ISSN: 2148-9955

International Journal of Research in Education and Science (IJRES)

www.ijres.net

U.S. Middle School Mathematics Teachers' Perceptions of the Standards for Mathematical Practice by Textbook Type

Jon D. Davis[1], Jeffrey Choppin[2], Corey Drake[3], Amy Roth McDuffie[4], Cynthia Carson[2]
[1]Western Michigan University
[2]University of Rochester
[3]Michigan State University
[4]Washington State University Tri-Cities

To cite this article:

Davis, J.D., Choppin, J., Drake, C., Roth McDuffie, A., & Carson, C. (2018). U.S. middle school mathematics teachers' perceptions of the standards for mathematical practice by textbook type. *International Journal of Research in Education and Science (IJRES), 4*(1), 55-68. DOI:10.21890/ijres.382934

International Journal of Research in Education and Science
Volume 4, Issue 1, Winter 2018 DOI:10.21890/ijres.382934

U.S. Middle School Mathematics Teachers' Perceptions of the Standards for Mathematical Practice by Textbook Type

Jon D. Davis, Jeffrey Choppin, Corey Drake, Amy Roth McDuffie, Cynthia Carson

Article Info

Article History

Received:
19 April 2017

Accepted:
20 July 2017

Keywords

Standards
Teacher perceptions
Mathematical processes

Abstract

An important component of the Common Core State Standards for Mathematics (CCSSM), used by the majority of states in the U.S., has the eight standards for mathematical practice (SMPs). While surveys have investigated teachers' perceptions of the CCSSM few have investigated middle school mathematics teachers' (MSMTs') (grades 6-8) perceptions of the SMPs. Similar to other countries, teachers in the United States frequently use mathematics textbooks. Two different types of textbooks have been in use in the United States, conventional and standards-based. The latter were designed on the basis of earlier standards documents including versions of the SMPs. As a result of these changes to the educational system in the U.S., we sought to characterize MSMTs' perceptions of the SMPs and investigate if these perceptions differed by the type of textbook teachers used. We found that MSMTs struggled in naming the SMPs, misinterpreted the modeling SMP, and conflated components of reform-oriented instruction with the SMPs. MSMTs using a standards-based textbook were more likely to view their textbooks as aligned with the SMPs, $\chi^2(3) = 7.708, p = .026$ and to view the SMPs as an instructional philosophy for the CCSSM, Fisher's Exact Test = 3.881, p = .05. The implications of these results are discussed.

Introduction

The Common Core State Standards for Mathematics (CCSSM) were adopted by a majority of states in the United States in 2010 (Common Core State Standards Initiative, 2010). Teachers are key change agents in implementing the CCSSM and consequently, this study focused on teachers' perceptions of these standards, specifically the Standards for Mathematical Practice (SMPs). The SMPs are similar in nature to the aims appearing in the standards of other countries (e.g., National Council for Curriculum and Assessment, 2012). While a number of surveys have examined teachers' perceptions of the CCSSM (Choppin et al., 2013; Davis et al., 2013; EPE Research Center, 2013), less research exists on teachers' perceptions of the SMPs. As a result, this study examines the perceptions that a group of middle school mathematics teachers (MSMTs) reported about the SMPs as ascertained from semi-structured interviews. Based on prior research on standards implementation (e.g., Spillane, 2004), we operated from the perspective that there would be substantial variation in teachers' interpretations of the SMPs and that these varying interpretations would influence teachers' enactment of the SMPs in their planning and instruction. Additionally, we hypothesized teachers' perceptions of the SMPs would be influenced by the type of textbook they were using, standards-based or conventional.

Background

History of the Standards for Mathematical Practice

As Martin and Hart (2012) pointed out, previous standards documents have contained versions of the SMPs. For instance, the *Curriculum and Evaluation Standards for School Mathematics* (CESSM) (National Council of Teachers of Mathematics [NCTM], 1989) contained four standards that have much in common with the SMPs: mathematics as problem solving; mathematics as communication; mathematics as reasoning; and mathematical connections. These were organized within the document as standards on the same footing as content standards. The *Principles and Standards for School Mathematics* (hereafter referred to as PSSM) (NCTM, 2001) contained five content standards in addition to *process standards*. These process standards at each grade band consisted of

the following: problem solving; reasoning and proof; communication; connections; and representation. Thus, similar to the CESSM, the PSSM elevated process standards to the same level as content standards.

Standards-Based and Conventional Curriculum Types

The CESSM were used as a framework from which the National Science Foundation (NSF) funded a total of fifteen curriculum programs spanning grades K-12 (Senk & Thompson, 2003). Many of these curricula were revised using the PSSM as guidance (Hirsch, 2007). These curricula have been referred to as *standards-based programs* and stand in contrast to *conventional textbook programs* that did not use these reform documents to guide design (Stein, Remillard, & Smith, 2007). In summarizing standards-based programs, Stein and colleagues noted that these programs tended not to deliver concepts to students, but instead they were designed to engage students in tasks from which concepts emerge. Standards-based programs tended to focus more on concepts and problem-solving before students develop mathematical fluency while conventional programs tended to focus more on providing opportunities for students to develop facility with mathematical procedures.

