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Abstract

In this article, we report on the findings of a study investigating the effects of an intervention designed to provide students in Grades 3–5 with opportunities to work with dynamic and static models of angles in a dynamic geometry environment. We utilized the microgenetic method in this cross-sectional study to observe and document changes in participating students' conceptions of angle. Our results indicate that the intervention had a positive effect on all of the students' terminology, third graders' estimations of angle measures, and third and fourth graders' drawings of obtuse angles. Sliders to vary the ray length, angle orientation, angle location, and angle openness had a positive effect on half of the participants in our study who struggled with misconceptions related to what effects the measure of an angle.

Introduction

Angle measurement is an important topic in school mathematics because of its significance in high school and undergraduate mathematics content, such as trigonometric functions. However, the study of angle measurement, and students' subsequent development of angle concepts, is more complex than memorizing definitions and using protractors. Multiple studies have identified several student misconceptions related to angle measurement. For example, research indicates that students think that angle size is based on the ray length (Clements, 2003; Foxman & Ruddock, 1984; Keiser, 2004) or ray orientation (Mitchelmore, 1998; Noss, 1987). Fyhn (2008) and Foxman and Ruddock (1984) required their participants to select the largest angle and then smallest angle out of a provided set of angles that included visual distractors, such as variations in ray length and orientation. Results based on these assessments indicate that middle and secondary school students also struggle with these visual distractors.

To provide students with opportunities to challenge their misconceptions, some researchers have turned to technology. Clements, Battista, Sarama, and Swaminathan (1996), Clements and Burns (2000), Noss (1987), and others utilized LOGO with some success. In Noss' (1987) study, Grades 3, 4, and 5 students learned geometrical concepts; one group learned through LOGO programming and the other without the use of LOGO. Group differences were assessed, in part, using a paper-pencil test. One item with three parts required students to determine which of two angles was bigger or if they were the same size. Noss found no significant difference between the two groups when the angles were oriented differently. However, when the angles had rays of different lengths, he did notice a significant effect in favor of the experimental group.

Clements et al. (1996) investigated third graders' understanding of turn and geometric paths in a modified LOGO environment. They reported that the immediate feedback provided by the computer allowed the students to compare and contrast different turn commands and thus reflect on their ideas about turn and turn measurement. These researchers also found that the students demonstrated some improvement in assigning numbers to turns by using benchmarks. Clements et al. asserted that in a LOGO environment, the reaction from the turtle to a turn command is less apparent or clear to students than the reaction from the turtle to a forward or backward command and posited that this was due to how turns and the result of turns are represented in that environment: "The turning motion itself usually does not leave a trace" (Clements, Battista, Sarama, & Swaminathan, 1996, p. 332). Despite LOGO's relative success in supporting elementary students' learning about geometrical and measurement concepts (including angle measurement concepts), it is becoming more obsolete with the emergence of dynamic geometry environments, such as Geometer's Sketchpad and GeoGebra.

For example, Crompton (2015) utilized an add-on program of Sketchpad Explorer (2012). Fourth grade students identified angles in the playground, took pictures of those angles using mobile devices (i.e., iPads), and then marked and measured those angles with the program's dynamic protractor. Crompton (2015) asserted that "Changes were made to the instructional plans to have students label the benchmark to support students in internalizing these benchmark measures" (p. 27), however, she did not discuss the effects of these modifications.

To extend the current literature on angle measurement, we designed an intervention. This intervention was enacted in a dynamic geometry environment (DGE) utilizing the computer software, Geometer's Sketchpad, to provide students with opportunities to work with movable angle situations as well as reflect on dynamic (the motion of an angle sweeping open) and static (the resulting image of an angle after sweeping open) angle models. Although other researchers have considered the sequencing of static images to be dynamic angle situations (e.g., Clements et al., 1996; Devichi & Munier, 2013), we argue that to be truly dynamic, the sequencing of these static images needs to be continuous. We also wanted to extend the literature on possible student misconceptions related to angle measure based upon ray length or rotation of the angle (Devichi & Munier, 2013). We posed the research question: In what ways does interacting with dynamic and static angle models affect students' understanding of angle measurement in Grades 3, 4, and 5?

Theoretical Perspective

We approached this research study from a theoretical perspective of constructivism. According to Simon (1995),

> Constructivism derives from a philosophical position that we as human beings have no access to an objective reality, that is, a reality independent of our way of knowing it. Rather we construct our knowledge of our world from our perceptions and experiences, which are themselves mediated through our previous knowledge. Learning is the process by which human beings adapt to their experiential world. (p. 115)

Hence, we worked under the premise that we as teacher-researchers cannot transfer our knowledge to the children. Instead, children must make sense of their own reality, which includes their interactions with the teacher-researchers on movable angle situations within a DGE. How a student makes sense of her reality is mediated through her perception and interpretation of her individual prior experiences, which includes in and out of school learning.

Method

Setting

This cross-sectional study took place during the fall and winter of 2014 with 18 students from three different grade groups: Grades 3, 4, and 5 (ages 9–12). In each grade group, six students were selected from two classes in a Midwestern, suburban public school in the United States. Participants were selected based on parental consent, student assent, and teacher input regarding their mathematical abilities in order to get a group of students per grade with a range of ability. Although Grades 3, 4, and 5 curricula at this school include angle measurement topics, none of the teachers had covered angle measurement prior to the completion of this study.

