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Characteristics and Changes in the Mathematics Textbooks for the Secondary School in Argentina along 67 Years

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Abstract

This work analyses the changes in the relationship between arguing and images from the mathematics textbooks for the secondary school in Argentina (students 12-17 years old) along 67 years. The textbooks have been published in the period 1940 thru 2007. The analysis is done by (N=137) textbooks based on three meta-categories in an inductive way. These meta-categories are: characteristics of the arguing, relationship between images and arguing as well as characteristics of the textbooks (the date of publishing, the epistemological conceptions about mathematics and the educational level which the textbook is oriented). A factorial analysis of multiple correspondences was performed to find the main similarities and differences between the textbooks and to make a cluster analysis and one possible classification. The main differences are given by the changes in the epistemological conception of mathematics and in the characteristics of the images, especially in the newest edition books.

Introduction

This paper analyses the characteristics of mathematical textbooks used in secondary school in Argentina, from 1940 to 2007. In 1994 an educational reform was carried out, the program was modified and the schoolbooks adapted to the new educational system. We analyse a set consisting of N = 137 textbooks edited before and after the educational reform. In this work developing a categorization based on the characteristics of textbooks, ideas about arguing and characteristics of images. Some categories developed by Otero, Moreira & Greca (2002) are used to describe the images and naive ideas of teachers related to images and their benefits, with the aim of describing the main changes given by the characteristics of the images and their relation with the Arguing.

The problem of the Mathematical Arguing has been approached from the Formalist Vision of the Mathematics, the Epistemological Vision of the Didactics of the Mathematics (Arsac, et. Al., 1992; Arsac 1987; Recio, 1997; Godino & Recio, 2001), from the Socioepistemology in Mathematical Education (Crespo Crespo & Farfán, 2005; D'Amore, 2005), and also other theories of arguing that based on cognitive conflict as Schwarz & Linchevski (2007); Asterhan & Schwarz (2016); Arzarello & Sabena (2011); Driver, Newton & Osborne (2000). On the other hand, and in relation to arguing, the study of the relations between language, discourse, knowledge and arguing is recognized (Bakhtin, 1998; Banks Leite, 2004; Candela, 1998; Colinvaux, 2007; Goulart, 2004, 2007; Leitão & Banks-Leite, 2006). In this research, we adopt an idea of arguing more general that emphasizes the relevance of the divergences between the different points of view and the epistemological function of the arguing, proposed by Leitão (2007). The central ideas of conception of Arguing in this research are developed below.

The data obtained of the textbooks was analysed by means of multivariate techniques like the Factorial Analysis of Correspondences Multiples (Lebart & Morineau, 2000) which allows a classification of the textbooks in three classes, mainly defined by the characteristics of the arguing and the use of the images.

Conception of Arguing

This research adopts an idea of arguing that emphasizes the relevance of the divergences or conflicts between different points of view and the epistemological function of arguing (Leitão, 2007; Leitão, Ramirez, Guevara & Fernandes, 2012). Following Leitão, arguing has to be analysed based on three elements: "argument, argument against and response" in order to generate confrontation between argument and argument against, to achive construction of knowledge and transformations of perspectives in the subject (response). These processes occur

into face to face situations, and also, when someone is involved into a negotiation of divergences with himself, in negotiations of the different points of view with us in this case.

In this conception, it is proposed that Arguing is always constructed between a dialogue, the proposer of an argument and an opponent, to whom the proposer directs his argument and whose counterarguments he is determined to answer (for example, production of a written text, reading a textbook), or when the individual argues with himself or herself (Leitão et al. 2012). In that sense the counterargument is made by an alternative viewpoint or a disagreement with the proposed textbook. Specifically here, we are interested to consider the negotiations of the different points of view with ourselves, when we are reading a textbook that allows us to review and to modify our own perspectives. This process is named by Leitão (2007) as "self-directing arguing".