A number of case studies have been completed on teachers' use of standards-based curricula (e.g., Collopy, 2003; Frykholm, 2004; Keiser & Lambdin, 1996; Lloyd, 1999; Remillard & Bryans, 2004; Wilson & Lloyd, 2000). In summarizing these studies, Stein and colleagues (2007) identified four factors that influence how teachers take up and use these programs: teachers, students, context, and curriculum. In particular, teachers' beliefs about mathematics, how mathematics should be taught, and students as learners of mathematics (Chavez, 2003; Lloyd & Wilson, 1998; Remillard, 1999; Romberg, 1997) influenced teachers' use of these programs. Additionally, evidence indicated that teachers' use of these curricula can also influence their beliefs (Clarke, 1997). This connection between documents intended to change the direction of mathematics education or promote reform such as the *Standards*, PSSM, and standards-based curriculum materials have led researchers to label instruction aligned with these curricula or standards as *reform-oriented instruction* (Anderson & Bobis, 2005; Schoen, Cebulla, Finn, & Fi, 2003).

Previous Surveys Regarding Teachers' Perceptions of the CCSSM

Although the SMPs are considered an important component of the CCSSM (McCallum, 2012), only a few studies have examined teachers' perceptions of this component of the standards. While several studies have examined teachers' perceptions of the CCSSM, most have not focused on the SMPs (c.f., Cogan, Schmidt, & Houang, 2013; EPE Research Center, 2013; Kane et al., 2016; Primary Sources, 2013, 2014). Exceptions include surveys that we conducted (Choppin et al., 2013; Davis et al., 2013), as well as Perry and colleagues (2015), and Opfer, Kaufman, and Thompson (2016). We begin by describing our prior findings with regard to teachers' perceptions of the SMPs.

In 2012, we piloted a survey examining MSMTs' perceptions of the CCSSM. We asked a group of 33 MSMTs to select the three most important practices out of the list of eight SMPs. These results are seen in Table 1. It is interesting that this list is nearly identical to the listing of the SMPs in the CCSSM. That is, the most frequently mentioned SMP is the first one that is listed in the CCSSM and so on. The only exception comes at the end of the list as the seventh SMP (structure) was the practice that was least mentioned. Although the sample is small, these findings suggest that teachers imbue the order in which the practices are mentioned as suggestive of a priority.

Table 1. MSMTs' three most important practices

SMP	Frequency (Percentage)
SMP 1: Make sense of problems and persevere in solving them.	27 (75.8%)
SMP 2: Reason abstractly and quantitatively.	17 (51.5%)
SMP 3: Construct viable arguments and critique the reasoning of others.	16 (48.5%)
SMP 4: Model with mathematics.	14 (42.4%)
SMP 5: Use appropriate tools strategically.	10 (30.3%)
SMP 6: Attend to precision.	8 (24.2%)
SMP 8: Look for and express regularity in repeated reasoning.	5 (15.2%)
SMP 7: Look for and make use of structure.	4 (12.1%)

A revised version of the survey piloted in 2012 was administered in February 2013 to a national sample of 403 MSMTs. The frequency and percentages disagreeing and agreeing with each question appear in Table 2. The majority of MSMTs in this sample perceived the SMPs as the biggest innovation of the CCSSM and agreed that these practices were essential in helping students to learn mathematics. MSMTs believed that students could focus on more than one practice at a time. However, the majority of MSMTs viewed successful participation as requiring students to first understand content. This contradicts MSMTs' perceptions that participation in the SMPs is essential for students to learn mathematics. Teachers might have interpreted the word "mathematics" differently from "content," perceiving that the former involves both mathematics content as well as the SMPs.

Table 2. Survey items involving SMP

Survey Question	Disagree/Strongly Disagree	Agree/Strongly Agree
The focus on mathematical practices is the biggest innovation of the CCSSM	119 (29.6%)	284 (70%)
It is necessary to focus on only one mathematical practice at a time.	304 (75.4%)	99 (24.5%)
Participating in the practices is essential for students to learn mathematics.	19 (4.7%)	384 (95.3%)
Successful participation in the practices requires that students first understand the content.	103 (25.6%)	300 (74.4%)

We conducted two more national surveys. The first survey of 366 MSMTs occurred in April – May 2013 (Davis et al., 2013) while the second survey included 1241 MSMTs and was conducted in May-June 2015. The questions involving SMPs as well as the frequencies and percentages of teachers in the sample that agreed/strongly agreed with these statements across both surveys are shown in Table 3. The results in this table illustrate that not only did teachers see the SMPs as the most innovative aspect of the CCSSM, they also believed that the SMPs are more rigorous than previous state curriculum frameworks. Teachers were not asked to define rigorous, so it is unclear what they considered this terminology to mean. McCallum (2012), in describing the CCSSM to an international audience, defined rigor as consisting of a balance among conceptual understanding, procedural fluency, and mathematical applications. Also, the sample viewed the CCSSM as incorporating more communication and student exploration than previous state standards. Moreover, they reported agreement with the statement that CCSSM-aligned assessments would assess all of the SMPs, though the level of agreement with this statement was notably lower in Survey 3 than Survey 2. In addition to these questions, we also asked survey participants about their level of agreement/disagreement with the following statement: The CCSSM provides more opportunities for students to struggle while solving problems. A total of 989 (79.7%) MSMTs agreed or strongly agreed with this statement.