Microgenetic Method

Because we wanted to give a detailed account of the effects of the intervention on students' understanding of angle measurement, we utilized the microgenetic method (Siegler & Svetina, 2006) to observe and document that change. The microgenetic method has three main tenets:

> (1) observations span the whole period of rapidly changing competence; (2) the density of observation within this period is high, relative to the rate of change; and (3) observations of changing performance are analyzed intensively to indicate the processes that give rise to them. (Siegler & Svetina, 2006, p. 1000)

We used the microgenetic method to inform our design: In an attempt to increase the probability that we would be present when the students exhibited a change in their understanding, we designed an intervention composed of a sequence of individual trials that gave students the opportunity to reflect on dynamic and static angle models. Consistent with the microgenetic method, observations of the highly concentrated trials were dense and spanned the observed period of change. We defined a trial to be a task-intervention pair (Siegler & Crowley, 1991), constructing an angle of a specified measure (task) and watching a ray sweep from the initial ray of the angle to the terminal ray of the angle the student had been instructed to create after which the measure of the angle constructed was displayed (intervention).

Procedures

Each student was interviewed individually three times by one of the three authors using a structured interview protocol. Each interview was recorded using screen-capturing software, Screencast-o-matic, to record the computer screen as well as audio during the interview. During the first interview, the student took a written initial survey. This survey required the student to give a definition of angle, estimate the measure of a given angle, and construct an angle. There were also six multiple choice items in which the student was asked to select one out of three angles that (a) had a specified measure, (b) was the angle with the largest measure, or (c) was the angle with the smallest measure. These items were similar to those used in Fyhn (2008), Foxman and Ruddock (1984), and Noss (1987) and included visual distractors.

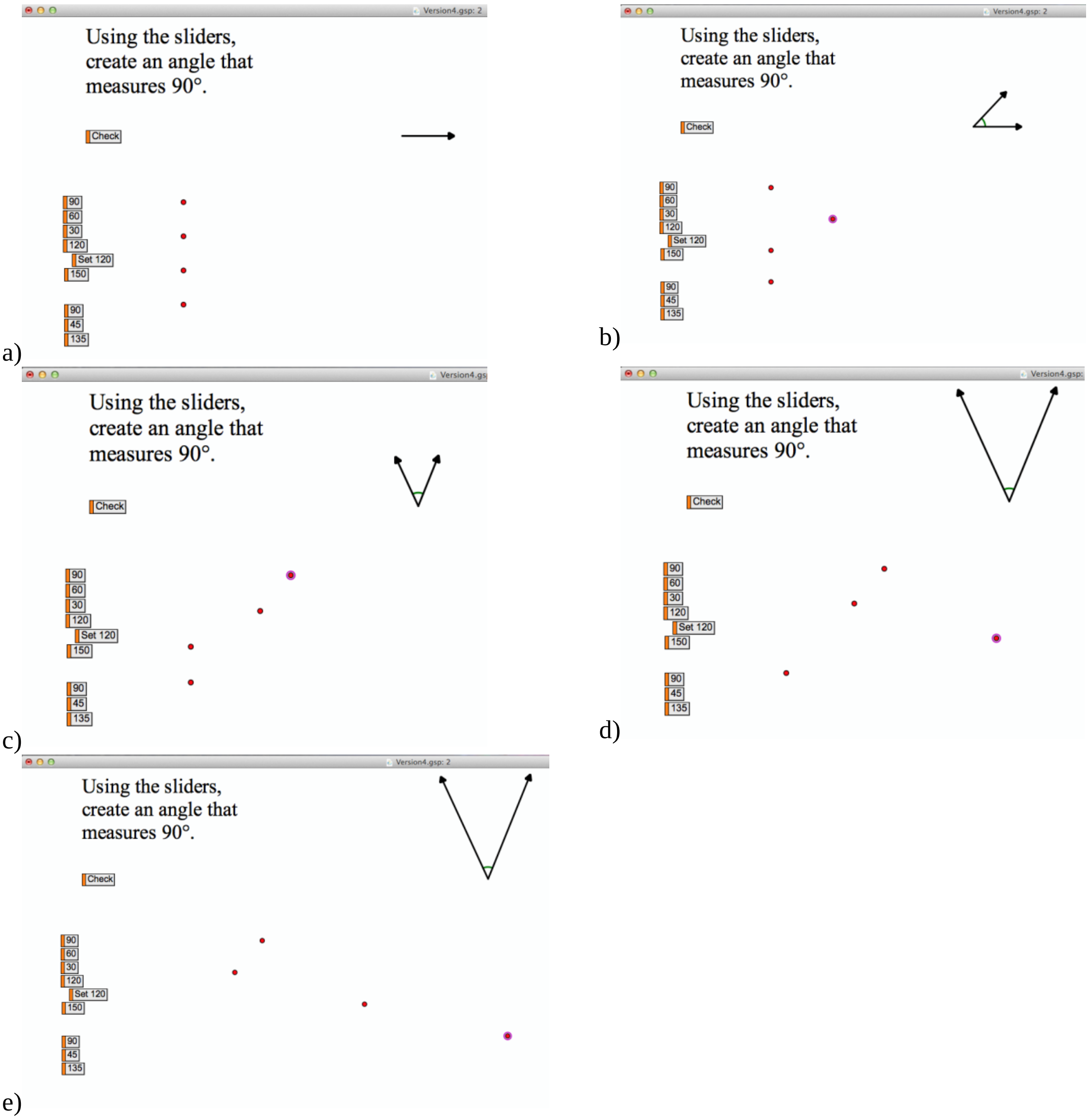

Figure 1. Screen shots a) of sliders in initial positions, b) after one slider opened the angle, c) after one rotated the image of the angle counterclockwise, d) after one lenghtened the rays, and e) after one translated the image right