The conception of arguing mentioned before, the theoretical and methodological orientations for the images and the naive conceptions of teachers about the images proposed by Otero et. Al. (2002) are used to develop the categories of analysis which are described as follows.

Methodology and Categories of Analysis

A set of (N=137) mathematics textbooks are selected by means of purposive sampling techniques. The textbooks have been published in the period 1940 thru 2007. The most employed textbooks in the secondary school were considered, and also the ones less employed which added some didactical differences in a certain period. The analysis was performed starting from a previous qualitative inductive categorization based on three meta-categories: the characteristics of the arguing, the relationship between the images and the arguing and the characteristics of the textbooks. The categories and sub-categories of analysis are defined as follows:

Characteristics of Arguing (A):

A1- Commencement of arguing: it is about the way in which each chapter begins. The following subcategories were identified:

A1.1- Questions: textbooks formulating a question or situation, which generally will be answered later.

A1.2- Definition: textbooks using definitions to introduce knowledge.

A1.3- Examples: used to introduce a content, through which the knowledge can be formulated and generalized.

A2- Type of arguing. It is about the type of argument adopted by the textbooks: deductive arguments being formal or informal, or inductive arguments.

A2.1- Deductive formal: textbooks using deductive mathematics argument of character more or less formal, identifying by: definition, theorem, hypothesis, theory, demonstration, reciprocal theorem and application exercises.

A2.2- Deductive informal: textbooks that also use deductive arguing's; but without reaching the formalism like the demonstrations founded in the previous subcategory.

A2.3- Inductive: textbooks generalizing from the analysis of examples or by means of transformations about only one example taken as a representative.

A3- Degree of arguing: it is about the cognitive conflict promoted by the text, which could be or not be solved later on, or if the text only seeks to inform. We identified three degrees of arguing:

A3.1- High: belonging to textbooks that try to generate explicitly some type of confrontation, without any solution of divergence in the text.

A3.2- Low: it corresponds to textbooks trying to generate explicitly a cognitive conflict, which is solved later.

A3.3- Absent: characterized by the textbooks that inform without questioning.

Relationship between the Images and Arguing (B): We consider the characteristics of the images according to the theoretical and methodological trend proposed by Otero, et. Al. (2002).

B1- Use of the image: we distinguish the images used to facilitate the comprehension of the text, of the images which have an esthetical aim.

B1.1- Ornamental: images which characterize the books that principally use images with a decorative aim, and not strictly related to the content.

B1.2- Argument: images that in the books are used as source of information, from which knowledge can be derivative.

B2- Type of image: two types are identified.

B2.1- Mathematical representations: it is about images linked with mathematical systems of representation.

B2.2- Non-mathematical representations: it is about images that are not related with mathematical content.

B3- Grammatical style of the images. Kress & Van Leween (2006) classifies the images, using the categories:

B3.1- Conceptual: images representing relations and fixed characteristics between the represented elements.

B3.2- Narrative: images that allow identifying actions between the objects and to build a story representing a relation between the objects involved in the image.

B4- Relationship with the "real world". The images of the textbooks are classified in accordance or not to aspects of the empirical world.

B4.1- Naturalist: images referring to the empirical world. They are detailed and complex.

B4.2- Abstract: images not referring to the world that we experience.

Characteristics of the Textbooks (C):

C1- Date of publishing: Refers to the date of the first publishing of the textbook.

C1.1- Period 1: textbooks published in the period 1940 thru 1973.

C1.2- Period 2: textbooks published in the period 1974 thru 1994.

C*1.3- Period 3:* textbooks published in 1995 thru 2007, after the educational reform mentioned before.

C2- Educational level: Refers to the educational level in which textbooks are aiming.

C2.1- Level 1: textbooks aiming to the first three years of the secondary school, (students between 12 and 14 years old).

C2.2- Level 2: textbooks aiming the last three years of the secondary school, students between 15 to 17 years old.

C2.3- Level 3: textbooks aiming the transition between secondary school and university, students older than 18 years old.

