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Yasemin Yilmaz, Soner Durmus, Hakan Yaman
Abant Izzet Baysal University

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Abstract

This study investigated the pattern problems posed by middle school mathematics preservice teachers using multiple representations to determine both their pattern knowledge levels and their abilities to transfer this knowledge to students. The design of the study is the survey method, one of the quantitative research methods. The study group was composed of 30 middle school mathematics preservice teachers attending their last year at a state university in Turkey. The Pattern Problem Posing Test with Multiple Representation (PPPTMR), developed by the researchers, was used as the data collection tool. It was identified that preservice teachers' performances when posing pattern problems based on different forms of representations differed according to their knowledge levels and a student level. It was observed that preservice teachers were more successful in using tables in the pattern problems they posed at their levels. Also, they were more skillful at using figures as representations when they posed the pattern problems at a student level. It was also determined that preservice teachers' performances when posing pattern problems using different representations at their own levels was higher than their performances when posing problems at a student level.

Introduction

Mathematics is the science of patterns (Steen, 1988). Patterns, which assist in structuring many mathematical subjects and fill the gaps between mathematical ideas, are defined as the cornerstones of mathematics and as the "soul of mathematics" (Liljedahl, 2004). Since mathematical theories are shaped based on finding patterns and explaining the relationships between these patterns (Steen, 1988), mathematical structures can be obtained by exploring patterns and relationships (Yaman, 2010). Identifying relationships between structures is possible through pattern exploration. Pattern exploration work allows for the formation of rich algebraic environments in terms of variables and the unknowns, algebraic expressions, domain and range of expressions equations, solving the unknown (Lee & Freiman, 2006). In this context, it is observed that the foundation of algebra, which examines variables and the relationships between variables, is established by finding patterns. Algebra is a field of mathematics that allows the acquisition of higher level skills based on concrete student experiences (Ministry of National Education [MoNE], 2013). As a matter of fact, National Council of Teachers of Mathematics (NCTM, 2000) report titled "Principles and Standards for School Mathematics" describes algebra as comprehension of patterns, relationships, and functions; analysis and representation of mathematical situations and structures; and comprehension and representation of quantitative relationships by using mathematical models.

Algebraic thinking requires the use of mathematical symbols and tools to transfer the knowledge embedded in real-life experiences to a new situation; to represent information by using words, tables, diagrams, schemas, graphics, and equations; to find the unknown; to test hypotheses; and to determine functional relationships (Herbert & Brown, 1997). The foundations of algebraic thinking are laid starting from the first years of primary school by making real-life situations and the order of life mathematical. Approaching this order, which is called a pattern, by using different forms of representations may prepare rich thinking environments for students and help them interpret the structure of patterns in a comprehensive manner. In this respect, students should be provided with opportunities to verbally express and use representations in general so that they can recognize these relationships in different contexts in addition to acquiring higher level skills such as recognizing patterns and relationships and testing hypotheses (Ferrini-Mundy, Lappan, & Philips, 1997).

Forms of representation are tools that assist students in understanding mathematical concepts and relationships; allow them to share mathematical approaches, arguments, and perceptions with others; play important roles in forming connections between mathematical concepts; and help adapt mathematics to real problem situations (National Council of Teachers of Mathematics [NCTM], 2000). In short, representations act like a bridge in organizing and recording mathematical ideas; in communicating and transforming representations; in forming models; and in interpreting physical, social, and mathematical facts and real life situations. Forms of representations are classified as internal and external representations. Internal representations are the images generated in our brain for mathematical concepts and operations (Cuoco & Curcio, 2001), and they cannot be directly observed (Goldin & Kaput, 1996). In this respect, internal representations focus on the cognitive processes of the individual to structure of mathematics. External representations such as equations, geometric drawings, and figures are the forms of representation that leave marks on paper, and they facilitate communication (Cuoco & Curcio, 2001). Goldin & Kaput (1996) define external representations as concepts that can be physically observed, such as words, graphics, figures, equations, etc. Edwards (1998) regards external representations as a component of "microworlds," which are virtual leaning environments that allow for forming connections between mathematical objects via multiple representations. In this sense, external representations can be defined as tools that play critical roles in understanding mathematical conditions by presenting them symbolically as tables, graphics, figures, etc.

Forms of representations are functional tools that play roles in problem-solving. Problems are defined as "inconvenient or uncomfortable situations against which individuals seek solutions with the help of their experiences and knowledge" (Baki, 2006, p. 146). In mathematical terms, a problem is the cluster of conditions that explains the relationships between the known and the unknown with the help of written, verbal, symbolic, or graphic presentations based on a specific subject (Gonzales, 1998). Problem-solving is defined as "knowing what to do when what needs to be done is unknown" (Altun, 2011, p. 58). Problem solving, which is included in a mathematics curriculum among the basic skills that are crucial to be gained to students, is a skill that forms a strong connection with other skills and that affects all other learning areas (Ministry of National Education [MoNE], 2015). Understanding of a mathematical knowledge and building relationships are shaped during the problem solving process (Soylu & Soylu, 2006). In order for the problem-solving process to take place in a healthy way, it is important to have a clear and intelligible problem statement. Understanding the problem, which is the first step in Polya's (1957) problem-solving model, is an indicator that lays the foundation of posing problems. Silver (1994) mentioned that problem posing can occur before and after the problem solving, and during the problem-solving process. It can occur *before* the problem solving "if problems were generated from a given contrived or naturalistic situation". According to him, *after* refers "when one might examine the conditions of the problem to generate alternative related problems". Finally, *during the problem solving process* refers "one can intentionally change some of the problem's goals or conditions" for him. In addition, problem finding has a resultant role for the problem solving (Dillon, 1982). Lin & Leng (2008) expressed that problem posing is an important companion to problem-solving. Therefore, problem posing cannot be considered without problem solving, so it is an integral part of problem solving (El Sayed, 2002).

