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Abstract

This study investigated the effect of van Hiele Phase-based Instruction (VHPI) on Ghanaian Pre-service Teachers' (PTs') geometric thinking in terms of the van Hiele Levels. A pre-test post-test quasi-experimental design was employed. There were 75 PTs each in the experimental group and the control group. Van Hiele Geometry Test (VHGT) was administered to all PTs as both pre-test and post-test. PTs in the experimental group were taught two-dimensional geometry using VHPI while the control group was instructed by conventional instruction. Chi-square results showed that the PTs in both groups had increment in their post-VHGT as compared to the pre-VHGT. However, the PTs in the experimental group achieved better levels of geometric thinking as compared to those in the control group ($\chi^2 = 58.949$, $p<0.05$). Again, results from paired samples t-test indicated a significant difference in mean scores between control and experimental groups favouring PTs in the experimental group ($t=30.776$, $p<0.05$). The significant improvement in the performance of the experimental group having more PTs at level 3 and 4 than at level 0, 1 and 2 suggest that the VHPI served a useful pedagogical approach, impacted positively on PTs geometric thinking levels and has the potential of improving teaching and learning of geometry in schools than the conventional approach.

Introduction

Geometry is an essential area in the school mathematics curriculum throughout history. It has had great importance in people's lives, originating with the need of human beings to specify quantities, to measure figures, land and earth, and make maps (Sunzuma, Masocha & Zezekwa, 2013). The study of geometry offers many foundational skills and helps to build the thinking skills of logic, deductive reasoning, analytical reasoning and problem solving. Geometry is also linked to many other areas in mathematics such as measurement, algebra, calculus, and trigonometry and is used daily by architects, engineers, physicists, land surveyors and many more professionals (Russell, 2014).

Students' mathematical competencies have been closely linked to their levels of geometric thinking (Atebe & Schafer, 2008; Senk, 1989). Thus, in Ghana, the college of education mathematics curriculum emphasizes geometric thinking as one of its objectives. The college of education mathematics course outlines provide for the teaching of content, dealing with the subject matter of mathematics and methodology aspect that deals with the pedagogical skills of the subject matter. The choice of the content in the course outlines is based on needs of the Pre-service Teachers (PTs) and the learners they would be expected to teach in the basic schools after their training. Moreover, the selection of the respective topics was also based on the assumption that the PTs had had a sound foundation in mathematics in the basic concepts in their first and second cycles of education. By the end of their three year pre-service program, the PTs are expected to have a sound knowledge and foundation in mathematics and geometry in particular to teach it effectively (Etsey, 2004).

In order to obtain the full benefits of geometry in the college of education mathematics curriculum, classroom instructions should aim at enhancing PTs' geometric thinking. However, it is observed that the teaching and learning culture of mathematics including geometry in Ghanaian schools have the following characteristics which have contributed to the mathematics underachievement of Ghanaian students: students accept whatever the teacher teaches them. The teacher is the sole authority of mathematical knowledge in the classroom, while the students are mere receptors of mathematical facts, principles, formulas, and theorems. Thus, if the teacher makes any mistakes the students would also make the same mistakes as the teacher made. It is believed that this teaching culture in most Ghanaian mathematics classrooms has significantly shaped the mathematics learning culture highlighted above (Fredua-Kwarteng & Ahia, 2004). This is supported by De Villiers (2012) when he indicates that majority of geometry teachers follow the conventional way of teaching where teachers only "provide students with ready-made content (definitions, theorems, proofs, classifications, and so on) that they merely have to assimilate and regurgitate in tests and exams" (p. 13). Meanwhile, there is enough evidence suggesting that current conventional textbook-chalkboard teaching approaches promote inadequate spatial experience (Alex & Mammen, 2016; Erdogan & Durmus, 2009; Fletcher & Anderson, 2012). Sakyi (2014) emphasized that "there is also the need to shift away from rote-learning and the current exam-centred educational system in Ghana, which produces robotic products who cannot solve problems or apply their knowledge to creative endeavours" (p. 4).

Studies (Halat & Şahin, 2008; Halat, 2008; Pandiscio & Knight, 2010) have indicated that the geometric thinking levels of PTs are not sufficient to teach at basic schools. The situation is not different in Ghana as confirmed by Armah, Cofie and Okpoti (2017). There seems to be something wrong with the way mathematics including geometry is learnt and taught in Ghana. This is because the performance of students in both internal and external mathematics examinations has remained consistently poor. The Trends in International Mathematics and Science Study (TIMSS) in 2003, 2007 and 2011 (Gunhan, 2014; Mullis, Martin & Foy, 2008) revealed that Ghanaian students' performance in the subject was abysmally low with mean scores far below the international average. The reports affirmed that students' performance in geometry was the lowest in the five domains the test covered. In Addition, the West African Examination Council (WAEC) Chief Examiner's annual reports for the West African Senior School Certificate Examination (WASSCE) from 2012 to 2015 indicated that candidates were weak in 2 and 3-dimensional geometrical problems. Also, the chief examiner's annual report for Diploma in Basic Education (DBE) End-of-Second Semester Mathematics Examination in geometry, from the years 2012 to 2014 makes it clear that generally, the performance of candidates in the content part was weak and their presentations of solutions to most of the 2 and 3-dimensional geometrical problems were poor and majority of them had problems solving questions that require the use of properties of geometrical shapes. In 2015 and 2016, the examiner's report once again stated candidates' lack of adequate knowledge in geometry and application of geometric concepts. According to the reports, College tutors should revise their instructional approaches to help reduce the abstractness of geometry, and thence remove the PTs' lack of interest and fear of the subject. As a result, it becomes relevant to look for interventions that could be manipulated in order to find their effects on geometry learning outcomes. This could address the problems of teaching and learning of geometry in schools.

One way of addressing the problems of teaching and learning of geometry (especially two-dimensional geometry) is through the use of van Hiele Phase-based Instruction (VHPI). It is widely recognized in literature that the VHPI is one of the most significant theoretical frameworks to understand students' learning processes, places learners at the center of learning and has interventions that can effectively improve learners' geometric thinking levels (Al-ebous, 2016; Erdogan & Durmus, 2009; Howse & Howse, 2015; Mostafa, Javad & Reza, 2017; Ramlan, 2016). Integrating hands-on investigations with manipulative concrete materials in the VHPI can help learners to build concrete concepts, and also provide learners ample opportunities to unleash their own creativity as they explore geometric concepts (Zhang, 2003 cited in Siew & Chong, 2014).