Following the survey, the student was guided through a tutorial on how to use the sliders in a movable angle situation within Geometer's Sketchpad to create angles. This platform afforded us the ability to display the initial ray, the final ray, as well as the transition from the initial to final ray (cf. Clements et al., 1996), which provided the students with opportunities to work with static and dynamic angle models. Additionally, the DGE microworld did not require the user to provide textual inputs (e.g., "rt 45" in LOGO). Instead, the students in this study clicked and dragged sliders. Each of the sliders had a different effect, and the order of the sliders varied. One slider opened and closed the angle, one translated the image left and right, one rotated the image of the angle, and one lengthened and shorted the rays (see Figure 1 for screenshots a) of sliders in initial positions, b) after one slider opened the angle, c) after one rotated the image of the angle counterclockwise, d) after one lengthened the rays, and e) after one translated the image right.). After the tutorial, the student was asked a sequence of questions about the effects of the four sliders. Specifically, "What will happen to the measure of this angle if I move this [first] slider? Will the measure of the new angle be larger than this one, smaller than this one, or the same as this one? Why do you think that?" The researcher then recorded the student's response, moved the slider to the initial position, and repeated this question for the second, third, and fourth slider.

During the second interview, the student first went through eight trials. In the middle of the fourth trial, after the student constructed the angle, they were asked the same sequence of questions about the effects of the four sliders, but this time, the researcher provided the student with an opportunity to check their responses. During the check, the research clicked a check button and benchmark rays appeared at each 30-degree interval until the ray stopped at the terminal side of the angle. The measure of the angle the student constructed was also briefly displayed, providing students with feedback that the measure of the angle did not change. During the third and final interview, the student went through nine trials. Then at the end of the interview, the student was asked the sequence of questions about the effects of the four sliders a third time (but without an opportunity to check), and the students were given a written final survey, which had the same items as those on the initial survey.

We provided the students with three opportunities to construct 90-degree angles, twice during Interview 2 (Trials 1 and 6) and once during Interview 3 (Trial 9). The purpose for this was twofold. The first was educational in nature: We wanted to reinforce the importance of a 90-degree angle as a benchmark not only as the boundary between acute and obtuse but also as a quarter of a turn around a circle and as an angle composed of three 30-degree angles. The second purpose was practical in nature in that we wanted repeated trials of constructing the same angle to identify changes in precision and accuracy. We also asked the students to construct other angles that measured to be multiples of 30 (i.e., 30, 60, 120, 150) during both Interviews 2 and 3 (see Figure 2) for similar educational and practical reasons. (Note that for all figures that follow, the trials with bench mark rays of 30 degrees appearing as the terminal ray sweeps are denoted with *.)

	Interview 2								Interview 3								
Trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Desired Angle	90	60*	30	120*	150	90*	45	135*	90	80*	30	40*	60	70*	120	110*	150

Figure 2. Order of the desired angle per trial

For the construction of all 17 angles (task component of the 17 trials) during Interviews 2 and 3, the student started with one initial horizontal ray and four sliders. For each angle, the interviewer told the student to use the sliders to create an angle of a specified measure. When the student indicated that he or she was ready to check, the check button was clicked, and a ray swept from the initial ray of the angle to the terminal ray of the angle the student had been instructed to create. Based on Clements et al.'s (1996) conjecture that students may struggle with angle as turn because the turning motion does not usually leave a trace, the timing was manipulated so that the ray left a trace, which faded with time, as it swept across the screen.

By allowing students to construct angles with the sliders, we provided the students with opportunities to work with dynamic angles. The purpose of the sweeping (rotating) ray was to promote the idea of angle as measured with reference to a circle. The ray sweeping from the initial to terminal ray, as well as the display of benchmarks every 30 degrees, provided immediate feedback to the students regarding their constructed angles (cf. Clements et al., 1996). The four sliders worked in tandem with this immediate feedback to offer students the opportunity to vary the ray length, angle orientation, angle location, and angle openness and then to determine how each affected the angle measure. The purpose of the additional sliders was to challenge the misconceptions that angle measure is based on ray length or orientation.

Data Analysis

We first analyzed students' responses on the written initial and final survey items. To compare student responses, we (the authors) independently read through all of the surveys. Then, for the first item on the survey (i.e., what is an angle), we sorted the responses into categories using the constant comparative method (Glaser & Strauss, 1967). After this initial sort, we met to discuss the themes that emerged from the sorting process. We also ordered the responses in terms of what we considered to be increasing in sophistication. For the second and third items on the survey, we compared the students' estimates to the desired, correct response and described similarities and differences. The remaining six items were multiple choice, and we scored these as correct or incorrect.