Silver (1994), who defined posing problems as generating new problems or reorganizing and formulating given problems, regards student-generated problems as a crucial part of instruction. When students pose their own problems, they can develop mathematical ideas and become open to critical thinking, and they may improve their operational skills by discovering an innate curiosity about some mathematical concepts (English, 1997), and so give them the feeling of becoming more engaged in their education (Lavy & Shriki, 2007). Students engaged in problem posing activities will have a creative, active learning identity with enterprising spirit (Lavy & Bershadsky, 2003). Therefore, in mathematics education, problem posing is expressed as an important activity (Kojima, Miwa & Matsui, 2009). In their study, Lavy & Shriki (2007) aimed to examine the development of mathematical knowledge of preservice teachers through problem posing. The study showed that preservice teachers developed their ability in terms of examining the definition and properties of mathematical objects, revealing the relationships between them, testing the validity of an argument, and enriching mathematical problems. However they tended to pose simple and common problems and were aifraid of their inability to prove them. According to Crespo & Sinclair (2008), students of all age groups tend to solve the problems they have seen in their textbooks or posed by their teachers. This is reflected in the fact that students have limited experience in posing their own problems. It is due to the middle school students or preservice teachers should be given the opportunity to pose their own problems. When this is provided, researchers have stated that they would be generated artifacts consistent with their normative understandings of what a school mathematics problem should look like (including its linguistic characteristics and its objectives). If this is not provided, young learners will continue to believe that posing mathematical problems is always the "someone else's business" (Ellerton, 2013). Indeed, Cunningham (2004) revealed that providing opportunity to pose problems enhanced students' reasoning and reflection. Also Crespo and Sinclair examined preservice teachers' behavior of problem posing and found that the quality of the posed problems is indicative of a mathematically richer understanding. Barlow & Cates (2006) aimed to examine the effect of problem posing on teachers' beliefs about mathematics and mathematics teaching. As a result of the study, they came to the conclusion that problem posing had an effect on teachers' beliefs meaningful and positive. In addition, researchers have reached the evidence to explain this positive effect in teachers' written explanations such as problem posing enables students to develop a deeper understanding and teachers to assess the students' understanding; develops high-order thinking skills in the students and gives them some ownership of the mathematics. Problem posing is an important element of mathematical exploration and contributes to explore students' thoughts from different perspectives (Cai, 2003) and is a tool that helps to strengthen and improve the critical thinking skills (Nixon-Ponder, 1995). To sum up in their meta-analysis on problem posing activities, Rosli, Capraro & Capraro (2014) reported that problem posing activities contribute to mathematics achievement, acquisition of problem-solving skills, attitudes towards mathematics, and levels of mathematical problems that are posed.

When the studies in the national field on patterns, representation forms and problem posing subjects were examined, Gürbüz & Durmuş (2009) tried to determine the proficiency levels of mathematics teachers according to the transformation geometry, geometric objects, pattern and ornamentation sub-learning areas in the mathematics curriculum. As a result, the teachers' proficiency levels of geometry, geometric objects, pattern and ornament were found to be 79%, 56% and 56% respectively in terms of sub-learning areas. Çakmak, Baş & Bekdemir (2014) aimed to evaluate the mathematical language skills of the mathematics preservice teachers on the topic of pattern in verbal and symbolic language. As a result of the study, they found that the performances of preservice teachers in verbal language use and pattern finding were higher than those of symbolic language use. Dündar & Yılmaz (2015) aimed to determine the performances of mathematics preservice teachers in tests consisting of problems prepared according to different forms of representation on the integral subject. In their study, despite the fact that the problems have the same numerical solution, preservice teachers' performance in the tests given in different forms of representation differed. While the preservice teachers were able to demonstrate their highest performances in the symbolic representation, they found that the performances in the other forms of representation were in the visual and verbal representations, respectively. Işık & Kar (2012) aimed to determine the problem posing skills of classroom preservice teachers through semi-structured situations. They pointed out that preservice teachers' number of problem posing differed from each other in the test, which consisted of a situation in which open-ended narratives emphasizing verbal representation and a symbolic representation. However, in both verbal and symbolic test items, preservice teachers' number of problem posing is different. In particular, the number of problem posing for verbal situations in the test of preservice teachers varied more than the symbolic ones.

In addition, preservice teachers are limited to different types of problems that can associate mathematical concepts given to them. On the other hand, preservice teachers have expressed that they prefer problems that can be solved by simple calculations. Korkmaz & Gür (2006) aimed to determine the problem posing skills of both classroom and mathematics preservice teachers. As a general result of the study, the problems that the preservice teachers have posed are more solvable. Besides, they have found that the problems that the preservice teachers pose are composed of the problems in the textbooks based on four operations. They also found that a large part of the preservice teachers were unable to transfer the expressions that were in their thinking and which they should have had during problem posing to their practices. Yıldız (2014) aimed to determine the perspectives, experiences and skills of mathematics preservice teachers on problem posing. She also aimed to examine the effect of teaching on problem posing preservice teachers' problem posing skills and metacognitive awareness. When the results of her study are examined, the preservice teachers have positive aspects of problem posing studies and knowledge levels of problem and problem posing are sufficient. In addition to these results, it has been found that the preservice teachers had achieved a success rate of 50% -65% over the average in the problem posing process in general. Also she has reached a conclusion that preservice teachers significantly increase their problem posing skills and metacognitive awareness levels with studying on problem posing.

The Aim and Importance of the Research

The foundations of algebraic thinking in students are laid during the primary grades. The foundations of algebra, which is one of the learning areas in the curriculum and which includes important skills should be gained to the students, are laid with the help of patterns. Therefore, it is crucial for mathematics teachers to acquire sufficient and necessary experiences and competences about patterns. In their study on middle school mathematics teachers, Gürbüz & Durmuş (2009) reported teacher competences related to patterns and ornaments included as sub-learning areas of algebra as at an average level of 56%. In this respect, it is believed that teacher competences are important both for their knowledge levels and for transferring this information to students by posing problems using different forms of representation. Some studies on problem posing report the problems as teachers' lack of involvement with problem posing and solving activities and their use of books and other sources in that regard (Albayrak, İpek, & Işık, 2006; Ellerton, 2013) and the tendency of preservice teachers to prepare the same kinds of problems (Çıldır & Sezen, 2011; (Lavy & Shriki, 2007).

In this context, guiding problem posing activities and providing momentum for them can be done by increasing the variety of pattern problems by using different forms of representation. In addition, competences of mathematics teachers play an important role in problem posing, which is regarded as one of the basic skills both in primary and middle school mathematics curriculums and in NCTM (2000). In this respect, it is seemed to be important to gain awareness about patterns for primary and middle school mathematics teachers. Qualified teachers should be trained to encourage students to pose their own problems and to provide experience in using different representations (written, verbal, symbolic, figures, tables, etc.) of mathematical ideas. It is considered that developing the ability to pose problems and running the related activities with them as one of the ways in which pre-service mathematics teachers gain competence (Tichá & Hošpesová, 2009). In this context, the knowledge about the pattern of middle school mathematics preservice teachers, which can help shape the basis of algebraic thinking, is gained importance in the context of problem posing with different forms of representation how to convey these knowledge to the students. This study examined the problems posed by middle school mathematics preservice teachers by using different forms of representations to determine their skills in the posing of pattern problems at their own level as well as their skills in problem posing at a student level. With this aim in mind, answers were sought to the following questions:

1) What are the problem posing skill levels (their undergraduate level) of middle school mathematics preservice teachers for patterns based on the forms of representations?
2) What are the problem posing skill levels (student level in middle schools) of middle school mathematics preservice teachers for patterns based on the forms of representations?