Consequently, several researchers in other countries have applied the theory to improve geometry instruction (Abdullah & Zakaria, 2013; Howse & Howse, 2015; Mostafa, Javad & Reza, 2017). However, in Ghana there are limited studies on the van Hiele theory and that very little studies have applied the van Hiele Phase-based Instruction to improve geometry instruction. In an attempt to seek a teaching strategy that can improve PTs' geometric thinking, this study investigated the effect of the use of van Hiele Phase-based Instruction on PTs' achievement in geometry which is taught in Ghanaian Colleges of Education.

Most importantly, PTs are being trained to teach at the basic level, as a result, if measures are not put in place to address their insufficient knowledge and poor performance in geometry, it may create great difficulties for the Ghanaian mathematics education, especially at the basic level. Instead of memorizing properties and definitions, it is suggested that PTs should be given opportunities to work with concrete manipulative materials, drawings and symbolic notations in order to progress tremendously through the levels of geometric understanding as defined in the van Hiele framework. This calls for an alternative teaching approach where the VHPI can be used to improve PTs' geometric thinking levels and problem-solving skills.

The van Hiele Theory

The van Hiele theory emerged from the separate doctoral works of a husband-and-wife team of Dutch mathematics educators, Dina van Hiele-Geldof and Pierre van Hiele, which were completed simultaneously at the University of Utrecht, Netherlands in 1957. According to Malloy (2002), the couple, having been concerned with the difficulties their students encountered with secondary school geometry, began to think that the content they were teaching was too advanced for many of their students to fully understand. The van Hiele theory comprises three main aspects, namely: Levels of geometric thinking, properties of the Levels and phases of learning (the van Hiele Phase-based Instruction) which offers a model of teaching that teachers could apply in order to promote their learners' levels of understanding in geometry (van Hiele, 1986).

The van Hiele Levels of Geometric Thinking

The van Hieles described five sequential and hierarchical discrete Levels (see Table 1) that characterized the thinking of learners as they become more refined in their understanding of geometric relationships. This is the most prominent feature of the theory. The van Hiele Levels are: *Visualization/Recognition, Analysis, Informal Deduction/Order, Deduction,* and *Rigor* (Alex & Mammen, 2016). The levels describe "how one thinks about, rather than how much knowledge one has" (van de Walle, 2001, p.309). The van Hiele Levels can be described as follows:

Table 1. van Hiele levels of geometric thinking

Levels	**Characteristics**
Level 1 (Visualization)	Students recognize figures by their appearance. They make decision based on intuition not reasoning.
Level 2 (Analysis)	Students recognize figures by their properties. They can analyze and name properties of figures, but they cannot make relationships between these properties.
Level 3 (Informal deduction)	Students can distinguish between necessary and sufficient conditions for a concept. They can form meaningful definitions and give informal arguments to justify their reasoning.
Level 4 (Deduction)	Students can construct theorems within an axiomatic system. They know the meaning of necessary and sufficient conditions of a theorem.
Level 5 (Rigor)	Students understand the relationship between various systems of geometry. They can compare, analyze and create proofs under different geometric systems.

Source: Karakuş & Peker, 2015, p. 339.

The van Hiele levels were originally defined from Levels 0 to 4. However, studies (Alex & Mammen, 2016; Howse & Howse, 2015; Siew & Chong, 2014) have changed to the levels from 1 to 5. This according to Mason (1998) allows for a sixth Level, Pre-recognition Level to be assigned Level 0. Clements and Battista (1990) also recommend the existence of level 0 which they call Pre-recognition; Level for learners who have not yet achieved even the basic Level 1. In this level, students initially perceive geometric shapes, but have an inability to distinguish between figures. For instance students "may recognize the difference between triangles and rectangles, but may not be able to distinguish between a rhombus and a square" (Karakuş & Peker, 2015, p. 339). This study used the Level 1 to 5 numbering scheme to allow utilization of Level 0.

The van Hiele Phase-based Instruction (VHPI)

Van Hiele (1986) emphasized that cognitive progress in geometry can be accelerated by instruction. According to him, an effective process of learning geometry is not the same as the teaching and learning process of other topics in mathematics such as numbers, algebra and probability. As a result, the method and organization of geometry instruction, as well as the content and materials used, are important areas of pedagogical concern. In order to deal with these issues, he proposed that the learning process leading to complete understanding at the next higher level has five phases namely: *information/inquiry, guided/directed orientation, explicitation, free orientation,* and *integration.* The approach used in these five phases provides a structured lesson by giving clear explanations of how the teacher should proceed to guide students from one level to the next (Crowley, 1987; Usiskin, 1982; van Hiele, 1986; van Hiele, 1999). The phases include;

Phase 1: Information/Inquiry
This phase of learning involves a two-way teacher-student interaction which is essential in understanding certain geometrical shapes such as making observations, asking questions and understanding the vocabulary for a particular geometrical shape.

Phase 2: Guided/Directed Orientation
In this phase, the teacher guides the students to explore the object of instruction by assigning carefully structured but simple tasks such as folding, measuring, or constructing that the teacher has carefully sequenced.

Phase 3: Explicitation
During this phase, students share their opinions on the relationships they have discovered in the hands-on activities. They describe what they have learned about the topic using their own language. However, the teacher ensures that the accurate and appropriate terminology is developed and used.

Phase 4: Free Orientation
In this phase, students solve more complex tasks independently which brings them to master the network of relationships in the material. The tasks designed here are to provide the students with problems that are open-ended and have multi-path solutions.

Phase 5: Integration
Students now have a clear sense of purpose and can review and summarize what they have learned with the goal of forming an overview of the new network of objects and relations.

Effect of the VHPI on Students' Geometric Thinking Levels

Apart from using the van Hiele model to identify students' Levels of geometric thinking, the model can also be used for planning effective geometry instruction. Abdullah and Zakaria (2013) argue that the interventions using the VHPI can be applied in classrooms in order to positively and effectively improve students' thinking Levels and help students achieve better level of geometric understanding. In the view of Crowley (1987) the VHPI is the way for teachers to provide more opportunities for students to experience geometry units or topics associated with the van Hiele model. The purpose of this section is to look at the effects of the VHPI on the development of learners' geometric thinking Levels.