After our analysis of the survey data, we performed a trial-by-trial comparison. The microgentic method affords us the ability to investigate not only overall change, but also to study what was happening before, during, and after that change. By performing a trial-by-trial comparison, we were able to make sense of these changes and examine what factors may have given to them. In this article, we discuss our findings from our analysis of slider use per trial. For example, on Trial 1 to create a 90-degree angle, Octavia_5 (subscripts are used to denote grade) first used Slider 1, which rotated the initial ray from horizontal pointing right to vertical pointing up. Second, Octavia_5 used slider 2 to open the angle to approximately a right angle. Next, she used Slider 3 to increase and decrease the ray lengths slightly (but did not return the slider to the initial position) and then said she was ready to click the check button. We coded this as three sliders used (the fourth slider was not moved), three slider moves (no sliders were repeated), and three sliders active (no sliders were returned to the start position). In contrast, on the same trial, Molly_4 first used Slider 1 to rotate the initial ray approximately 360 degrees before she returned the slider to its initial position. She said she "needed to use another one [slider]." Second, she used Slider 2 to open the angle to approximately a right angle. Third, Molly_4 used Slider 3 to increase and decrease the ray lengths (but did not return the slider to the initial position). Next, Molly_4 used Slider 4, which moved the angle right and left. As she moved this fourth slider, she stated, "that one just moves it, it doesn't really do anything." Then she moved Slider 4 back to its initial position and indicated that she was ready to check the measure of the angle she constructed. We coded this as four sliders used (all four sliders were used at some point), six slider moves (Slider 1 moved, Slider 1 moved back to initial position, Slider 2 moved, Slider 3 moved, Slider 4 moved, Slider 4 moved back to initial position), and two sliders active (only two sliders were active during the check because the other two sliders were returned to the start position).

In the sections that follow, we provide numeric results to supplement the descriptions of students' drawings, constructions, and estimates of angles. However, we did not conduct statistical tests of significance on these numeric results due to our sample size and the nature of the study as designed. Instead, the descriptive findings reported in the next section provide evidence of how an intervention designed in a DGE has the potential to promote growth and understanding.

Results and Discussion

In order to determine the ways in which the intervention affected students' understanding of angle measurement, we looked at comparisons of static images on the initial and final surveys as well as how or if the students' abilities to create angles of a specified measure in the dynamic environment changed over the course of the trials. In this section, we discuss findings for the students by grade level. We first describe comparisons between the initial and final written surveys.

Written Survey Comparisons

What is an Angle?

On the initial assessment, the third graders did not have consistent responses, but half of the group displayed an increase in understanding of angle through their responses to this question. Three third grade students who showed improvement drew a geometric figure, either an angle or a line segment, on the initial survey but did not answer the question with a written description of what an angle is. Following the completion of the trials, these three third graders provided a meaningful description of what an angle is on the final survey. For instance, one stated, "It is 2 lines that meet together to make like a corner" whereas another stated, "a angle is 2 lins moving apart." Our design decision was to include multiple sliders that varied the angle orientation and ray length in order to challenge the misconceptions that angle measure is affected by or defined by these characteristics. We intended for students to learn through trial and error that only one slider had an effect on the measure of the angle—the slider that opened the angle. Hence, we interpret the second student's reference to the motion of "lines [rays] moving apart" as evidence that his interaction with dynamic angle models did, indeed, influence his description of what an angle is.

Similar to the third graders, half of the fourth graders drew a geometric figure without writing any words on the initial survey. Two of these three fourth graders showed little growth from the initial figures given, but one of those students described an angle as "a line that slants" and drew an angle with an arrow pointing to the vertex. The other three fourth graders displayed some conception of angle on the initial and the final surveys. They were able to write a description of an angle as turn or the distance between two lines. The fifth graders had focused instruction on angles in fourth grade, thus it was not surprising that all of these students wrote a description of what an angle is. Three of the six fifth graders did not change their response from the initial to the final survey. Interestingly, the other half of the fifth graders improved the vocabulary they used in their descriptions of what an angle is, shifting from phrases such as "an angle is a line with a vertex (I'm pretty sure) in the middle" to "an angle are two rays that meet in the middle (vertex I'm pretty sure)." Even though the students' descriptions of angle were not mathematical definitions, we saw improvements across the grade levels in how the students described an angle following the intervention. We find these changes particularly interesting when we consider that there was no instruction on terminology during the study.

Draw an Angle

On Item 2 of the initial and final surveys, students were asked to draw a 115-degree angle without using a protractor. Four of the third graders left this item blank on the initial survey, but only one of those students left the item blank on the final survey. In addition, none of the six third graders drew an obtuse angle on the initial survey, but four of the six third graders drew an angle that measured greater than or equal to 90 degrees and less than 180 degrees on the final survey. On this written task, the third graders showed progress with the mean absolute error improving from 64 degrees to 28 degrees.

The fourth grade students only had slightly better performance than the third graders because only two of the six fourth graders drew an obtuse angle on this item on the initial survey. Yet these students also displayed improvement as four of the six fourth graders drew an obtuse angle on the final survey. Without the aid of a protractor for the task, the fourth graders' mean absolute error improved from 49 degrees to 8 degrees.

The fifth graders displayed no growth on this task following the intervention. All of the fifth graders drew obtuse angles on both the initial and final surveys, and there was no difference in the mean absolute error (17 degrees) on the initial and final. In summary, 16 of the 18 students provided more accurate sketches of an angle of a specified measure on the final in comparison to the initial survey. These results indicate that the intervention helped the third and fourth grade students create more accurate sketches of obtuse angles of a specified measure.

Angle Estimation

Item 3 stated, "make a good guess" about the measure of the angle shown. Because this question asked the students for an estimate, our analysis was a bit different. First, we determined how many of the students responded with either 90 degrees or an acute angle measure because the angle was 75 degrees. Next, we determined how many of the students in each grade level were within 10 degrees of the correct angle measure.