Table 3. SMP-related survey items from Survey 2 and Survey 3

Survey Question	Survey 2 (N = 366) Agree/Strongly Agree	Survey 3 (N = 1241) Agree/Strongly Agree
Compared to your state standards before the adoption of the CCSSM, the CCSSM Mathematical Practice Standards are more rigorous.	310 (84.7%)	1090 (87.8%)
Compared to your state standards before the adoption of the CCSSM, the CCSSM will require you to emphasize communication more with your students.	310 (84.7%)	956 (77%)
Compared to your state standards before the adoption of CCSSM, the CCSSM will require you to incorporate more student exploration.	311 (84.9%)	915 (73.7%)
The new state assessments will assess each of the eight CCSSM mathematical practice standards.	307 (83.8%)	750 (60.4%)

Perry and colleagues (2015) examined the CCSSM perceptions of 990 K-12 mathematics teachers, 122 site administrators, and 33 district administrators. Unlike the majority of other CCSSM surveys, this survey examined teachers' perceptions about the SMPs. Overall, teachers felt confident that their lesson plans contained opportunities for students to learn the SMPs (69%) and only 23% reported that they received few professional development opportunities to learn about the SMPs. Additionally, the majority of teachers stated that they did not need assistance in aligning curriculum to the CCSSM content and SMPs, did not need

assistance in creating lesson plans that embody the SMPs, and that they possessed a firm understanding of the SMPs.

Opfer and colleagues (2016) examined the perceptions of a nationally representative sample (N = 2,577) of K-12 public school teachers in the US with regard to the CCSSM as well as the Common Core English Language Arts and Literacy (CCELA) standards. The researchers used the following text from SMP 4 as a definition of modeling: "know[ing] how to solve problems arising in everyday life, society and the workplace" (CCSSI, 2010, p. 7). Less than half of the respondents (42%) chose a definition of modeling that was consistent with this wording. Moreover, they found differences between grades 9-12 teachers and elementary math teachers working in grades K-5. The former was statistically significantly ($p < .01$) more likely to define modeling in a way that was consistent with the wording of this standard in the CCSSM while the latter was more likely to consider modeling to involve the use of hands-on manipulatives to find the solution to problems involving mathematics. Cirillo, Pelesko, Felton-Koestler, and Rubel (2016) describe the former as mathematical modeling while the latter is defined as modeling mathematics.

Mathematical modeling begins with the real-world and seeks to use mathematics to better understand problems set within this realm. Modeling mathematics, in contrast, is set within the mathematical world. The perceptions reported by Opfer and colleagues may be due to the fact that the CCSSM does not clarify the differences between modeling mathematics and mathematical modeling. Indeed, in grades K-8 the CCSSM often references models such as rectangular arrays while in the high school conceptual category and SMP 4 model refers to mathematical modeling (Cirillo et al., 2016).

Opfer and colleagues (2016) also found that over half of the respondents stated that their students engaged daily or almost daily in the following practices: use mathematical language precisely; explain and justify work; and make sense of and persevere in solving problems. A much lower percentage of teachers asked students to utilize structure or engage in the construction of arguments and critique others' reasoning. Secondary teachers were less likely than elementary teachers to ask students to engage in some of the SMPs such as the appropriate use of tools. Teachers with less experience were more likely to ask students to engage with the practices on a daily basis.

Shaughnessy, Ball, Mann, and Garcia (2015) have argued that students need instruction to learn the SMPs and that a component of this instruction involves explicitness. Following Selling (2016) we define explicit to include those actions of the teacher that involve direct instruction (Gersten & Carnine, 1984) or the actions of teacher and students in reflective discourse or collective reflection (Cobb, Boufi, McClain, and Whitenack, 1997). Selling identified eight different teacher moves that helped to make the SMPs explicit. A sample of these included: naming SMPs; highlighting students' interactions with SMPs; evaluating student engagement in SMPs; and explaining the goal of students' engagement in the SMPs.

Summary

The predecessors of the SMPs include statements such as *mathematics as problem solving* and *mathematics as communication* appearing in the CESSM (NCTM, 1989). These standards later became known as process standards in the PSSM and were joined by another standard, *representation*. Our national surveys on MSMTs' perceptions suggest that teachers view the SMPs as the biggest innovation of the CCSSM, that these practices are essential to learning mathematics, but that students can only engage in the practices once they have mastered content. Teachers are generally confident that the SMPs will be assessed by CCSSM-aligned assessments. Research by Perry and colleagues suggests that teachers are fairly confident that they can develop lessons that incorporate the SMPs and believe that the professional development they have received around the CCSSM has addressed the SMPs. Opfer and colleagues found that less than half of the teachers they sampled engaged students in the SMPs on a daily basis and emphasized some SMPs more than others.

Although a number of surveys have been conducted regarding teachers' perceptions of the CCSSM, very few have addressed their views on the SMPs or if they have, the perceptions of MSMTs have not been disaggregated from the perceptions of elementary teachers (e.g., Opfer et al. 2016). Also, what we know about teachers' perceptions of the SMPs have come from large-scale surveys. While these data involve large sample sizes they are lacking the richness that comes through semi-structured interviews. Finally, we know that the use of standards-based curricula can influence teachers' beliefs (Clarke, 1997), but few studies have investigated the relationship between teachers' curriculum use and their perceptions of the CCSSM.