Erdogan and Durmus (2009) investigated the effects of instruction based on the VHPI on 142 senior PTs' geometric thinking Levels. There were eight classes of senior PTs, two of them were randomly assigned as experimental groups which were instructed with the VHPI and the other two were randomly assigned as control groups which were instructed with traditional instruction. An instruction consistent with geometric thinking Levels of van Hiele model was applied to experimental groups whereas traditional method was applied to control groups throughout the study. The activities applied in experimental groups were carried out with a method in which the concepts of discussion, group work, collaborative learning approaches were implemented in a related web in accordance with geometric thinking Levels of van Hiele whereas, in control groups, the activities were applied with traditional approaches in which the students follow the instruction the teacher gives and active participations are not promoted. Significant difference was found between the pre-test and post test scores of van Hiele Geometry Test of experimental groups. However, when the pre-test and post-test scores of van Hiele Geometry Test of PTs in control group were taken into consideration, it was seen that there was no significant difference between the results. It can be claimed that, while instruction given with traditional method provided limited geometric thinking Levels of PTs, instruction consistent with VHPI has a positive effect on the geometric thinking Levels of PTs.

Siew and Chong (2014) also conducted a study to determine the effects of VHPI on learners' creativity using tangrams activities among grade three primary school learners. A total of 144 Grade three learners took part in the study that employed a pre-test and post-test single group experimental design. The learners were taught two-dimensional geometry and Symmetry through the van Hiele's five phases of learning using tangrams. Paired samples t-tests which compared the mean scores of pre- and post- test indicated significant differences in mean scores between pre- and post- test. After a series of statistical analysis, the finding revealed that creativity can be fostered through the instruction using tangrams which was based on VHPI. The authors further indicated that hands-on activities enable students to develop knowledge and properties of polygons and their creativity. The study showed that the tangram, when integrated with VHPI is able to foster learners' creativity in geometric lessons.

In his study of the correlation between students' level of understanding geometry according to the van Hiele Model and students' achievement in plane geometry, Yazdani (2007) involved one hundred and sixty nine students in an experimental study. A pre-test was administered to all participants at the beginning of the semester. The pre-test consisted of the following two selected response assessment instruments: The "Plane Geometry National Achievement Test" and the "van Hiele Geometry Test". The same battery of tests was employed as a post-test and was administered to the participants after 6 weeks of instruction using the VHPI. A measure of the linear relationship between students' level of understanding geometry according to the van Hiele Model and students' achievement in geometry found a correlation coefficient of 0.8665 for the post-test. The results indicated that there was a strong positive correlation between the advancement of the van Hiele level of understanding geometry and achievement in geometry. The VHPI has significant implications for teaching geometry. It is therefore suggested that educators responsible for geometry instruction and professionals in charge of teacher training programs incorporate the principles upon which the VHPI is based into instructional and curricular design.

Purpose of the Study and Research Hypothesis

The purpose of the present study was to investigate the effect of the use of van Hiele Phase-based Instruction on PTs' geometric thinking in terms of the van Hiele Levels. In pursuance of this purpose, the research hypothesis below was formulated to guide the study:

- There is no statistically significant difference among PTs' performance in van Hiele level of thinking, regarding the methods by which they were taught (conventional instruction versus VHPI).

Significance of the Study

This study is unique and significant as it represents, as far as the researchers have been able to ascertain, the first attempt to investigate the effectiveness of the VHPI in Colleges of Education in Ghana. The measure and description of Ghanaian PTs' van Hiele Levels of understanding in geometry shall be of great interest to mathematics teacher educators, assessment developers and PTs' curriculum developers. As the school curriculum is a major factor in shaping the quality of education, the findings that will come out of this study can be used to help curriculum developers and college of education tutors on how to use the van Hiele model in order to improve the geometric thinking Levels of students.

In this research it is expected that PTs will be engaged in mathematical communications where they explain their ideas clearly and also follow each other's reasoning rather than just the tutor's instruction. As a result, the researchers are optimistic that the use of the VHPI will not only improve PTs geometric thinking Levels and achievements in geometry but would also offer PTs enhanced opportunities of varied forms of mathematical communications which are absent in other forms of teaching approaches such as the tutor-centred approach. Moreover, the research would help dispel PTs' general negative perceptions about mathematics and geometry in particular which would influence their teaching of mathematics positively at the basic level after completing their program. Also, the study would add new knowledge to mathematics education as well as serve as base-line data for future studies on the van Hiele model.

In summary, the van Hiele model can provide a framework on which geometry instructions can be structured and taught in schools: teachers could, for example, attempt first to raise their learners' van Hiele geometric thinking Levels through the instructional phases of the van Hiele model in order to improve the mathematical performance of their learners. Also, the study will aid PTs to overcome some barriers they encounter in using their informal strategies in the process of acquiring more sophisticated strategies. It is hoped that this will in turn sustain interest in geometry as PTs progress to higher classes and make them derive the full benefit of having a good knowledge of geometry.

Method

Research Design

The study was designed in line with Fraenkel and Wallen's (2006) description of pre-test post-test non-equivalent quasi-experiment groups design.

Type of Group	**Pre-test**	**Treatment**	**Post-test**
Experimental	T_1	X	T_2
Control	T_3	C	T_4

The pre-tests (T_1 and T_3) were done to determine the initial entry points and compare difference between experimental and control group before treatment. The post-tests (T_2 and T_4) were administered to examine the treatment effect after experimental group received van Hiele Phase-based Instruction (X) and the control group received the conventional instruction (C).

Participants

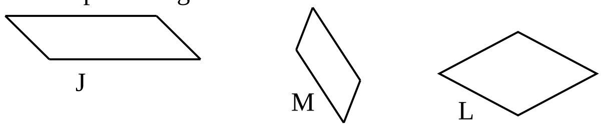
The study was carried out at Wesley College of Education and Accra College of Education in the Ashanti Region and Greater-Accra Region of Ghana respectively. 150 second year PTs were randomly selected from both colleges; 75 PTs in the experimental group and the other 75 PTs in the control group. The participating PTs in Wesley College of Education were randomly assigned as the experimental group while those in Accra College of Education were randomly assigned as the control group. PTs in these colleges of education are admitted from all over the ten administrative regions in Ghana. The participants are all graduates from the senior high schools in Ghana with an average age of 22 years. The researchers are of the view that the two colleges of education were ideal for this study because the researchers were able to monitor the progress of the PTs during treatment.