Method

This section provides information about the research design, study group, data collection tool, and data analysis.

Research Design

Since the study set out to describe preservice teachers' performances in problem posing of patterns based on the forms of representations they used with the help of data collected via quantitative methods, it is a descriptive study. Hence, the survey method, which is used to describe a situation as is, was used in the research (Karasar, 2014, p. 77).

Study Group

The study group was composed of 30 middle school mathematics preservice teachers attending their last year at a state university. These preservice teachers were selected for the study because they had already completed the courses related to mathematics education. The convenience sampling method, which is one of the purposeful sampling methods, was utilized in determining the study group. The reason behind utilizing this method was related to the fact that the researchers and preservice teachers included in the study were at the same university. It is also believed that the convenience sampling method is useful because it allows researchers to deal with a situation that is close and easy to access, giving speed and practicality to the research (Yıldırım & Şimşek, 2013, p.141). Twenty-three of the 30 preservice teachers selected for the study were females, and seven were males. Real names of the preservice teachers were not used in the study; instead, they were coded by the researchers as PT1, PT2, …, PT30.

Data Collection Tool

The Pattern Problem Posing Test with using Multiple Representation (PPPTMR), developed by the researchers, was used as the data collection tool (also see in Appendix A). The test is composed of two parts and has four questions in each part. Each part asks the preservice teachers to pose a problem about patterns by using "Figures," "Tables," "Verbal," and "Symbolic" forms of representations. The first part of the test requires the preservice teachers to pose the problems at their own level, whereas the second part asks the preservice teachers to undertake the same task at a student level. Since there was no appropriate criterion for calculating the validity coefficient, expert opinion was sought in this regard. These people consist of a total of 5 people, two of whom are teaching professors in mathematics education, two are mathematics education specialists, and one is an expert in the field of measurement and evaluation. On the other hand, the reliability measures of first and second parts of PPPTMR were obtained by calculating the Cronbach alpha coefficients. The Cronbach alpha coefficients of the first and second parts were found to be 0.82 and 0.86, respectively. The fact that these values are greater than 0.70 indicates that the data collection tool is reliable (Büyüköztürk, 2014).

Firstly, preservice teachers were informed about the purpose of the research and why this research was conducted. It was stated that the research consists of two parts and said that they need to think separately according to these parts. They should think about in the first part their own level of pattern knowledge, in the second part middle school student level. The first part of the PPPTMR requires preservice teachers to pose four problems about patterns at their own level by using “Figures,” “Tables,” “Verbal,” and “Symbolic” forms of representations. The second part asks preservice teachers to pose four problems about patterns by using “Figures,” “Tables,” “Verbal,” and “Symbolic” forms of representation by taking student levels into consideration.

Data Analysis

Preservice teachers were told about the criteria they should pose in problems. It was also stated that the problems would be evaluated by the researchers over these criteria. In the study, “teacher candidate level” meant the level of the mathematics teacher or instructor or undergraduate level, and “student level” meant the level of students in 5th through 8th grades. Both implementation parts for the PPPTMR were conducted during two separate sessions and one class hour. The criteria included in the scoring instructions developed by Yıldız (2014) were used in the data analysis to identify the criteria to assess the problems posed by preservice teachers (see Table 1). The problems posed by preservice teachers were assessed according to the following criteria, which were included in the scoring instructions: *being mathematical, data quality, grammar and expression, level appropriateness, instructions for the problems and amount of data*, and *possibility of being solved*.

Table 1. Criteria for the problems and what these criteria mean

CRITERIA	EXPLANATION
Being mathematical	Use of mathematical expressions and concepts
Data quality	Logical and mathematical suitability of data included in the problem and the meaningfulness of the result that can be obtained with the help of data provide in the problem
Grammar and expression	Appropriateness of the text in terms of grammar, lack of ambiguity or misspelling, and use of punctuation marks
Level appropriateness	Appropriateness of the problem to the level specified in the instructions
Instructions for the problems and amount of data	Appropriateness of the steps or conditions specified in the instructions
Possibility of being solved	Whether the problem can achieve the intended result (i.e., can it be solved?)

The scoring key provided in Table 2 was used by the researchers in scoring. This process was done as a scoring of randomly selected 5 papers of the preservice teachers’ papers by the first researcher and a different researcher who is an expert in mathematics education. Firstly, the differences between the scores of the two researchers were tried to be eliminated, and this situation was maintained until the two scorers agreed on the same opinion. After eliminating the differences between the scorers, the first researcher scored the rest of the data. The minimum score for each part of the test is 0. Since it is possible to obtain a maximum of two points for each of the four forms of representation and the six criteria, the maximum score that can be obtained for each part is 48 (4 x 6 x 2 = 48). Also, the minimum and maximum scores for 30 preservice teachers for each assessment criteria in the related representation of each part are 0 and 60 respectively. For instance, a teacher candidate receives two points for the being mathematical criteria in the problem posed by figure representation, and since there are 30 preservice teachers, for the criteria of figure representations and being mathematical all preservice teachers can obtain a maximum of 60 points.

Table 2. Scoring key for problems posed by preservice teachers

Explanation	Score
Empty, meaningless, irrelevant, or incorrect situations	0
Partial use of data; containing incomplete data	1
Providing all of the required conditions	2

It is tried to explain how the problem posing criteria in Table 1 are evaluated by the researchers according to the scoring key in Table 2 through an example given in Figure 1.

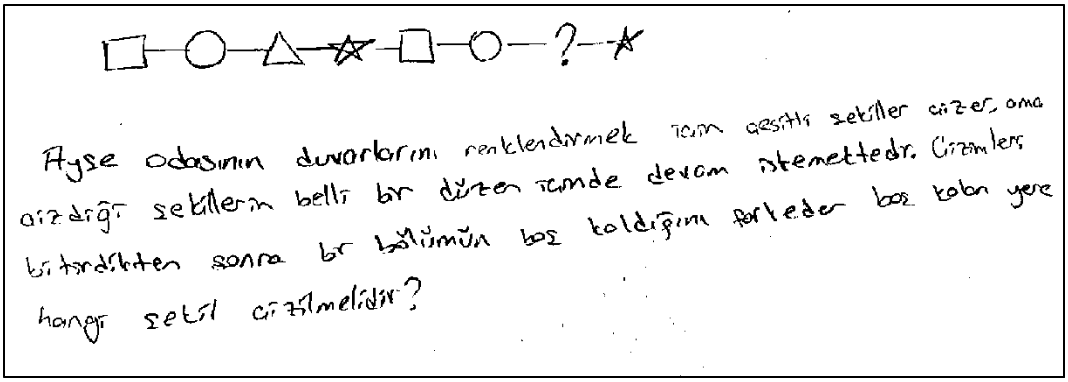

Figure 1. The pattern problem that PT12 pose in the form of figures representation in the first part of PPPTMR
(PT12's Problem: Ayşe draws various shapes to color the walls of the room, but she wants to continue the shapes she draws in a certain order. After finishing the drawing, notice that a section is empty what shape should be drawn at the empty position?)