Purpose and Research Questions

This study examines teachers' perceptions of the SMPs via background interviews with 76 MSMTs. Specifically, the following research questions were examined in this study.

1. What perceptions about the SMPs do participating MSMTs report?
2. Do perceptions of the SMPs differ between MSMTs using standards-based and conventional curriculum types?

Method

Participants and Data Sources

A total of 76 MSMTs were interviewed during two waves: 2012-2013 and 2013-2014. A total of 11 participants are male (14%) and 65 are female (86%). We used Stein, Remillard, and Smith's (2007) terminology of conventional (C) and standards-based (SB) to categorize the curricula that teachers were using at the time of the interview. A total of 39 teachers were using C programs and 37 teachers were using SB programs. There were a total of three different curricula that were categorized as SB and ten programs in use by MSMTs that were labeled as C.

The data comprising this study come from background interviews with MSMTs as part of a larger study funded by the National Science Foundation (DRL #1222359). This larger study was designed to develop principles to support MSMTs' capacity to use curriculum resources to design instruction that addresses the CCSSM. As an initial step in developing these principles we were interested in teachers' general perceptions of the CCSSM.

Analysis

A mixed methods research design (Creswell, 2014) was used in this study. Qualitative data analysis methods of analytic induction and constant comparison were used to identify patterns within the background interviews. As conjectures were developed we further examined the data to confirm or refute them (Corbin & Strauss, 2008; Miles, Huberman, & Saldana, 2014). The first step in data analysis involved identifying stanzas. Each stanza consisted of the interviewer's question, teacher's response, and additional text required for understanding the context. Next we created a codebook consisting of descriptions and decision rules associated with each code. A sample of these codes is as follows: preparations for implementing the CCSSM; curriculum materials in use or planned for use; descriptions of planning processes for a lesson, unit, or year; state assessments; and standards for mathematical practices. All interviews were coded in the qualitative analysis software, HyperResearch. To insure consistency in coding, two researchers independently coded each transcript. A third researcher was involved to discuss and resolve coding discrepancies. The next phase of data analysis involved identifying themes in teacher responses to this question, text coded as standards for mathematical practice, and foci associated with our analysis of the background literature (e.g., explicitness).

In identifying the SMPs that participating teachers actually mentioned, we used key words associated with a particular SMP as seen in Table 4. For instance, teachers' mentioning of the word persistence was sufficient to identify the first SMP. If teachers mentioned verbs that were associated with several SMPs this was not sufficient to identify these SMPs. For instance, the word "explain" occurs in the description of SMP 1: Make sense of problems and persevere in solving them and the description of SMP 3: Construct viable arguments and critique the reasoning of others. While these instances were not coded for a particular SMP, we kept track of the specific terminology that teachers used that was associated with the practices.

Quantitative analyses involving independent samples t-tests, Chi Square tests, and Fisher Exact Tests were used to examine differences in the frequencies of mentioned SMPs or among coded themes between MSMTs using C and SB curricula. Fisher's Exact Tests were used to analyze categorical data with at least one cell with a frequency of less than five. Chi Square tests were used on all other categorical data. Analyses of average number of SMPs were conducted with independent samples t-tests. An alpha level of .05 was used to determine significance for all tests.

Table 4. Keywords associated with SMPs

SMP	Keywords
1	Perseverance, Making sense of problems, Understanding problems and perseverance in solving them, solve a problem and struggling, Stick with a problem long enough, Persevere in solving them
2	Reason abstractly, Abstract thinking, Think abstractly, Move them toward the abstract
3	Constructing arguments, Critique the reasoning of others, Construct a viable argument, Critique the work of others, Critique others' arguments, Strengths and weaknesses of that argument
4	Modeling, Model with mathematics
5	Using tools strategically, Using tools appropriately, Tools that make the most sense, Choose a tool, Tool use, Selecting the right tools
6	Precision, Precise, Precision of communication, Precise in our language
7	Structure, Looking for structure
8	Pattern, Looking for patterns, Repeated reasoning, Regularity

Results and Discussion

In characterizing MSMTs' perceptions of the SMPs, we examined their use of terminology associated with the SMP when specifically asked about the SMPs. Table 5 shows the number of SMPs mentioned by MSMTs. The most frequent category was zero SMPs mentioned. The majority of MSMTs mentioned at most one SMP. The mean number of SMPs mentioned by the participants was 1.6. Overall, the participants seemed to lack familiarity with the actual wording of the SMPs in the CCSSM. These differences in mean number of SMPs mentioned during the background interview were not statistically significant, $t(74) = -.794$, $p = .430$.

Table 5. SMPs mentioned by curriculum

SMPs Mentioned	C	SB
0	15	7
1	8	13
2	4	8
3	4	7
4	3	1
5	3	3
Mean	1.5	1.8

As shown in Table 6, we further explored the actual SMPs that interviewed teachers mentioned during interviews. The most frequently mentioned SMP was SMP 1: Make sense of problems and persevere in solving them. The next most frequent SMP was attending to precision (SMP 6). SMP 5, using appropriate tools strategically, was only occasionally mentioned by MSMTs. The SMP mentioned least frequently involved the identification and use of mathematical structure (SMP 7). Teachers using C and SB textbooks identified SMPs 1, 3, and 8 at the same frequency. SB teachers more frequently mentioned SMPs 2, 5, and 7. More teachers using C textbooks mentioned SMP 4 than teachers using SB textbooks. These differences between curriculum types shown in Table 5 were not statistically significant as calculated through a Fisher's Exact test = 4.941, $p = .691$.