Research Instrument

PTs in experimental and control groups were given van Hiele Geometry Test (VHGT) as pre-post-tests to identify their geometric thinking levels. The VHGT was taken from the study of Usiskin (1982). The initial thought to develop this test was to determine the van Hiele Level of understanding geometry of a student. Each item was written as a means to identify the behaviours, using quotes from the van Hieles, of the students at each Level. This is a well-known geometry test which has been used in several research works since it was developed. The VHGT has been shown to be valid and reliable over the years (Usiskin, 1982; Burger & Shaughnessy, 1986; Knight, 2006; Atebe & Schafer, 2008; Baffoe & Mereku, 2010). Reported reliability estimates of the test ranged from .69 to .78. In this study, four college of education mathematics teachers who were invited to check for validity confirmed that the test could be used to measure PTs' geometric thinking Levels. Cronbach Alpha reliability coefficient was also computed to examine the internal consistency of PTs test scores. Alpha value of .75 was obtained indicating that the scores were consistent and reliable for further analysis.

The test consists of 25-item multiple choice test (part A) and is organized sequentially in blocks of five Items. Items 1-5 measure the ability of students to recognize the geometric figures according to personal observation and ability and measure student understanding at Level 1, Items 6-10 measure the ability of students to identify the properties of the figures, this measure students' understanding at Level 2. Items 11-15 measure the ability of students to recognize the figures by analyzing their properties and the relationships between these properties, this also measure students' understanding at Level 3. Items 16-20 measure the ability of students to understand the proof of theorems and measure students' understanding at Level 4, while Items 21-25 measure the ability of students to understand the differences between axiomatic systems, this measure student understanding at Level 5. The researchers included a second part (part B of the VHGT) consisting of 3 items where participants were expected to provide written responses. This was designed to further explore the problem-solving abilities of the PTs. These items included some commonly found in texts and examination papers set for these learners. Item 1 required the PTs to calculate a missing value in a given geometrical shape; item 2 also required the PTs to find the surface area of a geometric figure; and item 3 required the PTs to write a complete proof of a theorem in geometry giving reasons.

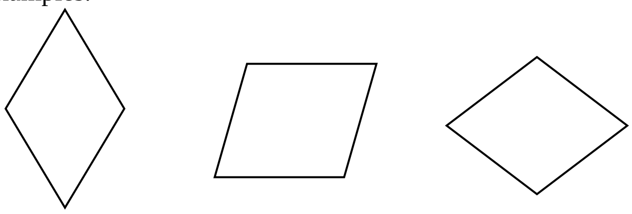
Item 5: Which of these are parallelograms?



A. J only
B. L only
C. J and M only
D. All are parallelograms
E. None of these are parallelograms

Figure 1. Sample of the items in subtest 1

Item 8: A rhombus is a 4- sided figure with all sides of the same length. Here are three examples.



Which of (A) – (D) is not true in every rhombus?
A. The two diagonals have the same length.
B. Each diagonal bisects two angles of the rhombus.
C. The two diagonals are perpendicular.
D. The opposite angles have the same measure.
E. All of (A) – (D) are true in every rhombus.

Figure 2. Sample of the items in subtest 2

Administration and Grading of the VHGT

The VHGT was used, both as the pre-test and post-test of the study. The test was meant to be written by all second year PTs who were participating in the study. All participants' answer sheets from the VHGT were read and scored by the researchers. Scoring of the part A of the VHGT was done as indicated below;
First grading method: Each correct response to the 25-item multiple-choice test was assigned 1 point. Hence, each pre-service teacher's score ranged from 0–25 marks.

Second grading method: the second method of grading the VHGT was based on "3 of 5 correct" success criterion suggested by Usiskin (1982). By this criterion, if a pre-service teacher answered correctly at least 3 out of the 5 items in any of the 5 subtests within the VHGT, the pre-service teacher was considered to have mastered that level. Using this grading system developed by Usiskin (1982), the PTs were assigned weighted sum scores in the following manner:

- 1 point for meeting criterion on items 1-5 (Level-I, Recognition);
- 2 points for meeting criterion on items 6-10 (Level-II, Analysis);
- 4 points for meeting criterion on items 11-15 (Level-III, Ordering);
- 8 points for meeting criterion on items 16-20 (Level-IV, Deduction);
- 16 points for meeting criterion on items 21-25 (Level-V, Rigor).

Thus, the maximum point obtainable by any pre-service teacher was 1 + 2 + 4 + 8 + 16 = 31 points. The method of calculating the weighted sum makes it possible for a person to determine upon which van Hiele Level the criterion has been met from the weighted sum alone. For example, a score of 7 indicates that the learner met the criterion at Levels I, II and III (i.e.1+ 2 + 4 = 7). The second grading system served the purpose of assigning the learners into various van Hiele Levels based on their responses. For the Part B, each of the 3 items was assigned 10 points. Thus, PTs' scores ranged between 0 and 30 marks.

Treatment

The Van Hiele Phase-based Instruction (VHPI) was applied to the experimental group whereas conventional instruction was applied to the control group throughout the study. PTs in both groups were taken through three different lessons. The topics treated covered properties of angles formed by parallel lines and their transversal, properties of quadrilaterals and lastly, the relationship between quadrilaterals.

The lessons taught in the experimental group were carried out with a method in which the concepts of discussion, group work, hands-on investigations and collaborative learning approaches are implemented in a related web in accordance with geometric thinking Levels of van Hiele. In line with the VHPI, Phase 1 (Information/Inquiry Phase) of each of the three lessons involved reviewing PTs' previous knowledge on the various topics and further holding a conversation with the PTs concerning the topic, in well-known language symbols making the context clear. This two-way teacher-student interaction was essential in understanding certain geometrical shapes such as making observations, asking questions and understanding the vocabulary for that particular geometrical shape. This also helped the researcher learn what prior knowledge the students have about the topic, and also informed PTs what direction further studies will take (Crowley, 1987).