When the pattern problem that PT12 pose according to the figures representation is evaluated, the scores on which the problem has been posed are given below in paranthesis:

Being mathematical (1), data quality (2), grammar and expression (1), level appropriateness (0), instructions for the problems and amount of data (2), possibility of being solved (2).

This posed problem presents a clarity when it is considered visually. However, in the problem, the expressions in the 6th and 7th words are needed in a broader sense. In this respect, it has received 1 point from the criterion of "being mathematical" because of the fact that the presentation of mathematical expressions and concepts can not be fully realized and that the knowledge contained in the problem is partially presented. The fact that there were punctuation errors in the statements of the first sentence of PT12 and the expression was not correct caused the preservice teacher to get 1 point from the "grammar and expression" criterion. On the other hand, PT12 has posed a kind of problem that may be more appropriate for middle school or even primary school students than their own level. For this reason PT12 received a score of 0 on the criterion of "level appropriateness". Other criteria such as "data quality", "instructions for the problems and amount of data", and "possibility of being solved" criteria were able to fully express the statements in the description of the criteria mentioned in Table 1. Thus, PT12 received 2 points out of each of these three criteria. After scoring was completed, preservice teachers' performances in the first (in terms of their level) and second (in terms of student level) tests were categorized by the researchers as low, medium, and high by using Alamolhodaei's (1996) formula (Figure 2).

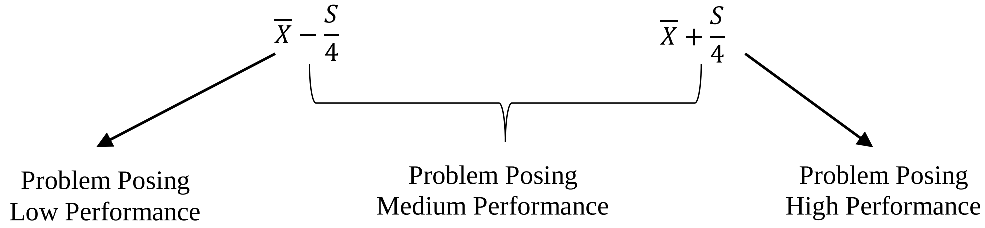

Figure 2. The formula used to determine preservice teachers' problem-posing performance

Findings

This section presents findings related to the research questions.

Findings Related to the First Research Question

Table 3 presents the performances of preservice teachers when posing pattern problems at their own level by using different forms of representation.

Table 3. The performances of preservice teachers in the first part of the PPPTMR

	N	$\bar{X}$	Sd
1st Part	30	37.03	7.64

Table 3 displays the general performance means of preservice teachers in the first part of the PPPTMR as 37.03 out of 48. When the performances of preservice teachers in posing pattern problems at their own levels by using different forms of representation were examined, it was observed that PT28 obtained the lowest score with 17, and PT23 obtained the highest score with 46. Table 4 displays the problem posing performances of PT23 and PT28 based on the forms of presentation and assessment criteria (also see the examples of PT4 and PT23 preservice teachers' papers in Appendix B).

Table 4. The performances of T23 and T28 at their own levels

Criteria	PT23				PT28			
	Forms of Representations				Forms of Representations			
	Figures	Tables	Verbal	Symbolic	Figures	Tables	Verbal	Symbolic
Being mathematical	2	2	2	2	1	1	0	0
Data quality	2	2	2	2	1	0	0	0
Grammar and expression	1	2	2	1	2	0	0	0
Level appropriateness	2	2	2	2	2	2	0	0
Instructions for the problems and amount of data	2	2	2	2	2	2	0	0
Possibility of being solved	2	2	2	2	2	2	0	0
General evaluation	11	12	12	11	10	7	0	0
TOTAL	**46**				**17**			

Table 4 shows that PT23, who received the highest score in the PPPTMR for posing problems at their levels, was able to pose pattern problems in accordance with all criteria by using tables and verbal forms of representation. PT23 partially met the grammar and expression criterion in the problem posed by using figures and symbolic representations. According to Table 4, PT28, who received the lowest score in the PPPTMR for posing problems at teachers' levels, was not able to meet any of the criteria for posing problems using verbal and symbolic forms of representation but fully met level appropriateness, instructions for the problems, and amount of data and possibility of being solved criteria in using figures and tables as forms of representation.

The scores obtained from the PPPTMR show that the numbers of teachers who had low, medium, and high problem posing performances were 10, 3, and 17, respectively. On the other hand, preservice teachers were also assessed for performance in using forms of representation and problem posing criteria at their own levels. Table 5 and Table 6 provide these performances.

Table 5. Performances of preservice teachers in terms of criteria related to different forms of representations*

Criteria	Forms of Representations				
	Figures	Tables	Verbal	Symbolic	TOTAL
	%	%	%	%	%
Being mathematical	16.00	16.44	14.87	18.91	16.56
Data quality	18.18	17.81	16.73	18.55	17.82
Grammar and expression	15.64	13.36	18.22	9.45	14.13
Level appropriateness	8.73	15.07	13.75	16.73	13.59
Instructions for the problems and amount of data	21.09	17.47	18.22	17.09	18.45
Possibility of being solved	20.36	19.86	18.22	19.27	19.44
TOTAL	100.00	100.00	100.00	100.00	100.00

**Maximum score was 60, and the table presents scores in percentages. For example, in the form of figures representation, the total score of preservice teachers according to the problem posing criteria is 275. According to the mathematical criterion in this representation form, preservice teachers' total score is 44. When the figures representation is taken into account, the percentage of performance of the preservice teachers in the mathematical criterion is calculated as 44x100 / 275 = 16%. The numerical percentages in the other cells in Table 5 are similar to this calculation.*

Table 5 shows that using figures as a form of representation, preservice teachers displayed the highest performance in the instructions for the problems and amount of data (21.09%) criterion and the lowest performance in the level appropriateness (8.73%) criterion when they posed pattern problems at their own level. Preservice teachers displayed the highest performance for using both tables and symbolic forms of representations in the possibility of being solved criterion and the lowest performance in the grammar and expression criterion. Table 5 presents that preservice teachers displayed the highest performance in own-level pattern problems posed by verbal representations in the grammar and expression (18.22%), instructions for the problems and amount of data (18.22%) and possibility of being solved (18.22%) criteria. However, they presented the lowest performance in the level appropriateness (13.75%) criterion.