C3- Traditions: Klimovsky & Boido (2005) identify three different ways to conceive and justify mathematics in textbooks.

C3.1- Computational tradition: textbooks showing mathematics involved with the resolution of problems and calculation, or in numbers, and the operations done with them.

C3.2- Axiomatic tradition: textbooks using correct forms of reasoning, and presenting demonstrations steps.

C3.3- Structuralist tradition: textbooks showing the mathematical work as a search of regularities that meet the same conditions.

Using this categorization, a qualitative description of the characteristics of the textbooks, arguing and images is made, which originated a first analysis. Then, the categorization is transformed in a group of nominal variables and modalities using Exploratory Data Analysis (Lebart, Morineau, 2000). An Analysis of Contingency among the variables is performed, producing a re-categorization of the texts with the variables significantly associated. A Factorial Analysis of Multiple Correspondences allowed the selection of one possible classification in three classes, in agreement with the interpretative analysis was realized. In addition, a test of randomness to analyse the reliability of the sample was performed using the statistical software SPAD 8.2 (Coheris-Spad, 2017).

Factorial Analysis of Correspondences Multiples and Categorization

The study is based on techniques from the Multivariate Data Analysis, using the statistical software SPAD 8.2 (Coheris-Spad, 2016, 2017). The factorial reduction technique provides a simplification and synthesis of the information that allows analyzing the main conjunctions-oppositions between the variables. The graphic representation of the different modalities of the variables in factorial planes provides a direct and global view of the main aspects of the information to be taken into account (Otero, Moreira & Greca, 2002).

An Analysis of Contingency was performed between the variables, and it was decided to fuse the modalities of some variables, making them dichotomous and reduced the number of modalities to those that did present a significant association. The modalities that decide to fuse correspond to the variables *Type of Arguing* and *Style of Arguing* (before *Degree of Arguing*). The variable *Type of Arguing* has two associated modalities that describe textbooks that opt for deductive mathematical arguments (formal or informal) or inductive. The variable *Style of Arguing* has now associated two modalities: Confrontational (Degree of arguing high and low) and Non-confrontational (Degree of arguing: absent). To perform the Factorial Analysis of Multiple Correspondences, active and nominal variables are determined and all of them are entered into the analysis. They are selected as active variables: *Traditions*, *Commencement of arguing*, *Type of arguing*, *Style of arguing*, *Use of the image* and *Type of image*. The first three factors determine 68.76% of the variance explained. The variables that more they contribute to the conformation of the first factor are: "use of the image" and "style of arguing". This explains why the biggest differences found in the group of books analysed are represented in the ornamental use in opposition to the argumentative use of external images and in the explicit conflicts that some texts promote, unlike the ones that only inform the reader.

The classifications allowed selecting a partition of three classes (Benzécri, 1980) as it is showed in the Figure 1. In addition to describing the characteristics of each class, an example of a book selected as representative is proposed. Of the three books selected, we propose as an example the analysis about the remarkable points of the triangles, to describe the differences between the classes with the same theme proposed.