In Phase 2; Guided Orientation Phase, PTs were given hands-on activities that allow them to become familiar with the many properties of the geometric concept. PTs were guided to carefully explore the objects used in the instruction by assigning carefully structured but simple tasks including folding, measuring and constructing that the researcher has carefully sequenced. In this phase, the PTs were expected to observe features such as angles, sides, diagonals etc. PTs were allowed to use their own language, but occasionally right terminologies were introduced.

In Phase 3; Explicitation Phase, PTs expressed in their own words what they have discovered in the previous phase. The target here was to introduce relevant geometrical terminologies. PTs shared their opinions on the relationships they have discovered in the hands-on activities.

In Phase 4 which is Free Orientation Phase, PTs were asked to solve more complex tasks independently. These tasks were designed to provide the PTs with problems that are open-ended and have multi-path solutions. Some of the tasks were more complex and required more free exploration to find solutions. This helped the PTs to master the network of relationships in the material.

Lastly, in Phase 5 which is the Integration Phase, PTs had a clear sense of purpose and could review and summarize what they have learned with the goal of forming an overview of the new network of relations. The PTs were provided with an overview of everything they have learned. No new material was presented during this phase, but only a summary of what has already been learned.

The instructional materials used for this study were designed using such different sources as past research materials, worksheets and textbooks and the adaptation of these were done in discussion with the mathematics tutors from the participating colleges on the van Hiele theory. The materials were then revised to address deficits after piloting was done during geometry teaching in one college.

In the control group, the PTs were taught the same topics (as in the experimental group) but with conventional instruction in which the learners strictly follow the instruction the teacher gives and active participations are not promoted. The PTs were not given any hands-on activities to help them explore geometrical concepts and form their own knowledge. In other words, geometry instruction was not in line with the VHPI but mainly in lecture format and therefore instruction was tutor-centred.

Table 2. Areas of instruction involving VHPI

Lesson	Topic	Objective	Van Hiele Instruction
Lesson 1	Properties of Angles Formed by Two Parallel Lines and Their Transversal	• Identify some basic properties of parallel lines. • Discover the relationships between the angles formed by two parallel lines cut by a transversal.	Using Mathematical sets, cut-outs of cardboards, pair of scissors, and masking tape, guide PTs through the phases of instruction.
Lesson 2	Properties of Quadrilaterals – congruence, line of symmetry, rotational, diagonals of square, rectangles etc.	• Discover properties of Quadrilaterals (squares, rectangles, rhombuses and parallelograms). • Use relationships among sides and angles of Quadrilaterals (squares, rectangles, rhombuses and parallelograms). • Use relationships among diagonals of Quadrilaterals.	Using Mathematical sets, cut-outs of cardboards, pair of scissors, and masking tape, guide PTs through the phases of instruction.
Lesson 3	Relationships between Properties of Quadrilaterals	Define and classify special types of Quadrilaterals.	Using Mathematical sets, cut-outs of cardboards, pair of scissors, and masking tape, guide PTs through the phases of instruction.

Results and Discussion

In this study, the geometric thinking levels of PTs in experimental and control groups, before and after the instructions were determined. Charts, Chi-square tests and t-tests were used to determine whether there is a significant difference between the geometric thinking levels and van Hiele Geometry Test scores of PTs in experimental and control groups. The study seeks to use quantitative analysis to find out the effect of VHPI on Ghanaian PTs' geometric thinking levels. The researchers therefore implemented an intervention based on the VHPI to address the PTs' difficulties in learning plane geometry.

Analysis of the Pre-VHGT

In this section, the results of the van Hiele geometry pre-test are presented.

Levels Reached by Pre-service Teachers in the Pre-VHGT

In Figure 3, the van Hiele Levels of geometric thinking attained by the PTs after the van Hiele Geometry pre-test in both the control and experimental groups are presented in a bar chart.

As shown in Figure 3, 24% and 22.67% of PTs attained van Hiele Level (VHL) 0 (i.e. the Pre-recognition Level or Level for those who have not yet attained any van Hiele Level) in the control and experimental groups respectively. For VHL 1, 30.67% and 33.33% of PTs in the control and experimental groups attained that Level respectively. 36% of PTs each in the control and experimental groups attained VHL 2. In addition, 9.33% and 8% of PTs attained VHL 3 in the control and experimental groups respectively. However, no PT attained VHL 4 and 5.

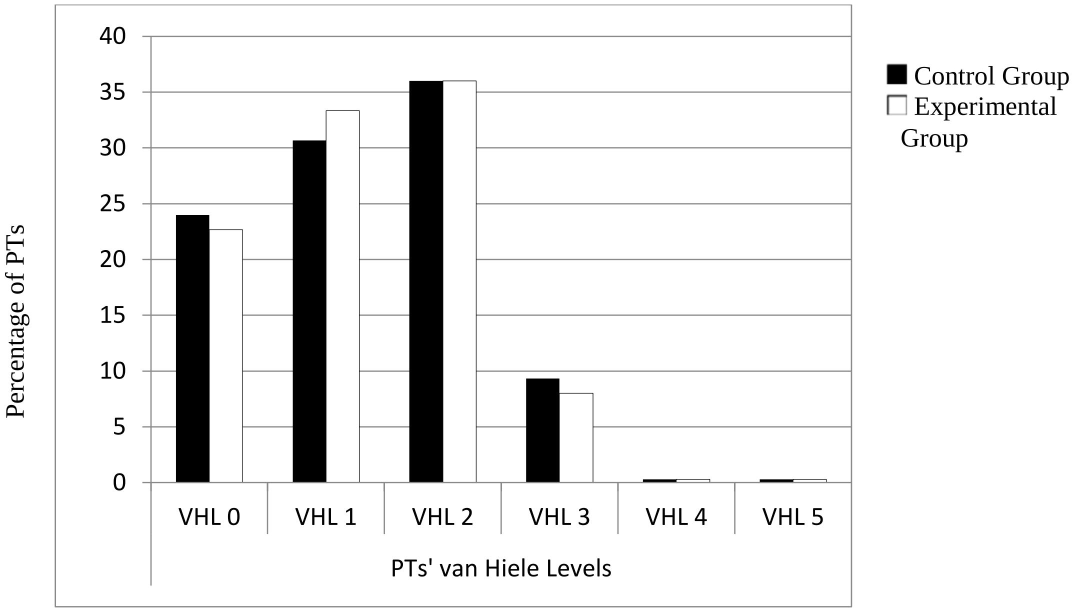


Figure 3. A bar chart showing participants' van Hiele levels of geometric thinking in the pretest

Comparison of Pre-test Scores of Control and Experimental Groups

Independent samples t-test statistic was conducted on the pre-test scores in both control and experimental groups. The results of the independent samples t-test on the participants' scores in the pre-test are presented in Table 3.