Preservice teachers' scores were obtained in terms of criteria related to posing pattern problems at their own levels by using tables as a form of representation. Verbal forms of representation also provided similar results. However, performances in using figures and symbolic forms of representations highly differed in terms of problem posing criteria, especially in level appropriateness and grammar and expression. Scores obtained from these criteria were found to be lower than those obtained from other criteria. Teachers were also found to display higher performance in general in the possibility of being solved (20.36%) criterion compared to other criteria regarding the use of different forms of representations in posing pattern problems. In this context, it can be argued that when basing them on their own level, teachers tended to pose problems that could be solved.

Table 6. Performances of preservice teachers at their levels in terms of criteria related to problem posing criteria*

Criteria	Percent	Forms of Representations				
		Figures	Tables	Verbal	Symbolic	TOTAL
Being mathematical	%	23.91	26.09	21.74	28.26	100.00
Data quality	%	25.25	26.26	22.73	25.76	100.00
Grammar and expression	%	27.39	24.84	31.21	16.56	100.00
Level appropriateness	%	15.89	29.14	24.50	30.46	100.00
Instructions for the problems and amount of data	%	28.29	24.88	23.90	22.93	100.00
Possibility of being solved	%	25.93	26.85	22.69	24.54	100.00
TOTAL	%	24.75	26.28	24.21	24.75	100.00

** Maximum score was 60, and the table presents scores in percentages. Similar calculations were made as shown in Table 5.*

When the being mathematical criterion was taken into consideration in teacher-level pattern problems posed by preservice teachers, as seen in Table 6, teachers were observed to display the highest performance in using the symbolic form of representation (28.26%) compared to other forms of representation. In terms of the data quality criterion, preservice teachers were found to display the highest performance when using tables as a form of representation (26.26%) while posing teacher-level pattern problems, and they were found to display the lowest performance when using verbal forms of representation (22.73%). In terms of the grammar and expression criterion, teachers were found to display variable performances in posing pattern problems at their own level. They were found to display especially higher performances in posing pattern problems at their own level using verbal forms of representations (31.21%). Preservice teachers performed rather low in using symbolic forms of representations (16.56%). With regards to structure in the forms of representation, grammar knowledge is at the forefront in problems using verbal representations, and symbols are used in problems using symbolic representations.

In terms of the level appropriateness criterion, the highest performance was displayed by preservice teachers when using symbolic (30.46%) forms of representation, and the lowest performance was displayed when using figures as forms of representation (15.89%) while posing pattern problems at their own levels. In terms of the instructions for the problems and amount of data in problems criterion, teachers were found to display the highest performance when using figures as a form of representation (28.29%) and the lowest performance when using a symbolic form of representation (22.93%) while posing pattern problems at their own level. In terms of the possibility of being solved criterion, teachers were found to display the highest performance when using tables as a form of representation (26.85%) and the lowest performance when using a verbal form of representation (22.69%) while posing pattern problems at their own level.

Findings Related to the Second Research Question

Table 7 presents the performances of preservice teachers when posing pattern problems at a student level by using different forms of representation.

Table 7. The performances of preservice teachers in the second part of PPPTMR

	N	$\overline{X}$	Sd
2nd Part	30	35.87	8.50

Table 7 displays the general performance means of preservice teachers in the second part of the PPPTMR as 35.87 out of 48. When the performances of preservice teachers were examined in posing pattern problems at student level by using different forms of representation, it was observed that PT14 obtained the lowest score of 10, and PT26 and PT28 both obtained the highest score of 46. Table 8 displays problem posing performances of PT14, PT26, and PT28 at a student level based on forms of presentation and assessment criteria (also see the examples of PT28 and PT29 preservice teachers' papers in Appendix B).

Table 8. The performances of PT14, PT26, and PT28 at a student level

Criteria	PT14				PT26				PT28			
	Forms of Representations				Forms of Representations				Forms of Representations			
	Figures	Tables	Verbal	Symbolic	Figures	Tables	Verbal	Symbolic	Figures	Tables	Verbal	Symbolic
Being mathematical	0	0	0	0	1	1	2	2	1	1	2	2
Data quality	2	0	0	0	2	2	2	2	2	2	2	2
Grammar and expression	2	0	0	0	2	2	2	2	2	2	2	2
Level appropriateness	2	0	0	0	2	2	2	2	2	2	2	2
Instructions for the problems and amount of data	2	0	0	0	2	2	2	2	2	2	2	2
Possibility of being solved	2	0	0	0	2	2	2	2	2	2	2	2
General evaluation	10	0	0	0	11	11	12	12	11	11	12	12
TOTAL	**10**				**46**				**46**			

Table 8 shows that PT26 and PT28, who received the highest scores on the PPPTMR for posing problems for a student level, were able to pose pattern problems in accordance with all criteria by using verbal and symbolic forms of representation. Also, PT26 and PT28 partially met the being mathematical criterion in the problems posed by using figures and tables as forms of representation. According to Table 8, PT14, who received the lowest score on the PPPTMR when posing problems at a student level, was not able to meet any of the criteria for posing problems using verbal and symbolic forms of representation, and this teacher only met the being mathematical criterion in using figures as form of representation but could not display the same performance for the other criteria.

The scores obtained from the PPPTMR also show that the numbers of preservice teachers who had low and medium problem posing performances at a student level were equal (8), and the number of teachers who had high problem posing performances at a student level was 14. On the other hand, preservice teachers were also assessed for performance when using forms of representation and problem posing criteria at a student level. Table 9 and Table 10 provide these performances.