Table 6. SMPs mentioned

SMP	C	SB
1. Make sense of problems and persevere in solving them.	15	15
2. Reason abstractly and quantitatively.	4	8
3. Construct viable arguments and critique the reasoning of others.	11	11
4. Model with mathematics.	8	7
5. Use appropriate tools strategically.	2	7
6. Attend to precision.	10	14
7. Look for and make use of structure.	0	2
8. Look for and express regularity in repeated reasoning.	5	5

MSMTs did not frequently mention SMPs by name when they were asked questions about these standards. Some of the terminology used by MSMTs included verbs that appear in multiple SMPs (e.g., explain) as well as language that does not appear in the SMPs at all such as the words, *exploration* or *discovery*. These teachers

appeared to associate the SMPs with *reform-oriented instruction*. The five most frequently appearing categories broken down by curriculum type appear in Table 7. Moreover, the language teachers used around the practices suggested that they believed that the SMPs would be challenging for them to teach and for students to learn. For example, some teachers viewed problem-solving as involving complex problems for which students did not currently possess a procedure.

Table 7. Terminology used to refer to the SMP by curriculum type

Code	C	SB
Problem-Solving	11	5
Explanation	4	7
Student Autonomy	3	8
Real-World	8	4
Multiple Strategies	1	4

Other teachers reported that the SMPs asked students to explain their work, which they reported was more difficult for students than environments where this was not a classroom norm. Eleven teachers interpreted the SMPs as asking students to think for themselves (Student Autonomy in Table 7). Teachers reported that students thinking for themselves involved more work on the teachers' part, thus increasing the level of difficulty of implementing the CCSSM when compared to previous standards. Twelve teachers stated the presence of real-world contexts in the SMPs, perhaps as a result of the modeling practice, increased teachers' difficulties in implementing the CCSSM. They believed that problems set within a real-world context were much more difficult for students to solve.

When teachers using a C curriculum type mentioned challenge with respect to the SMPs it was linked to problem-solving and real-world contexts. Teachers using a SB curriculum type were more likely to associate the difficulty of implementing the CCSSM with explanation, student autonomy, and multiple strategies. Statistical analyses, however, revealed that the differences between curriculum categories were not statistically significant (Fisher's Exact Test = 8.098, p = .082).

Table 8 shows that teachers using SB curricula were more likely than teachers using C curricula to believe that their textbooks were aligned with the SMPs and these differences were statistically significant $\chi^2(3) = 7.708, p = .026$. Thirteen teachers provided us with mixed responses stating that the textbook was aligned with some of the practices but not others. This is seen in the following excerpt from Crane who was using a textbook categorized as conventional.

> Interviewer: Do you feel like this student textbook emphasizes those eight standards for mathematical practice in the Common Core?
>
> Crane: Some of them but not all of them, like the modeling part, I don't feel like is as strong as it needs to be because I feel like when a student should be able to model something and they have to have that concrete understanding because I think that is weak in that text (January 7, 2014).

Table 8. Teachers' perspectives on textbook alignment with SMP by curriculum type

Category	C (N = 21)	SB (N = 29)
Yes	8 (38%)	22 (76%)
No	3 (15%)	1 (3%)
Mixed	7 (33%)	6 (21%)
Don't Know	2 (10%)	0 (0%)

Curiously, two teachers were not sure if their textbook was aligned with the practices and were focused more on alignment with content standards, as seen in the following excerpt from Guinness who was also teaching from a conventional textbook.

> Interviewer: In your look at the material so far do they seem to emphasize the mathematical practices?
> Guinness: I haven't looked that closely in it, I've more looked for what topics are covered and when are they covered and can we fit those in in this quarter and how is that going to look for our testing dates and so it's just basically more of a framework, not looking directly at the content specific kind of stuff (May 17, 2013).

Table 9 documents the number of MSMTs who viewed the SMPs as different from previous state standards. This perspective is seen in the following excerpt from a teacher using a conventional curriculum.

> Well I think the major features would have to be like the practice standards [SMPs], because I mean the content is the content, if you had State Standards you had content, Common Core has the content, so what I feel is really different is the practice standards and integrating those with the content (Nichols, October 7, 2013).

The majority of MSMTs, regardless of type of curriculum, saw the SMPs as a change from previous standards. The percentages across these groups were very similar, thus the differences were not statistically significant, $\chi^2(1) = .007$, $p = .609$.

Table 9. Are SMPs a change from previous standards

Curriculum	Yes	No
C	14 (70%)	6 (30%)
SB	11 (69%)	5 (31%)

For those teachers who stated that the SMPs caused them to change their instructional practices, the change categories associated with each of these explanations appears in Table 10. The most frequent category of change was problem-solving. The instruction category referred to teachers stating that the SMPs caused them to change their instruction by moving them away from more traditional teaching techniques to more *reform-oriented instruction*. This is seen in the following excerpt.

> You can't teach [all of the] mathematical practices, I mean you can't stand up in front of a class and teach those mathematical practices to kids and have kids really truly understand them and learn them I think if you're not teaching on a more inquiry based way (Pless, November 7, 2012).