Table 3. Independent samples T-test of pre-test of control and experimental groups

Groups	N	Mean	Std. Dev.	t-value	df	p-value
Control	75	18.84	4.18	0.803	148	0.423
Experimental	75	18.32	3.74			

Table 2 presents the results of the independent-samples t-test performed on the pre-test scores of the 150 randomly selected PTs of the two independent groups (i.e. control and experimental groups). The results of this test (Table 2) revealed that there was no statistically significant difference in mean scores between the control group ($M = 18.84, SD = 4.18$) and experimental group ($M = 18.32, SD = 3.74$) conditions; $t\ (148) = 0.803,\ p = 0.423 > 0.05$. These results suggest that both the control group and the experimental group were almost at the same level of conceptual understanding of geometry before the start of treatment.

Analysis of the Pre-VHGT according to the van Hiele Levels

The chi-square test was further used to investigate whether there was a statistically significant difference in the van Hiele Levels of geometric thinking between the control and experimental groups before the start of intervention. Some PTs did not attain any van Hiele Level (VHL), the study utilized the 1 to 5 van Hiele numbering scheme to allow utilization of Level 0 (i.e. the Pre-recognition Level) for this category of PTs. Since no pre-service teacher attained Level 4 and Level 5 in the pretest, only the first three van Hiele Levels were measured. The investigation therefore involved a 2 × 4 design where van Hiele Level (VHL) was measured on four Levels (i.e. VHL 0, VHL 1, VHL 2 and VHL 3) and Group was measured on two Levels (i.e. Control and Experimental). Table 4 presents the 2 × 4 contingency table showing the actual and expected counts as well as within group and within van Hiele Level percentages of the distribution.

As can be seen from the table there are only slight differences in the distribution. For example, in the control group 18 PTs were classified as being at VHL 0 but less (i.e. 17.5) were expected, whereas, in the experimental group 17 PTs were classified as being at VHL 0 when more (i.e.17.5) were expected. Also the actual count for

PTs in VHL 1 in the control group was 23 with an expected count of 24.0, whilst in the experimental group, there were 25 PTs in VHL 1 with an expected count of 24.0. The actual count for PTs teachers in VHL 2 in the control group was 27 with an expected count of 27.0, similarly, in the experimental group, there were 27 PTs in VHL 2 with an expected count of 27.0. For the VHL 3 category, 7 were from the control group although 6.5 were expected, similarly, another 7 came from the experimental group when 6.5 were expected.

Table 4. Contingency table (cross tabulation) of group and van Hiele level (for pretest)

			Group		Total
			Control	Experimental	
Van Hiele Level	VHL0	Count	18	17	35
		Expected Count	17.5	17.5	35.0
		% within Van Hiele Level	51.4%	48.6%	100.0%
		% within Group	24.0%	22.7%	23.3%
		% of Total	12.0%	11.3%	23.3%
	VHL1	Count	23	25	48
		Expected Count	24.0	24.0	48.0
		% within Van Hiele Level	47.9%	52.1%	100.0%
		% within Group	30.7%	33.3%	32.0%
		% of Total	15.3%	16.7%	32.0%
	VHL2	Count	27	27	54
		Expected Count	27.0	27.0	54.0
		% within Van Hiele Level	50.0%	50.0%	100.0%
		% within Group	36.0%	36.0%	36.0%
		% of Total	18.0%	18.0%	36.0%
	VHL3	Count	7	6	13
		Expected Count	6.5	6.5	13.0
		% within Van Hiele Level	53.8%	46.2%	100.0%
		% within Group	9.3%	8.0%	8.7%
		% of Total	4.7%	4.0%	8.7%
Total		Count	75	75	150
		Expected Count	75.0	75.0	150.0
		% within Van Hiele Level	50.0%	50.0%	100.0%
		% within Group	100.0%	100.0%	100.0%
		% of Total	50.0%	50.0%	100.0%

The chi-square test was used to ascertain whether these differences are enough to be significant. Table 5 presents the results of the chi-square test for PTs pretest scores and as can be seen from this table the test reveals that there is no statistically significant difference in the van Hiele Levels of geometric thinking between the control and experimental groups at the pretest ($\chi^2 = 0.189; df = 3; p > 0.05$).

Table 5. Results from Chi-square Tests (for Pretest)

Chi-Square Tests			
	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	.189[a]	3	.979
Likelihood Ratio	.189	3	.979
N of Valid Cases	150		

* 0 cells (.0%) have expected count less than 5. The minimum expected count is 6.50.

Analysis of the Post-VHGT

In this section, the results of the van Hiele geometry post-test are presented.

Levels Reached by Pre-service Teachers in the Post-VHGT

The bar chart in Figure 4 provides a visual confirmation of the van Hiele Levels of geometric thinking attained by the PTs after the van Hiele Geometry posttest in both the control and experimental groups.

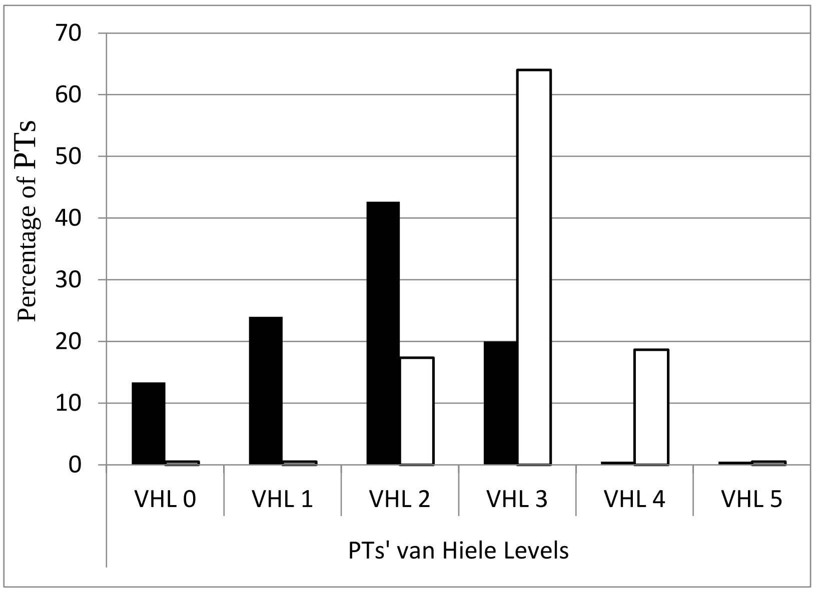


Figure 4. A bar chart showing PTs van Hiele levels of geometric thinking in the Post-test