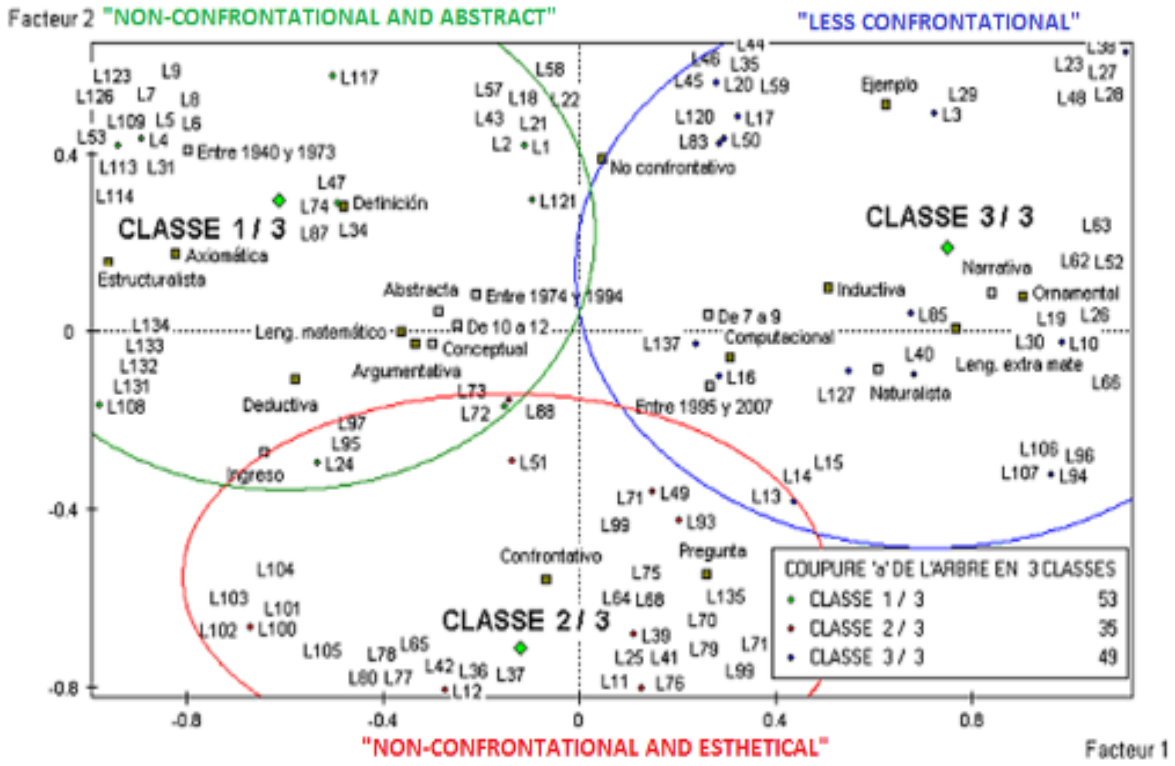
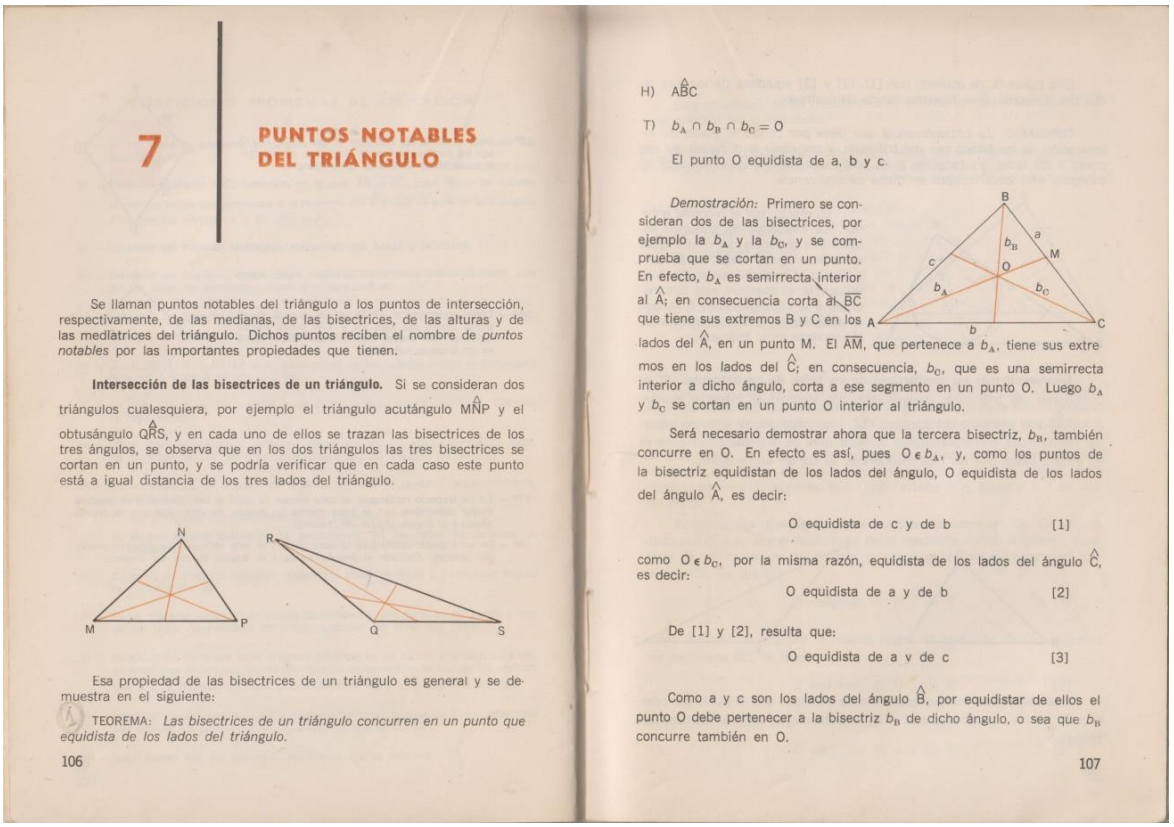