The real-world, group work, and precision categories were the least frequently mentioned changes to teachers' instructional practices. The differences reported in the table were not statistically significant (Fisher's Exact Fest = 6.887, $p = .316$).

Table 10. SMP changes from previous standards

Category	C ($N = 14$)	SB ($N = 10$)
Problem-Solving	10	7
Precision	1	3
Instruction	4	3
Reasoning	5	2
Group Work	3	0
Communication	6	1
Real-World	3	0

A total of 50 MSMTs were asked the following question as part of the background interview: What do you see as a primary instructional philosophy of the Common Core? All of the interviewed teachers perceived that the CCSSM embodied an instructional philosophy. We further examined these responses to see if this instructional philosophy was connected to the SMPs. These results by curriculum are seen in Table 11. Teachers using an SB curriculum type were more likely than teachers using C curriculum types to consider the SMPs as equivalent to an instructional philosophy as seen in the following quote: "But I know with the math practices that's where they're really trying to get us to, I guess the kids to do more, I guess the practice is how, I don't know, instructionally [sic]" (Allen, November 7, 2012).

These results were statistically significant (Fisher's Exact Test = 3.881, $p = .05$). Moreover, regardless of curriculum, for those teachers who saw the SMPs as a CCSSM instructional philosophy, that philosophy was reduced to SMP 1: making sense of problems and persevering in solving them. For the fifteen teachers using a C curriculum type who did not see a connection to the SMPs, their responses fell into two categories: understanding ($N = 8$) and applications ($N = 7$). Teachers who mentioned understanding as a philosophy of the CCSSM stated that the standards expected students to understand content in a deeper way than previous standards, learn mathematics conceptually, understand why, or engage in higher level thinking.

Teachers who felt that applications were the instructional philosophy of the CCSSM mentioned that students were asked to solve applications type problems, solve real-life problems, or engage in hands-on problem-

solving. For the five teachers using a SB curriculum that did not describe a connection between an instructional philosophy and the SMPs, three teachers connected an instructional philosophy to understanding and three teachers connected an instructional philosophy to applications (one teacher mentioned both understanding and applications). An example of understanding as an instructional philosophy is seen in the following excerpt:

> Teaching them the process behind why they do that, because they can get a procedure, they can follow steps, but knowing why they have to follow the steps as to why the Common Core is, I think it's more of a why do you do this, rather than just this is the procedure, how do I distribute (Blackburn, November 20, 2012).

An example of a teacher linking applications with an instructional philosophy is seen in this excerpt:

> I think I don't know as far as like the Common Core, I think maybe more so since I've been teaching it, I do now as I'm teaching reference more to some of that application type questions. Like you know, I'm currently teaching surface area so I just find anything like laying around my room that would represent things so that way it's not like I'm already planned. It's not, you know, just a cylinder it's actually, my can of pop that you see on my desk every day that I drink or this container of pretzels that I used for something else (Shaw, April 5, 2013).

Table 11. Instructional Philosophy and the SMP by Curriculum

Connection between Instructional Philosophy and the SMP	Curriculum	
	C	SB
Yes	10 (40%)	12 (71%)
No	15 (60%)	5 (29%)

In discussing SMPs during the background interview MSMTs also touched upon modeling. The themes related to modeling appear in Table 12. A total of three MSMTs gave responses that we were unable to code. As can be seen the majority of teachers discussed models when referring to the modeling SMP. These models either consisted of fraction strips, algebra tiles, area models for factoring quadratic expressions, pictures, graphs, tables, and centimeter cubes. Only five teachers mentioned modeling by providing a description of working with trying to describe a problem set within a real-world context. One teacher using a SB curriculum type drew on a specific activity involving modeling the strength of a bridge. The differences between the two curricula were not statistically significant (Fisher's Exact Test = 1.490, p = .570).

Table 12. Perceptions of the Modeling SMP by Curriculum

Category	Curriculum Type	
	C	SB
Ambiguous	1	2
Modeling	2	3
Models	10	6

Explicitness

A total of 25 MSMTs mentioned the topic of explicitness when discussing the SMPs. Themes appearing in teachers' responses around explicitness by curriculum category are seen in Table 13. The first theme that appeared in teacher responses around explicitness was naming SMPs. These teachers chose to specifically name student work with SMPs when it appeared during classroom lessons. Oftentimes this naming involved teachers pointing to posters that placed the SMPs in kid-friendly terms as seen in Figure 1.

Table 13. Themes appearing around explicitness

Theme	Curriculum	
	C	SB
Naming SMPs	5	2
Accountability	2	2
Student Motivation	4	3
Professional Development	1	1
Curriculum	1	3
Aligned with Beliefs	0	3

We have a series of posters up around our room that have all the practices laid out and they're constantly ripping them off the wall and showing children and looking at the overhead projector and various cameras so they're constantly being linked to the practices (Sanschmidt, November 17, 2013).

Figure 1. Posters displayed in Nichols' classroom

An equivalent number of teachers using SB and C curricula chose to make the SMPs explicit because they felt that the practices would be assessed on CCSSM-aligned assessments. Teachers also chose to make the SMPs explicit for student motivation reasons. This is seen in the following excerpt by a teacher using an SB curriculum.