Table 9. The performances of preservice teachers at the middle school level in terms of criteria related to different forms of representations*

Criteria	Forms of Representations				
	Figures	Tables	Verbal	Symbolic	TOTAL
	%	%	%	%	%
Being mathematical	12.50	14.96	15.41	21.72	15.89
Data quality	17.95	18.90	18.05	22.54	19.24
Grammar and expression	15.71	13.78	17.67	12.70	15.06
Level appropriateness	17.95	17.32	15.79	13.52	16.26
Instructions for the problems and amount of data	18.59	18.90	16.92	16.80	17.84
Possibility of being solved	17.31	16.14	16.17	12.70	15.71
TOTAL	100.00	100.00	100.00	100.00	100.00

** Maximum score was 60, and the table presents scores in percentages. Similar calculations were made as shown in Table 5.*

Table 9 shows that using figures as a form of representation, preservice teachers displayed the best performance in the instructions for the problems and amount of data (18.59%) criterion and the lowest performance in the being mathematical (12.50%) criterion when they posed pattern problems at student levels. Preservice teachers displayed the highest performance when using tables as forms of representations in the criteria data quality (18.90%) and instructions for the problems and amount of data (18.90%) and the lowest performance in the grammar and expression (13.78%) criterion. Table 9 presents that preservice teachers displayed the highest performance on student pattern problems posed by verbal representations in the data quality (18.05%) criterion. However, they had the lowest performance on the being mathematical (15.41%) criterion. It is observed that preservice teachers displayed the highest performance in pattern problems posed by symbolic representations in the data quality (22.54%) criterion, and they presented the lowest performance in the grammar and expression (12.70%) and possibility of being solved (12.70%) criteria. It is observed that preservice teachers' performance in the grammar and expression criterion required for posing pattern problems by using different forms of representations was lower than the performance they displayed for other criteria. Table 9 also displays that in general, preservice teachers performed better in the data quality criterion compared to other criteria in posing pattern problems at a student level.

Table 10. The performances of preservice teachers at a middle school level in terms of criteria related to problem posing criteria*

Criteria	Percent	Forms of Representations				
		Figures	Tables	Verbal	Symbolic	TOTAL
Being mathematical	%	22.81	22.22	23.98	30.99	100.00
Data quality	%	27.05	23.19	23.19	26.57	100.00
Grammar and expression	%	30.25	21.60	29.01	19.14	100.00
Level appropriateness	%	32.00	25.14	24.00	18.86	100.00
Instructions for the problems and amount of data	%	30.21	25.00	23.44	21.35	100.00
Possibility of being solved	%	31.95	24.26	25.44	18.34	100.00
TOTAL	%	29.00	23.61	24.72	22.68	100.00

** Maximum score was 60, and the table presents scores in percentages. Similar calculations were made as shown in Table 5.*

When the being mathematical criterion was taken into consideration in student-level pattern problems posed by preservice teachers in Table 10, preservice teachers were observed to display the highest performance in using a symbolic form of representation (30.99%) compared to other forms of representation. In terms of the data quality criterion, preservice teachers were found to display the highest performance when using figures as a form of representation (27.05%) in posing student-level pattern problems, and they were found to display the lowest performance when using tables (23.19%) and verbal (23.19%) forms of representation. In terms of the grammar and expression, level appropriateness, instructions for the problems and amount of data and possibility of being solved criteria, preservice teachers were found to display the highest performance when using figures as a form of representation and the lowest performance when using symbolic forms of representation.

In addition to research findings, researchers categorized preservice teachers' performances on both parts of the PPPTMR. Accordingly, preservice teachers PT2, PT8, PT9, PT12, and PT28 presented lower performances when posing problems at own-level by using different forms of representations, while they had higher performances at the student level. Preservice teachers PT7, PT14, PT19, and PT21 presented higher performances when posing problems at their own levels by using different forms of representations, while they had lower performance at the student level. Preservice teachers PT17 and PT19 presented low performances at both their own levels and the student level. Preservice teachers PT5, PT10, PT11, PT13, PT18, PT23, PT25, and PT26 presented high performance at both their own levels and the student level. Additionally, PT28 was one of the teachers who presented the highest performance in posing problems at the teacher level by using different forms of representations while presenting the lowest performance at the student level.

Discussion

This study aimed to determine preservice teachers' performances when posing pattern problems at their own level and at a student level by using different forms of representations (figures, tables, verbal, and symbolic). Preservice teachers were asked to pose four different problems using "figures," "tables," "verbal," and "symbolic" forms of representations provided in the two-part Pattern Problem Posing Test with Multiple Representation. In this research's findings identified that preservice teachers' performances when posing pattern problems by using different forms of representations differed at their own levels and at a student level. Preservice teachers' performances related to pattern problems they posed at their own levels using different forms of representation are tables, figures, symbolic and verbal, respectively. According to this result, preservice teachers displayed the lowest performance at their own levels in verbal forms of representation. The reason for this, preservice teachers may have been able to see and organize the operational knowledge and numerical connections more easily in figures, tables, and symbolic forms of representations, but may not have fully reflected this situation on the problem posing in verbal forms of representation. In their study, Dündar & Yılmaz (2015) reported that preservice teachers displayed their lowest performances in a verbal form of representation while solving integral problems. Starting from this, it can be thought that preservice teachers display low performances in posing and solving verbal problems.

Preservice teachers' performances related to pattern problems they posed at student levels using different forms of representation are figures, verbal, tables and symbolic, respectively. According to this result, mathematics preservice teachers displayed the lowest performance in posing pattern problems at a student level by using symbolic forms of representation. In MoNE (2013), it is encountered some explanations of the acquisition related to the pattern to include daily life situations, shape patterns and even be associated with using tables. For this reason, preservice teachers may have taken this into account so the highest performance in figures form of representation, then may have performed better in verbal and tables forms of representation. This may be the reason why preservice teachers showed their lowest performances at student level in symbolic form of representation. This situation overlaps with the findings of Çakmak, Baş & Bekdemir (2014) on the pattern of the study that the performances of the preservice teachers on verbal situations are higher than those of the symbolic ones. The researchers found that preservice teachers' use of verbal language in finding patterns was higher compared to their use of symbolic language. Also, this result supports the findings of Işık and Kar's (2012) study on preservice teachers. In Işık and Kar's study, preservice teachers posed richer problems in verbal representations compared to symbolic representations when posed problems appropriate to the data set.

When pattern problems posed by preservice teachers at their own level by using different forms of representations were taken into account, preservice teachers were found to display the highest performance in the possibility of being solved criterion. In this sense, it is seen that preservice teachers were more inclined to pose problems that could be solved when they posed pattern problems at their own levels by using different forms of representation. This result is consistent with Korkmaz and Gür's (2006) and Işık and Kar's (2012) results, which reported that preservice teachers tended to pose problems that could be solved thorough simple operations. Preservice teachers' performances in terms of other criteria were found to be instructions for the problems and amount of data, data quality, being mathematical, grammar and expression, and level appropriateness. Although the preservice teachers were studying their last year and had already studied all the instructional courses in their field, they displayed the lowest performance in the level appropriateness criterion when they posed pattern problems at their own levels. On the other hand, preservice teachers' pattern problems posed at student levels using different forms of representation were examined and the highest performance was observed not in the possibility of being solved criterion but in data quality criterion. The reason for this, it can be considered that the data in the problem is enriched with logical and mathematical sequences that are more meaningful. Indeed, it has been found that in the examined papers, they descend to the level of the students and place in real life pattern problems which are more understandable to them. A similar result has been reached in Crespo and Sinclair's (2008) study in that the quality of the problem provides richer understandings and Lavy and Shriki's (2007) study in that the appreciation of the richness underlines mathematical problems.