Figure 1. Representation of the individuals in every class and of the representative textbooks

The **Class 1** (N=53) gathers the "non- confrontational and abstract" textbooks. They are informative textbooks that do not give explicitly conflicts that can be addressed by the reader. The arguing always begins with definitions and the type of predominant arguments is deductive. They identify with the axiomatic tradition, using correct form of reasoning (deductive mathematics argument). The images showed are mathematical, conceptual and abstract representations. Principally the textbooks are edited between 1940 and 1973 corresponding to the First period of Edition. The style of arguing is non-confrontational. The Figure 2 is an example of the textbooks that belong to this class.

The book of the figure 2 is edited in the year 1967 for students between 13 and 15 years. The arguing begins with the definition of the notable points of the triangle, and then presents the theorem of the bisector of triangle, hypothesis, proposition and demonstration. All the theorems are enunciated with the respective demonstrations and the steps of the formal deductive arguments. All the images are strictly related to the content; contribute to

the development of the lesson. It is an example of an informative textbook, where there are no any differences of views proposed to the readers. Activities related to the notable points of the triangle are proposed at the end of the chapter.



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PUNTOS NOTABLES DEL TRIÁNGULO

Se llaman puntos notables del triángulo a los puntos de intersección, respectivamente, de las medianas, de las bisectrices, de las alturas y de las mediatrices del triángulo. Dichos puntos reciben el nombre de *puntos notables* por las importantes propiedades que tienen.

Intersección de las bisectrices de un triángulo. Si se consideran dos triángulos cualesquiera, por ejemplo el triángulo acutángulo $M\hat{N}P$ y el obtusángulo $Q\hat{R}S$, y en cada uno de ellos se trazan las bisectrices de los tres ángulos, se observa que en los dos triángulos las tres bisectrices se cortan en un punto, y se podría verificar que en cada caso este punto está a igual distancia de los tres lados del triángulo.

Esa propiedad de las bisectrices de un triángulo es general y se demuestra en el siguiente:

TEOREMA: *Las bisectrices de un triángulo concurren en un punto que equidista de los lados del triángulo.*

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H) $A\hat{B}C$

T) $b_A \cap b_B \cap b_C = O$

El punto O equidista de a, b y c.

Demostración: Primero se consideran dos de las bisectrices, por ejemplo la b_A y la b_C, y se comprueba que se cortan en un punto. En efecto, b_A es semirrecta interior al $\hat{A}$; en consecuencia corta al $\overline{BC}$ que tiene sus extremos B y C en los lados del $\hat{A}$, en un punto M. El $\overline{AM}$, que pertenece a b_A, tiene sus extremos en los lados del $\hat{C}$; en consecuencia, b_C, que es una semirrecta interior a dicho ángulo, corta a ese segmento en un punto O. Luego b_A y b_C se cortan en un punto O interior al triángulo.