On the initial survey, two of the third graders left this item blank, two answered with length measurements (i.e., 15 cm and 3 in), and two answered with angle measures (i.e., 15 degrees and 50 degrees). After the intervention, all of the third graders answered with angle measures (i.e., 110 degrees, 90 degrees, 90 degrees, 90 degrees, 80 degrees, and 60 degrees). Although we interpret these responses to indicate an improvement from the initial survey, only one of the six third grader students estimated a measure (i.e., 80 degrees) that was within 10 degrees of the correct measure of 75 degrees.

None of the fourth graders estimated an angle measure of 90 degrees on either survey. Initially, half of these students estimated the measure to be a value that corresponded to an acute angle. Following the intervention, four out of the six fourth graders made an estimated that was less than 90 degrees. In contrast to the third graders, half of the fourth grade students estimated an angle measure that was within 10 degrees of the correct measure on the final survey.

All six of the fifth grade students provided acute angle measures on both the initial and final surveys, yet we still note an improvement. Prior to the computer trials, four of the six fifth graders estimated an angle measure that was within 15 degrees of the given 75-degree angle, with a mean absolute error of 11.7 degrees. Following the intervention, all of the fifth grade students predicted an angle measure within 15 degrees of the correct measure, with a mean absolute error of 9.2 degrees. Although none of the fifth grade students reported a benchmark angle measure used in the intervention (i.e., a multiple of 30) on the initial survey, three of the students reported estimates of 60 degrees on the final survey. We infer that the students' proclivity to use the benchmark angle of 60 degrees on the final survey was a result of their interaction with the intervention.

Multiple Choice

The remaining items on the survey required the students to select one out of three angles that (a) had a specified measure, (b) was the largest angle, or (c) was the smallest angle. The multiple-choice questions, in which we asked the students to select an angle of a specified measure from a group of three angles, provided little information. On all but one question, at least two-thirds of the students at each grade level correctly identified the angle specified on either the initial or the final survey. Thus, there was very little change on these four questions on the surveys.

On the last two items of the survey, students were asked to select one out of three angles as the largest or the smallest. On Item 8, students were asked to select one out of three angles as the angle with the largest angle measure, but the largest angle (i.e., angle on right) had smaller rays than one of the options but longer rays than the other option (see Figure 3). On Item 9, students were asked to select one of three angles as the angle with the smallest angle measure, but the smallest angle (i.e., angle in the middle) had longer rays than the other two options (see Figure 4).

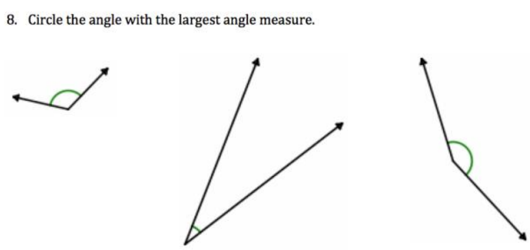

Figure 3. Item 8 on the surveys

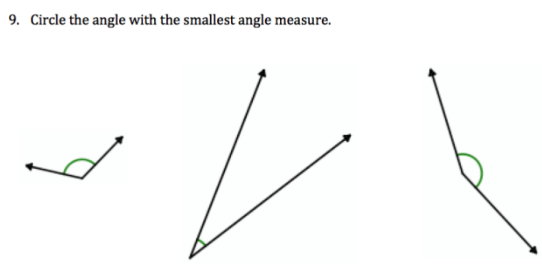

Figure 4. Item 9 on the surveys

If a student selected the angle with the shortest rays on Item 8 as the smallest angle as well as the angle with the longest rays on Item 9 as the largest angle, then we would have taken this as evidence that the student had the ray length misconception. However, only one fourth grade student repeated the error (i.e., on Items 8 and 9), and this occurred on the final survey. We identified 15 incorrect responses (out of 72 total responses) on Items 8 and 9, and 14 of those 15 incorrect responses were attributable to the ray length error (see Table 1). (One third grade student selected the angle with the largest angle measure as the angle with the smallest angle measure on Item 9 of the initial survey.) In addition, more students were distracted by ray length on Item 9 than on Item 8, perhaps because the ray length distractor was not as straightforward on Item 8 as it was on Item 9.

Table 1. Number of incorrect responses on survey items

	Grade 3		Grade 4		Grade 5		Total		
	Item 8	Item 9	Item 8	Item 9	Item 8	Item 9	Item 8	Item 9	Total
Initial	1	3	1	2	0	1	2	6	8
Final	0	3	1	2	0	0	1	5	6
Total	1	6	2	4	0	1	3	11	14

Ten of the 18 participants were distracted by ray length and selected the wrong angle on either Item 8 or Item 9 (or both) on either the initial or final survey (or both), but the susceptibility to this distraction was not evenly distributed across the three grades. Although five (of six) third graders and four (of six) fourth graders were distracted by ray length, only one (of six) fifth graders was. These results indicate that most of our younger participants were distracted by ray length on Items 8 and 9, which is somewhat consistent with the literature (cf. Clements, 2003; Foxman & Ruddock, 1984; Keiser, 2004; Mitchelmore, 1998; Noss, 1987).