Preservice teachers were found to effectively use the instructions for the problems and amount of data criterion in pattern problems at both levels by using figures as a form of representation. They displayed low performances in the level appropriateness criterion while posing pattern problems at their levels using figures as a form of representation and in the being mathematical criterion while posing pattern problems at a student level. Also, preservice teachers displayed the lowest performance in the grammar and expression criterion while posing pattern problems at both levels using tables as a form of representation. On the other hand, they displayed the highest performance in the possibility of being solved criterion while posing pattern problems at their own levels using tables as a form of representation and in data quality and instructions for the problems and amount of data criteria while posing pattern problems at student level.

In the study it was determined that 17 of the 30 preservice teachers showed high performance when posing pattern problems using different forms of representation at their own level. When these numerical values are converted into percentiles, it can be said that 56.6% of the participants displayed high performances in posing pattern problems using different forms of representation at their own level. This result is consistent with the results of Yıldız's (2014) study that reported similar success in preservice teachers' problem posing skills. On the other hand, 14 of the 30 preservice teachers showed high performance when posing pattern problems using different forms of representation at a student level. Therefore 46.6% of the preservice teachers were found to have high performance in posing pattern problems using different forms of representation at a student level. These results point to the fact that preservice teachers' performances when posing pattern problems using different forms of representation at their own level are higher than their performances when posing pattern problems using different forms of representation at a student level. As a reason for their high performance at their own level, we can say that the preservice teachers may have a wide range of thinking by gaining different perspectives from the courses they have taken and are taking in their education. This view can also be supported by the study of Cai (2003). Because preservice teachers can see a pattern in a trigonometric function or a derivative and reflect these structures in their problems, this case may also be an indicator of a high performance. On the other hand, going down to the level of the students and posing kinds of problem they can understand is a situation that requires attention. So, it may be a reason behind the relatively low performance of preservice teachers at the student level. Another result of the study is that there are only 8 preservice teachers in total, which demonstrate high performance in terms of problem posing in both parts of PPPTMR. Perhaps this number can be a sign that preservice teachers have difficulty in problem posing in general terms. In total there are 9 preservice teachers, in one of the part of PPPTMR showed high and the other one low performance. From this perspective, it can be assumed that preservice teachers are more likely to pose problems in one part and this situation is reversed in the other one.

Recommendations

Based on the finding that preservice teachers had better performance when posing pattern problems using different forms of representation at their own levels, it can be suggested that preservice teachers should be guided to pose problems at student levels in mathematics teaching courses. Activities to develop problem posing skills can be undertaken by discussing these problems with the course instructor and other preservice teachers. Changes in preservice teachers' performances when posing problems can be studied after they are provided with training on problem posing and the effectiveness of these types of training can be studied. Based on the result that preservice teachers performed differently with different forms of representations, the reasons for these differences can be examined through in-depth interviews with preservice teachers.

This study examined preservice teachers' performances when posing problems related to patterns. In-service teachers' performances when posing problems related to patterns can also be studied, and similarities and/or differences between these two groups can be presented to identify the factors that differentiate training and practice so that necessary adjustment and reorganizations can be undertaken.

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Author Information

Yasemin Yilmaz
Abant Izzet Baysal University
Institute of Educational Sciences
Contact e-mail: *yaseminyilmaz@ibu.edu.tr*

Soner Durmus
Abant Izzet Baysal University
Faculty of Education

Hakan Yaman
Abant Izzet Baysal University
Faculty of Education

APPENDIX A

Pattern Problem Posing Test with Multiple Representations (PPPTMR)

FIRST PART

You are undergraduate students. Answer the following question. For doing this, consider your level as a teacher, educator, mathematics educator.

Question 1. Pose problems that are appropriate for each forms of representation related to pattern. These forms of representation should be considered as figures (graphics, pictures, drawings, schemas, diagrams), tables, verbal (real life problems, stories), symbolic (equations, equality, formula, relation).

1.1. Figures

1.2. Tables

1.3. Verbal

1.4. Symbolic

SECOND PART

Question 2. Pose problems according to the following different forms of representation in order to be gained the pattern concept and the skill of patterning processes for 5-8th grade students.