Será necesario demostrar ahora que la tercera bisectriz, b_B, también concurre en O. En efecto es así, pues $O \in b_A$, y, como los puntos de la bisectriz equidistan de los lados del ángulo, O equidista de los lados del ángulo $\hat{A}$, es decir:

O equidista de c y de b [1]

como $O \in b_C$, por la misma razón, equidista de los lados del ángulo $\hat{C}$, es decir:

O equidista de a y de b [2]

De [1] y [2], resulta que:

O equidista de a y de c [3]

Como a y c son los lados del ángulo $\hat{B}$, por equidistar de ellos el punto O debe pertenecer a la bisectriz b_B de dicho ángulo, o sea que b_B concurre también en O.

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Figure 2. “non- confrontational and abstract” textbooks. Image taken from Repetto, C.; Linskens, M. Fesquet, H. (1966). Geometría 1. Kapeluz, Argentina

The **Class 2** (N = 35) brings together the "least contentious" characters. These textbooks present differences of view, which are resolved in the textbook below, but without analyzing alternative views. The argument begins with a question that indicates that these books try to question and other questions with a respective answer. They identify here the textbooks of "low degree of discussion" trying to generate explicitly a cognitive conflict, that is resolved later. The reader can decide whether he will solve the argument for himself or whether he will agree with the answer given by the textbook. Mainly identified with the structuralist tradition5. In this group of textbooks images are also used for argumentative purposes. The characteristics of the images are: mathematical, conceptual and abstract representations. Mainly the textbooks are published in 1974 to 1994.

In figure 3 we can see an example of the textbooks that belong to this class. The book selected as a representative was published in 1989 (period 2) for students between 14 and 15 years of age. They are "less contentious" because they propose a question that is answered below besides the other questions are raised that seek to study the perpendicular bisector and the bisector of a segment respectively, with the answers being precise and finalized in each case. Later there are other situations to verify that this "test" is valid for other cases. Images are abstract representations strictly related to content.