The post-test indicated a huge improvement in the van Hiele Levels of geometric thinking among PTs in the experimental group. In other words, the PTs in the experimental group attained higher van Hiele Levels of geometric thinking as compared to their counterparts in the control group. As shown in Figure 4, 13.33% of PTs attained VHL 0 (i.e. the Pre-recognition Level or Level for those who have not yet attained any van Hiele Level) in the control group. However, no PT attained VHL 0 in the experimental group. 24% and 0% of the PTs attained VHL 1 in the control and experimental groups respectively. In addition, 42.67% and 17.33% of the PTs attained VHL 2 in the control and experimental groups respectively. 20% of PTs in the control group attained VHL 3 as compared to 64% in the experimental group. Significantly, 18.67% of PTs in the experimental group attained VHL 4 as compared to 0% in the control group. This indicates a significant improvement in the geometric thinking Levels among PTs in the experimental group. However, no PT attained VHL 5.

Comparison of Pre- and Post-test Scores of Control and Experimental Groups

Paired samples t-test statistic was conducted on the pre and post-test scores in both control and experimental groups. The results of the paired samples t-test (Table 6) of PTs in the control group indicated that there was no significant difference in their mean scores for the pre-test ($M = 18.84$, $SD = 4.182$) and post-test ($M = 20.25$, $SD = 4.305$) conditions; $t(74) = -0.445$, $p = 0.658 > 0.05$. However, in the experimental group, there was a significant difference in their mean scores for the pre-test ($M = 18.32$, $SD = 3.739$) and post-test ($M = 30.79$, $SD = 4.294$) conditions; $t(74) = -30.776$, $p = 0.000 < 0.05$.

Table 6. Paired sample t-tests of Pre-and Post-tests of the two groups

	N	Mean	Std. Dev.	Mean Difference	t-value	df	p-value
Pre-test – Post-test (Control Group)	75	18.84 20.25	4.182 4.305	1.410	-0.445	74	0.658
Pre-test – Post-test (Experimental Group)	75	18.32 30.79	3.739 4.294	12.467	-30.776	74	0.000

Effectiveness of VHPI in Improving Pre-service Teachers' Geometric Thinking Levels

The chi-square test was used to investigate whether there was a significant difference in the van Hiele Levels of geometric thinking between the control and experimental groups after the post-test. The investigation involved a 2 × 2 design where van Hiele Level (VHL) was measured on two Levels (i.e. VHL0,1&2 and VHL3&4) and Group was measured on two Levels (i.e. Control and Experimental). Table 7 presents the 2 × 2 contingency table showing the actual and expected counts as well as within group and within van Hiele Level percentages of the distribution. As can be seen from the table there are differences in the distribution. For example, in the control group 60 PTs were classified as being at VHL0,1&2 when a lot less (i.e. 36.5) were expected, whereas, in the experimental group 13 PTs were classified as being at VHL0,1&2 when more (i.e. 36.5) were expected. For the VHL3&4 category, 15 were from the control group but a lot more (i.e. 38.5) were expected, while, 62 came from the experimental group when 38.5 were expected.

Table 7. Contingency table (cross tabulation) of group and van Hiele level (for Post-test)

			Group		Total
			Control	Experimental	
van Hiele Level	VHL0,1&2	Count	60	13	73
		Expected Count	36.5	36.5	73.0
		% within van Hiele Level	82.2%	17.8%	100.0%
		% within Group	80.0%	17.3%	48.7%
		% of Total	40.0%	8.7%	48.7%
	VHL3&4	Count	15	62	77
		Expected Count	38.5	38.5	77.0
		% within van Hiele Level	19.5%	80.5%	100.0%
		% within Group	20.0%	82.7%	51.3%
		% of Total	10.0%	41.3%	51.3%
	Total	Count	75	75	150
		Expected Count	75.0	75.0	150.0
		% within van Hiele Level	50.0%	50.0%	100.0%
		% within Group	100.0%	100.0%	100.0%
		% of Total	50.0%	50.0%	100.0%

The chi-square test was used to ascertain whether these differences are enough to be significant. Table 7 reveals that only 19.5% of PTs in the control group attained VHL3&4, whereas 80.5% of PTs in the experimental group attained VHL3&4. Table 8 presents the results of the chi-square test for PTs post-test scores and as can be seen from this table the difference highlighted above was statistically significant ($\chi^2 = 58.949; df = 1; p < 0.05$). Hence the null hypothesis that there is no statistically significant difference among students' performance in van Hiele level of thinking, regarding the methods by which they were taught (traditional instruction versus VHPI) was rejected.

Table 8. Results from Chi-square Tests (Post-test)

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	58.949[a]	1	.000	.000	.000
Continuity Correction[b]	56.467	1	.000		
Likelihood Ratio	63.607	1	.000	.000	.000
Fisher's Exact Test				.000	.000
N of Valid Cases	150				

*0 cells (0.0%) have expected count less than 5. The minimum expected count is 36.50.
Computed only for a 2x2 table

The analysis of the levels of thinking in Figure 3 showed that before instruction, most of the PTs were at van Hiele level 0, 1 and 2. This was an indication that the majority of PTs had difficulty in recognizing geometric figures according to personal observation and identifying figures in non-standard positions. PTs also had an insufficient understanding concerning the identification of the geometric shapes using their properties. This confirms the literature (Armah, Cofie & Okpoti, 2017; Halat & Şahin, 2008; Halat, 2008; Pandiscio & Knight, 2010) that majority of PTs geometric thinking levels are low thus, not sufficient to teach their target audience. These findings raise concerns about the delivery of instruction that is appropriate to learners' level of thinking. Several studies (Atebe & Schafer, 2008; Erdogan & Durmus, 2009; Howse & Howse, 2015; Mostafa, Javad &

Reza, 2017; Siew & Chong, 2014; Usiskin, 1982; Yazdani, 2007) have emphasized that learners' abysmal performance in geometry are significantly linked to the quality of classroom instruction.