> So to me the mathematical practices is sort of a window into those students who may not have that facility with mathematics and it's this idea that these are the things that you really have to do, this is like describing what it means to be a mathematician or to be thinking mathematically and I think that that can be helpful to those students who struggle… (Johnson, April 3, 2013).

Two teachers mentioned that they made the SMPs explicit due to the professional development they had experienced. Four teachers noted that their curriculum caused them to make the SMPs more explicit in their classrooms, but this was more common among SB curriculum users than those using a C curriculum. Last, teachers using a SB curriculum noted that they enjoyed making the SMPs explicit because these practices aligned with their beliefs about what it means to do mathematics. The differences between curriculum categories were not statistically significant (Fisher's Exact Test = 5.275, df = 5, p = .444).

Discussion

Overall, this study found that the MSMTs in our study struggled in naming the SMPs, with fewer than two standards named on average. It is important to note that the criteria we used to indicate a teacher's identification of the SMPs were quite liberal and focused on representation of the ideas associated with the SMPs. That is, if teachers mentioned one or more keywords associated with a specific SMP then we indicated that they had mentioned that mathematical practice. Moreover, teachers most often viewed the SMPs as problem-solving specifically and rarely mentioned SMP 7: Look for and make use of structure. This finding mirrors the results of a pilot survey we conducted as well as the results of a survey given to a nationally representative sample of K-12 teachers (Opfer et al., 2016). Indeed, as indicated in Table 9 most teachers saw problem-solving as the biggest change from previous state standards. Thus, teachers may simply focus on the SMPs as problem-solving and provide fewer opportunities for students to engage in other practices such as looking for and making use of structure.

Our data suggest that teachers see the SMPs as a repository and a call for *reform-oriented instruction*. This was seen in the teachers' use of terminology associated with reform such as exploration, teachers' association of inquiry-oriented instruction with the SMPs, group work, and real-world contexts. This certainly makes sense

given the connection between the SMPs and reform documents as mentioned earlier. However, while the SMPs are associated with reform, there are components of reform that are not part of the SMPs. Our study suggests that MSMTs are associating reform that is not part of the SMPs to these practices. When asked about the SMPs, teachers used terminology that cut across several SMPs. For instance, teachers associated "explanation" with the SMPs. Explaining occurs in the first, third, and sixth SMPs. Other teachers used terminology that was not specifically associated with any SMP, such as exploration. These findings are problematic, as explain and exploration are generic actions that students are engaged in, but they do not encapsulate the critical components of the SMPs.

In addition, a number of teachers stated that they used posters in their room describing the SMPs in more kid-friendly terms as seen in Figure 1. The phrase "check your work" is used to illustrate SMP 6 involving precision. It is difficult to argue that the sixth SMP is equivalent to asking a student to check his or her work. Opfer and colleagues (2016) also found that teachers possessed interpretations of the modeling SMP (SMP 4) that differed from its description in the CCSSM. This study also found that MSMTs conflated mathematical modeling and modeling mathematics. Taken together, these results suggest that teachers working in states that have adopted the CCSSM are in need of professional development that helps them to become more familiar with the wording of the SMPs as it appears that the descriptions of these practices are not sufficiently detailed to help teachers in interpreting them. It is difficult for teachers to draw students' attention to and nurture students' development of these practices if they themselves struggle in describing them. In addition, engaging students in explanation dilutes the SMPs in which this verb appears. For instance, students are engaged in explanations in SMP 1, SMP 3, and SMP 6, but these SMPs involve much more than explanation as they involve problem-solving, constructing viable arguments, and attending to precision, respectively. These results suggest that professional development could focus on more carefully describing each of these practices, providing teachers with examples of what these practices look like in the classroom, and, as other mathematics educators have argued (Heck et al., 2011), provide teachers with examples of what a particular practice would look like across grade levels. Additionally, this study suggests that the production of shorter, more student friendly descriptions of the SMPs would be welcome by MSMTs.

This study was partially motivated by examining the differences between standards-based and conventional textbook users' perceptions of the SMPs. We examined a total of eight differences by curriculum type in teachers' SMPs perceptions. Only two of these differences were statistically significant. The first statistically significant difference by curriculum group involved textbook alignment with the SMPs. Given the connections among the SMPs, *Standards*, PSSM, and standards-based curricula as noted earlier, it is not surprising that teachers using these textbooks were more likely to see these programs as aligned with the SMPs. At the same time, teachers using conventional curricula were more likely to perceive their textbooks as being unaligned with the SMPs. This finding suggests that future research needs to investigate how these teachers react to this perceived misalignment. Do these teachers supplement their textbook programs or do they simply follow their textbook feeling that CCSSM-aligned assessments are likely to focus on content and address the SMPs negligibly, if at all? If the former, what curricular activities do teachers locate and how do teachers' personal resources play a role in their decision making processes with regard to these materials?