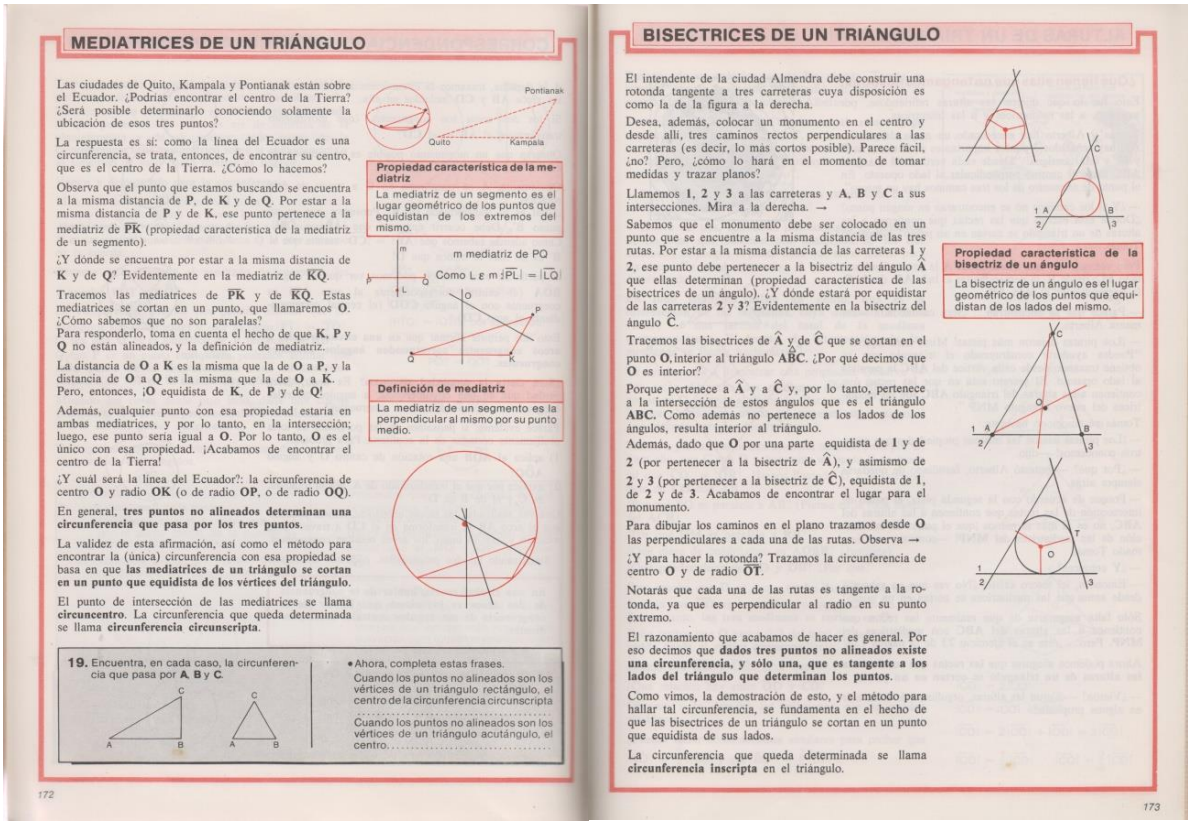

Figure 3. “less confrontational” textbooks. Image taken from Sadovsky, P.; Kass, M.; Panizza, M.; Reyna, M. (1989). Matemática 2. Santillana, Argentina

The **Class 3** (N = 49) brings together "non-confrontational and aesthetic" textbooks. The main purpose of this type of textbook is to inform, there are no questions. The argument begins with examples and definitions, mainly on a single example taken as representative. The results are relative to numbers and operations with numbers. These textbooks are identified with computational tradition. The predominant images found in these textbooks are used to decorate, and are categorized into narrative or naturalistic modalities. Images are not related to mathematical content and only have an ornamental function. These textbooks were published after the educational reform in 1994.

In Figure 4 we can see an example of textbooks of this class. The text was published in the year 2001 for students between 12 and 13 years old. The chapter begins with a situation that is answered later, but the content is always defined, exemplified and some activity is placed to "reproduce" the technique developed. The definition is proposed as a generalization from the single example. This book contains only examples, definitions and exercises; it is more an activity folder than a book to study. Most images are not related to mathematical content. Images and colours are mainly used with a decorative lens. For example, in this book: What does the child, the lantern and the image of the lord represent with a coat? And the coat hangers?

The main differences between the classes explain changes in the images and arguing from the textbooks analyzed, given by: textbooks that propose questions or only definitions and examples, the way of conceive and validate to mathematical knowledge, and mainly by the changes in the images and the relation between images and knowledge.