The analysis of the pretest scores from the independent-samples t-test showed that there was no significant difference between PTs conceptual understanding of plane geometry in experimental and control groups. However, in the posttest, it is evident from Table 6 that there was an increase in the mean scores of both groups. The experimental group's mean score increased from 18.32 to 30.79 and the control group's mean score also increased from 18.84 to 20.25. The increase in the mean score of the experimental group was higher than that of the control group. The paired samples t-test was used to find out whether there was a statistically significant difference between the two groups. Analysis from paired samples t-test statistic indicated that, that of the experimental group was significant at $p < 0.05$.

As shown in Figure 4, in the control groups' post-test, the percentage number of PTs at VHL 0 decreased from 24% to 13.33%, the percentage number of PTs at VHL 1 also decreased from 30% to 24%. However, the percentage number of PTs at VHL 2 increased from 36% to 42.67% and the percentage number of PTs at VHL 3 also increased from 9.33% to 20%. Whereas in the experimental groups' post-test, the percentage number of PTs at VHL 0 decreased from 22.67% to 0%, the percentage number of PTs at VHL 1 decreased from 33.33% to 0%, the percentage number of PTs at VHL 2 also decreased from 36% to 17.33%. However, the percentage number of PTs at VHL 3 increased significantly from 8% to 64%. Also, it was interesting to see 18.67% of PTs operating at VHL 4 in the experimental groups' posttest. Consequently, the experimental group recorded more PTs at the higher van Hiele Levels than those in the control group. This is an indication that the van Hiele Phase-based Instruction (VHPI) for the experimental group had a more positive effect than the conventional approach in the control group.

Moreover, the findings from the chi-square analysis indicated that there was no significant difference between PTs' geometric thinking Levels in the control and experimental groups at the pretest (i.e. before the start of intervention) at $p > 0.05$. This showed that both groups were at the same level of geometric thinking before treatment. However there was a significant difference between PTs' geometric thinking Levels in the control and experimental groups at the posttest (i.e. after the intervention) at $p < 0.05$ favoring PTs in the experimental group; 19.5% of PTs in the control group attained VHL 3 and 4, as compared to 80.5% of PTs in the experimental group who attained VHL 3 and 4. This finding confirms the claim (Al-ebous, 2016; Alex & Mammen, 2016; Erdogan and Durmus, 2009; Mostafa, Javad & Reza, 2017; Ramlan, 2016; Siew & Chong; 2014; Yazdani, 2007) that instruction consistent with VHPI has a positive effect on the geometric thinking Levels of learners.

The considerably low percentages of PTs at van Hiele Level 4 suggest that the PTs have difficulties understanding the proof of theorems. This has also been remarked by past researchers (Armah, Cofie & Okpoti, 2017; Erdogan and Durmus, 2009; Halat, 2008; Halat & Şahin, 2008; Pandiscio & Knight, 2010). Key to this study is the finding that none of the PTs attained van Hiele Level 5 in both pretest and posttest. These findings show that the cohorts of PTs were not prepared for formal geometric proofs in college of education. Although all the PTs in the experimental group were taught with VHPI and there was a significant increase in their van Hiele levels of geometric thinking in the post-test, it was realized that the geometric thinking levels of some PTs could not be raised to a level of thinking that is expected at the college of education level. A comparison between the pretest and the posttest of the PTs in the control group shows that there was advancement in their geometric thinking levels which could be attributed to the conventional method of teaching. However, the advancement was not as significant as that of the experimental group.

Conclusion

This study investigated the effect of VHPI on PTs' geometric thinking in terms of the van Hiele Levels. The researchers utilized pretest posttest quasi-experimental design. The sample comprised of 150 second year PTs, 75 in the experimental group and the other 75 in the control group. PTs in the experimental group were given instruction according to the VHPI while PTs in the control group received conventional instruction. The van Hiele Geometry Test (VHGT) was administered as both pretest and posttest of the study. Both groups showed increment in their post-VHGT as compared to the pre-VHGT. However, the PTs in the experimental group achieved better levels of geometric thinking compared to those in the control group. The significant improvement in the performance of the experimental group having more PTs at Levels 3 and 4 than at level 0, 1 and 2 suggest that the VHPI had a more positive effect on PTs' geometric thinking than the conventional approach. The hands-on activities using concrete manipulative materials provided an equal support for every pre-service teacher to eventually achieve an enhanced conceptual understanding of the geometric concepts taught. The use of the VHPI has the potential of helping PTs improve their geometric reasoning while enhancing their understanding of geometry concepts to enable them teach the subject diligently at the basic level later when they have completed college of education. If educators want to improve Ghanaian students' mathematics performance in international and national examinations like TIMSS, BECE, WASSCE and DBE examinations then the use of the VHPI is imperative. The information generated from this study therefore is available for policy makers, teachers and other stakeholders to help improve students' geometric thinking Levels in Ghana through the use of the VHPI.

Recommendations

It is therefore recommended that as is done in other countries such as United States, Netherlands and Russia, colleges of education mathematics tutors should revise their instructional methods to utilize the VHPI in planning and delivering lessons. In view of this, college of education mathematics tutors should be encouraged to use teaching learning materials in enhancing and developing the geometric thinking of PTs. PTs should also be provided with hands-on activities using manipulative concrete materials for discovering the properties of simple geometric shapes in different orientations. Seminars and workshops can also be organized for mathematics teachers, textbook writers and curriculum developers to appraise with the use of VHPI. The limitations in this study relate to the non-existence of information regarding the van Hiele model in the Ghanaian mathematics curriculum. The researchers were unable to draw from more local examples and knowledge. The insufficient duration of this study also produces an obvious limitation. The three consecutive lessons in this study definitely cannot achieve a very convincing result. A more lengthy study may produce persuasive results in examining the effectiveness of the VHPI. The research concentrated on investigating only two-dimensional figures using the van Hiele model. Future study employing this study design may consider using the model to investigate other areas of geometry such as three-dimensional figures, circle theorems and coordinate geometry. A study in this area can also be done to involve more colleges of education to obtain the general picture of how the VHPI improves PTs' geometric thinking Levels.

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