Trial Comparisons

Next, we analyzed the students' responses to the sequence of questions about the effects of three of the sliders. As discussed in the Method section, this sequence of questions was posed three times—at the end of Interview 1 (after the initial survey, before trials, and without a check), during the middle of Interview 2 (in between trials and with a check), and during Interview 3 (after all trials, without a check, and before the final survey). During the analysis stage, we noticed trends for groups of students that did not seem to be restricted to grade. Thus we grouped the participants based on their performance on the questions designed to elicit information about their understanding of the effects of the sliders on the angle measure.

Table 2. Responses to tasks related to the effects of angle orientation, ray length, and angle location on angle measure

Group	Name	Interview 1					Interview 2			Interview 3				
		Item8	Item9	O	RL	T	RL	T	O	O	RL	T	Item8	Item9
1	Molly_4	1	1	1	1	1	1	1	1	1	1	1	1	1
1	Peter_4	1	1	1	1	1	1	1	1	1	1	1	1	1
1	Lester_5	1	1	1	1	1	1	1	1	1	1	1	1	1
1	Octavia_5	1	1	1	1	1	1	1	1	1	1	1	1	1
1	Frank_5	1	1	1	1	1	1	1	1	1	1	1	1	1
1	Nina_5	1	1	1	1	1	1	1	1	1	1	1	1	1
2	Alaina_3	1	0	0	0	1	1	1	0	1	1	1	1	0
2	Lauren_3	1	0	1	0	1	0	1	1	1	1	1	1	0
2	Eric_3	1	1	0.5	0	1	0	1	1	1	0.5	1	1	1
2	Zander_3	1	1	1	1	1	1	1	0.5	0.5	0	1	1	0
2	Bob_4	0	1	1	0	1	0	1	1	1	1	1	1	0
2	Oscar_4	1	1	1	0	1	0.5	1	0	1	0	1	0	0
3	Brad_3	0	0	1	0	1	1	1	1	1	1	1	1	1
3	Patrice_3	1	0	1	0	1	0	1	1	1	1	1	1	1
3	Alan_4	1	0	1	0	1	0	1	1	1	1	1	1	1
3	Gina_4	1	0	1	0.5	1	1	1	1	1	1	1	1	1
3	Zane_5	1	0	1	0	1	1	1	1	1	1	1	1	1
3	Amber_5	1	1	0	1	1	1	1	1	1	1	1	1	1

Note: O = Angle Orientation (due to rotation), RL = Ray Length, T = Angle Location (due to translation), 0 = Incorrect, 1 = Correct, 0.5 = Changed answer from incorrect to correct during explanation.

Six of the 18 participants (i.e., Molly_4, Peter_4, Lester_5, Octavia_5, Frank_5, and Nina_5) correctly answered the sequence of three questions all three times (see Group 1 in Table 2). These students also correctly answered Items 8 and 9 on both the initial and final survey. We take this as evidence that either these students had already begun to confront the misconception that angle size was affected by ray length or orientation prior to Trial 1 for these students or they never had it.

Of the 12 remaining participants, six did not exhibit growth (i.e., Alaina_3, Eric_3, Lauren_3, Zander_3, Bob_4, and Oscar_4). Five of them (i.e., Alaina_3, Eric_3, Lauren_3, Bob_4, and Oscar_4) incorrectly answered Items 8 and 9 on the initial survey or the first time the sequence of three questions was posed, incorrectly answered items the second time the sequence was posed, and incorrectly answered Items 8 and 9 on the final survey or the third time the sequence was posed (see Group 2 in Table 2). The sixth, Zander_3, also did not exhibit growth but in a different way than the other five. He correctly answered Items 8 and 9 on the initial survey as well as all of the items the first time the sequence was posed. However, he did not correctly answer the items the second or third time the sequence was posed, and he incorrectly answered Item 9 on the final survey. We take this as evidence that these students did not exhibit growth in how they thought about angle size.

Six of the participants did exhibit growth. Brad_3, Patrice_3, Alan_4, Gina_4, Zane_5, and Amber_5 incorrectly answered at least one item on the initial survey or one of the items the first time the sequence was posed (see Group 3 in Table 2). Patrice_3 and Alan_4 also incorrectly answered one of the items the second time the sequence was posed. However, all six of these students correctly answered the sequence of three questions the third time they were posed as well as Items 8 and 9 on the final survey. We take this as evidence that change occurred. Because we utilized the microgentic method, we have a dense set of observations from "before the change began and…continue until a point of relative stability was reached" (Siegler & Crowley, 1991, p. 607). We analyzed this data to investigate these students' responses on all 17 trials, which included but was not limited to their use of sliders and their explanations on the 17 trials.

From our analysis of students' use of sliders (see discussion in Method section on analysis of slider moves, sliders used, and sliders active), we found that most students became more efficient in their use of sliders overall. From Trial 1 to Trial 17, students decreased in the number of slider moves and decreased in the number of sliders active at the time of check. We found it interesting that for Groups 1 and 2 for slider moves and Group 3 for sliders used there is a sharp increase at Trial 9 before there is an overall decrease again. Recall that Trials 1–8 occurred during Interview 2, whereas Trials 9–17 occurred during Interview 3. Although there was at most two days (one day for all but one student due to an absence) between interviews, we believe this is just the result of time between trials. Because it was hard to see trends in sliders used, we analyzed use of sliders per group. We calculated the mean number of slider moves (see Figure 5), sliders active (see Figure 6), and sliders used (see Figure 7) per trial per group.