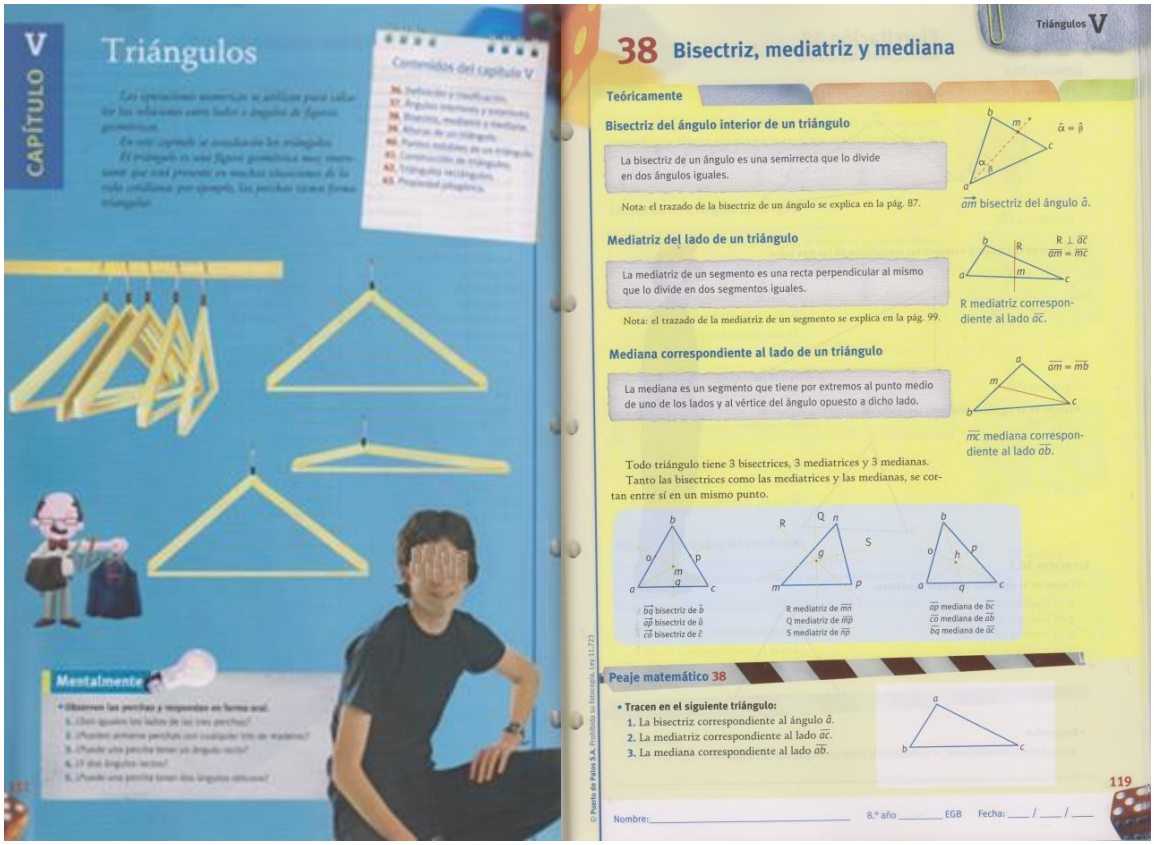

Figure 4. "non-confrontational and esthetical" textbooks". Image taken from Laurito, L.; Stisin, L.; Trama, E. Ziger, D. (2003). Matemática 8. Puerto de Palos, Argentina

Conclusions

The analysis of the textbooks that includes 67 years, allows identifying changes in the arguing and the images of the texts in each period of publishing. These changes could be linked with the naive ideas of teachers about the images and about mathematics. The textbooks of the period 1 are characterized by the Axiomatic Tradition, the demonstrations occupy a central position and all the affirmations are justified deductively. Besides the theorems, demonstrations and definitions there are exercises and activities. The most new textbooks, especially edited after the reform of 1994, propose a reduction of the mathematics to the numbers, the operations and results with numbers. The arguing is mainly inductive from the examples, and in most cases from a single example taken as representative. This shows that the most modern books are completely traditional since they reduce the mathematics teaching to the process: definition, example, application and mainly numerical results.

The characteristics of the images also change in the periods of edition of the textbooks. The esthetical use of images has increased with time, as well as in quality and in quantity, especially in the most recently published textbooks. These changes would be linked with the naive ideas of teachers about the images and about mathematics as a science. Teachers are the most important recipients of the editorial industry because they make the decision on which textbooks their students have to use and to buy. The goal of most textbooks seems to be informative. This explains: the absence of questioning and discussing about several points of view and the low level of arguing and differences of views founded. However, we consider that it is possible to write textbooks where the self-directing arguing is promoted, accordingly with the construction of knowledge and with teaching and learning by questioning and research.

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