2.1. Figures

2.2. Tables

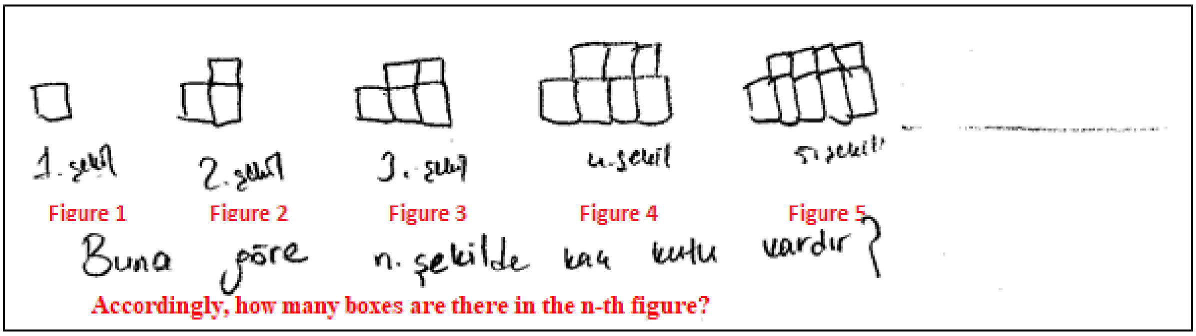
2.3. Verbal

2.4. Symbolic

APPENDIX B

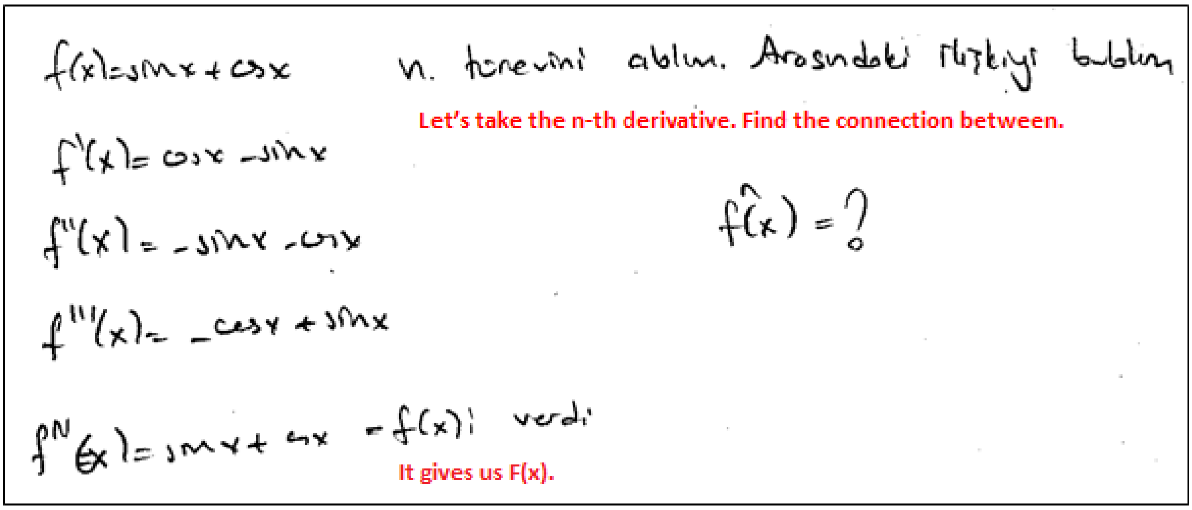
Examples of Preservice Teachers' (PT4, PT23, PT28, PT29) Papers

PT4



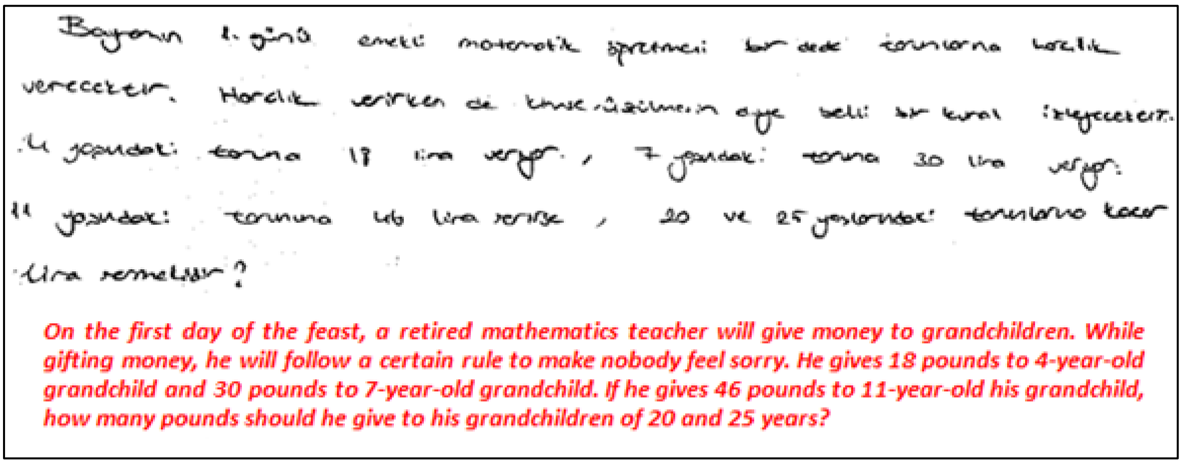
The average of preservice teachers in the first part of PPPTMR is 37.03. PT4 preservice teacher's performance (38) nearly represent the average of class. PT4's scores on which the problem has been posed in the form of symbolic representation are given below in paranthesis:
Being mathematical (1), data quality (2), grammar and expression (2), level appropriateness (0), instructions for the problems and amount of data (0), possibility of being solved (2).

PT23



The pattern problem that PT23 pose in the form of symbolic representation in the first part of PPPTMR is given above. PT23's scores on which the problem has been posed in the form of symbolic representation are given below in paranthesis:
Being mathematical (2), data quality (2), grammar and expression (1), level appropriateness (2), instructions for the problems and amount of data (2), possibility of being solved (2).

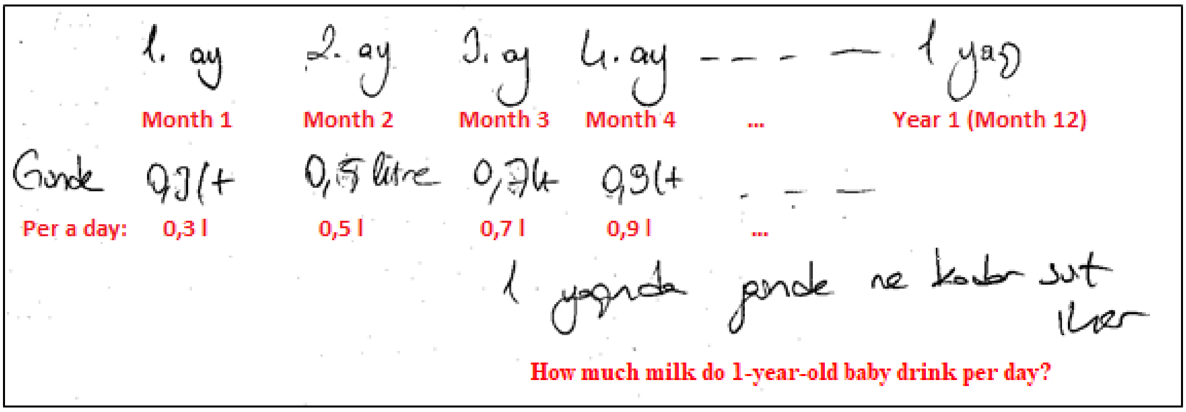
PT28



The pattern problem that PT28 pose in the form of verbal representation in the second part of PPPTMR is given above. PT28's scores on which the problem has been posed in the form of verbal representation are given below in paranthesis:

Being mathematical (2), data quality (2), grammar and expression (2), level appropriateness (2), instructions for the problems and amount of data (2), possibility of being solved (2).

PT29



The average of preservice teachers in the second part of PPPTMR is 35.87. PT29 preservice teacher's performance (35) nearly represent the average of class. PT29's scores on which the problem has been posed in the form of tables representation are given below in paranthesis:

Being mathematical (2), data quality (2), grammar and expression (2), level appropriateness (2), instructions for the problems and amount of data (2), possibility of being solved (2).