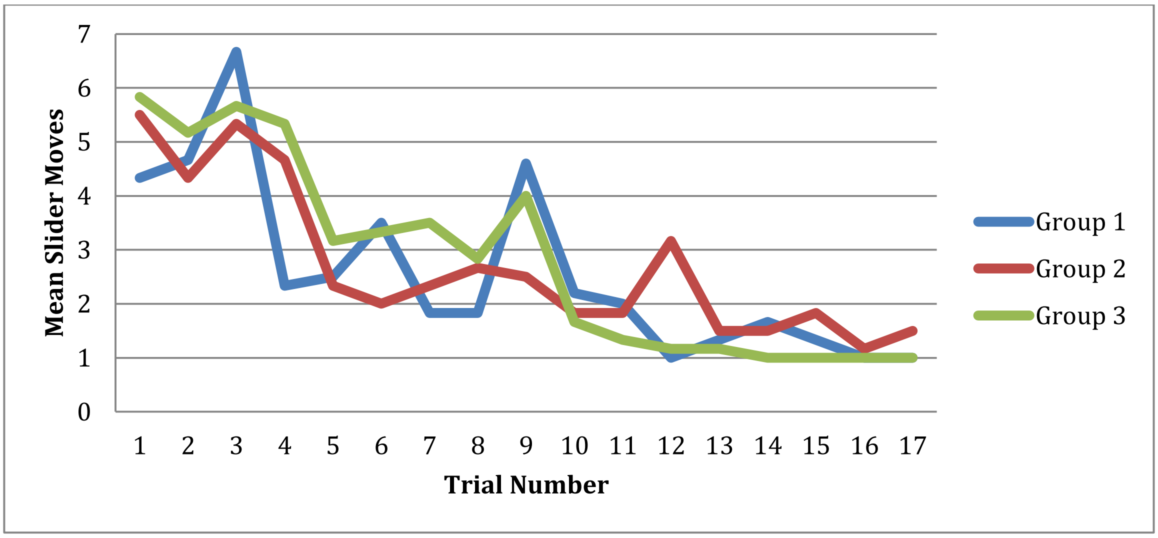

Figure 5. Mean slider moves per trial per group

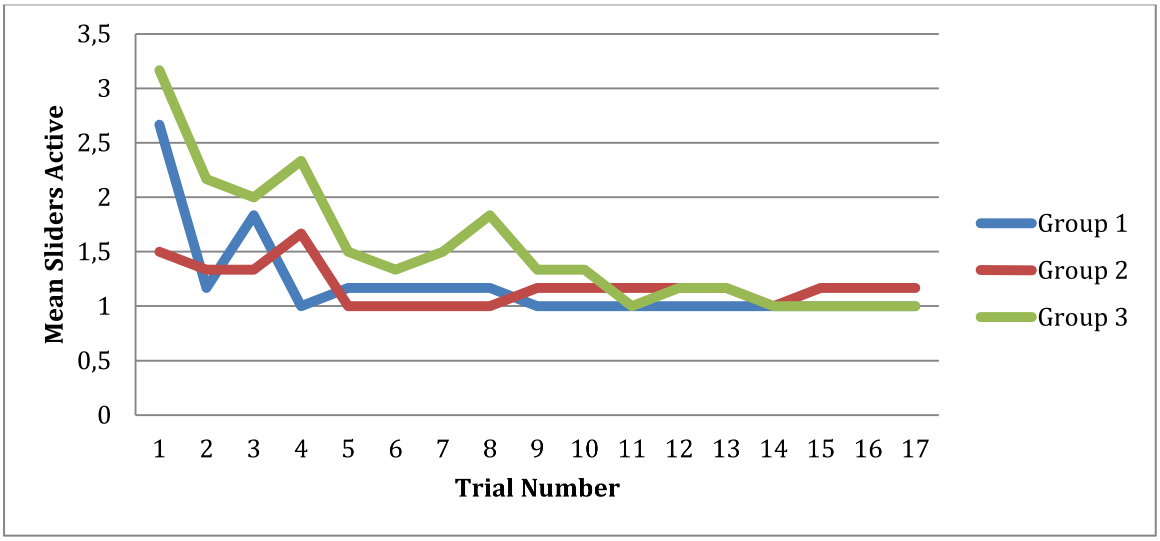

Figure 6. Mean sliders active per trial per group

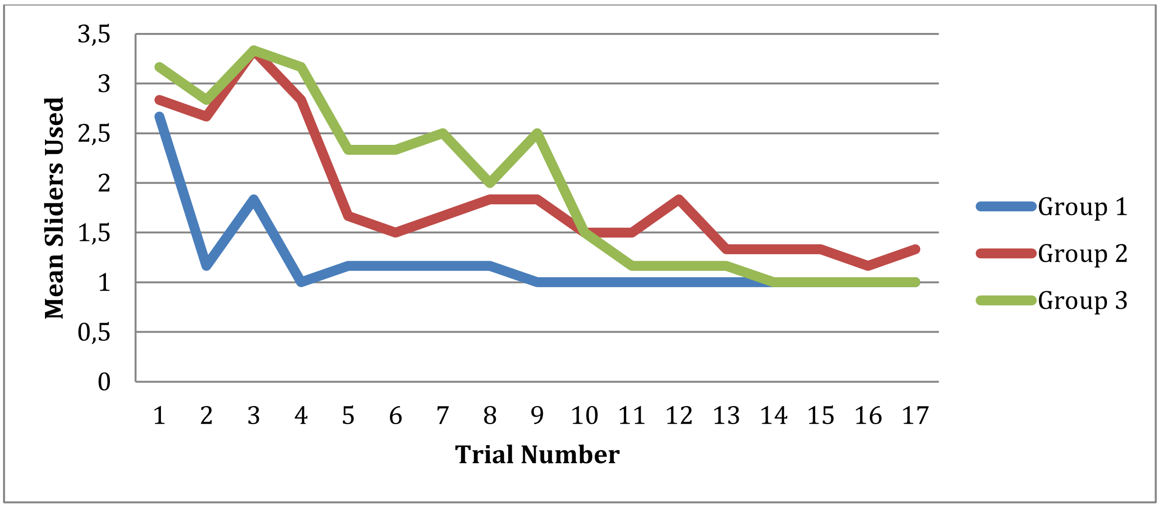

Figure 7. Mean sliders used (max 4) per trial per group

We found that from Trial 1 to Trial 17 each group decreased in the number of slider moves, decreased in the number of sliders active at the time of check, and decreased in sliders used. To analyze use of sliders per group further, we calculated the mean number of sliders used for Interview 2 (Trials 1–8) per group, the mean number of sliders used for Interview 3 (Trials 9–17) per group, and then found the difference per group. Using the same procedure, we computed the difference in means from Interview 2 to Interview 3 for slider moves and sliders active (see Table 3).

Table 3. Mean and standard deviation for sliders used, slider moves, and sliders active per group

	Interview 2		Interview 3		Difference
	Mean	SD	Mean	SD	
			Sliders used		
Group 1	2.29	1.11	1.33	0.58	0.96
Group 2	2.29	1.17	1.46	0.61	0.83
Group 3	2.71	1.17	1.28	0.68	1.43
			Slider moves		
Group 1	3.46	2.32	1.73	1.48	1.73
Group 2	3.65	2.6	1.87	1.45	1.78
Group 3	4.35	2.62	1.48	1.33	2.87
			Sliders active		
Group 1	1.42	1.01	1	0	0.42
Group 2	1.23	0.56	1.15	0.41	0.08
Group 3	1.98	1.83	1.11	0.42	0.87

There was more variability in slider moves for all three groups in both Interview 2 and Interview 3 than the variability in sliders used and sliders active, even though this variability decreased from Interview 2 to Interview 3. Although this greater variability may be due in part to participants “playing” with the sliders or trying to be overly precise in their final placement of the slider we also note that the count for slider moves was unbounded. This is in contrast to sliders used and sliders active, which were both limited to at most four (i.e., the total number of sliders). One explanation for a decrease in variability across all three groups in slider moves, sliders used, and sliders active is as all three groups became more familiar with using the sliders, the students became more efficient in their slider selection. Additionally, we take this decrease as an indicator that participants were becoming less distracted by the sliders, which controlled ray length, orientation, and position.

We also found that the difference in means for sliders used, slider moves, and sliders active from Interview 2 to Interview 3 was larger for Group 3 than for Groups 1 and 2. In other words, Group 3 became more efficient in their personal slider use. Although Group 1 was still more efficient overall in their slider use, this is additional evidence that change occurred for students in Group 3, all of whom showed growth on the survey items as well as the sequence of questions relating to the effects of angle orientation, ray length, and angle location on angle measure. Because students in Group 1 did not previously