prompted to place the base of the triangle over another one for constructing a kite. Thereafter, they might be asked to draw the diagonals in a kite to help them to explore its properties, e.g. the diagonals are perpendicular and that one of these diagonals is both an angle bisector and a line of symmetry. Thus, once mathematics teachers can explore the fundamental properties of a kite, they will be more able to recognize the kite family. Similarly, teachers can be asked to draw a parallelogram and rectangle and then to show the diagonals. When the properties of the diagonals are examined, it can be concluded that the diagonals of a parallelogram and rectangle are not perpendicular. Hence, mathematics teachers can more readily detect that a parallelogram and rectangle are members of the kite family. Similar exercises with a rhombus and square and their diagonals, can help teachers to investigate the properties of their diagonals in relation to a kite and appreciate why these shapes are members of the same family.

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