struggle with the misconception that angle measure is based on ray length or orientation but students in Groups 2 and 3 did, these results indicate that the use of sliders to vary the ray length, angle orientation, angle location, and angle openness had a positive effect on half of the participants in our study who did struggle with this misconception during Interview 1 and at least part of Interview 2. In other words, it helped six of 12 participants.

Conclusion

The intervention in this study was designed to provide opportunities for students to engage with dynamic and static angle models while also providing information about third, fourth, and fifth grade students' understanding of angle measurement. Our results from the written surveys and computer trials indicate that the intervention had some effect on all of the participants' terminology, third graders' abilities to estimate angle measures, and third and fourth graders' abilities to create accurate sketches of obtuse angles on the written survey.

The students in this study were offered the opportunity to vary the ray length, angle orientation, angle location, and angle openness with the sliders in order to challenge the misconceptions that angle measure is based on ray length or orientation. Our results indicate that most of the students in our study were distracted by ray length on the initial survey, but only six students did not exhibit growth and were distracted by ray length on the initial and final written surveys. These six students (i.e., Group 2) also continued to struggle with the sequence of questions about the effects of the four sliders into Interview 3. These results are consistent with the findings of Clements (2003), Foxman and Ruddock (1984), Keiser (2004), Mitchelmore (1998), and Noss (1987). Although Items 2 and 3 on the written survey (i.e., drawing an angle of a given measure and estimating a given angle) provided us with more information about student understanding before and after the intervention, we cannot make more conclusions without more questions of these types. Thus, it seems those seeking to use a pre- and post-survey to research third, fourth, and fifth graders' understanding of angle should consider focusing on construct and estimate items rather than multiple-choice items as others have done in the past (e.g., Foxman & Ruddock, 1984; Fyhn, 2008; Noss, 1987).

Although the purpose of the sweeping (rotating) ray was to promote the idea of angle as measured with reference to the circle, we do not have evidence that the participants in this study were able to reason about angle measure by comparing the fraction of the circular arc and the circle's circumference. In future studies, we plan to interview more students from each grade level, including more angle construction and estimation items, and study the effects of the display of benchmarks every 30 degrees has on students' angle constructions.

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