The second statistically significant difference between groups involved instructional philosophy and the SMPs. Teachers using curricula within the SB category were more likely to view the SMPs as an instructional philosophy than teachers using conventional curricula. This perhaps is not that surprising given the connections between previous standards and the SMPs and the fact that SB curricula were designed with components of the standards in mind. Teachers using SB curricula may be more likely to possess an instructional philosophy that is aligned with the SMPs. Given these connections these teachers may be more likely to provide students with opportunities to daily engage in the SMPs as they appear to be instantiated in how they should teach.
Teachers, who did not view the SMPs as an instructional philosophy of the CCSSM, connected the instructional philosophy of the standards to either understanding or applications. McCallum (2012) describes one of the fundamental principles on which the CCSSM was designed as rigor, consisting of conceptual understanding, procedural fluency, and meaningful applications of mathematics. Thus, while understanding is a laudable area in which to focus one's instruction it does not encapsulate the different components of the CCSSM. A similar critique applies to applications. Providing students with opportunities to apply mathematics is but one component of instruction under the CCSSM. Teachers who focus instruction around these components may not be interpreting the CCSSM in ways that are aligned with what the architects of these standards had intended.

In discussing the SMPs, a total of 25 MSMTs mentioned explicitness, in terms of how they presented SMPs to their students. Five different themes appeared around teachers' mentions of explicitness. Selling (2016) identified naming as a practice that teachers use to help make the SMPs explicit to students. Additionally, the

motivation theme was similar to Selling's move of explaining the goal or rationale. Our study makes a unique contribution in this area by describing why MSMTs made the SMPs explicit to students. These included accountability, student motivation, professional development, and the alignment between the SMPs and teachers' beliefs. Several teachers mentioned the act of naming SMPs when they became present in students' work. Although it is important to name the practices it was not clear that these teachers did anything more with the practices during classroom instruction. For instance, Shaughnessy and colleagues (2015) have advocated that students need to know how to persevere when challenged by what appears to be an insurmountable problem. It is not clear that the teachers in our study are providing students with support for how to engage in the SMPs or what a trajectory of learning a particular SMP might involve for middle grades students. These are additional areas that professional development could target for not only MSMTs but mathematics teachers at other grades as well.

Acknowledgements

This research was supported in part by the National Science Foundation under Grant DRL-1222359. The opinions expressed herein are those of the authors and do not necessarily reflect the views of the National Science Foundation.

References

Anderson, J., & Bobis, J. (2005). Reform-oriented teaching practices: A survey of primary school teachers. In H. Chick & J. Vincent (Eds.), *Proceedings of the 29th conference of the International Group of the Psychology of Mathematics Education* (pp. 26-72). Melbourne: University of Melbourne.

Chavez, O. (2003). *From the textbook to the enacted curriculum: Textbook use in the middle school mathematics classroom.* Unpublished manuscript, University of Missouri, Columbia.

Choppin, J., Davis, J. D., Drake, C., & Roth McDuffie, A. (2013). *Middle school teachers' perceptions of the Common Core State Standards for Mathematics and related assessment and teacher evaluation systems.* Rochester, NY: Warner Center for Professional Development and Education Reform. Retrieved from http://www.warner.rochester.edu/files/warnercenter/docs/commoncoremathreport2.pdf

Cirillo, M., Pelesko, J. A., Felton-Koestler, M. D., & Rubel, L. (2016). Perspectives on modeling in school mathematics. In C. R. Hirsch and A. Roth McDuffie, (Eds.), *Annual perspectives in mathematics education: Mathematical modeling and modeling mathematics 2016* (pp. 3-16). Reston, VA: National Council of Teachers of Mathematics.

Clarke, D. M. (1997). The changing role of the mathematics teacher. *Journal for Research in Mathematics Education, 28*(3), 278-308.

Cobb, P., & Jackson, K. (2011). Assessing the quality of the Common Core State Standards for Mathematics. *Educational Researcher, 40*(4), 183-185.

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics Education, 28*(3), 258-277.

Cogan, L., Schmidt, W., & Houang, R. (January, 2013). *Implementing the Common Core State Standards for Mathematics: What we know about teachers of mathematics in 41 states: Working paper #33*. East Lansing, MI: The Education Policy Center at Michigan State University.

Collopy, R. (2003). Curriculum materials as a professional development tool: How a mathematics textbook affected two teachers' learning. *Elementary School Journal, 103*, 287-311.

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. Retrieved from http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf.

Corbin, J., & Strauss, A. (2008). *Basics of qualitative research, 3e*. Thousand Oaks, CA: Sage.

Creswell, J. (2014). *Research design: Qualitative, quantitative, and mixed methods approaches*. Thousand Oaks, CA: SAGE.

Davis, J. D., Choppin, J., Roth McDuffie, A., & Drake, C. (2013). *Common Core State Standards for Mathematics: Middle school teachers' perceptions.* Rochester, NY: Warner Center for Professional Development and Education Reform. Retrieved from http://www.warner.rochester.edu/files/warnercenter/docs/commoncoremathreport.pdf

EPE Research Center. (2013). *Findings from a national survey of teacher perspectives on the Common Core*. Retrieved from http://www.edweek.org/media/epe_survey_teacher_perspctives_common_core_2013.pdf.

Frykholm, T. (2004). Teachers' tolerance for discomfort: Implications for curricular reform in mathematics. *Journal of Curriculum and Supervision, 19*(2), 125-149.

Gersten, R., & Carnine, D. (1984). Direct instruction mathematics: A longitudinal evaluation of low-income elementary school students. *Elementary School Journal, 84*(4), 395-407.

Heck, D. J., Weiss, I. R., Pasley, J. D., Fulkerson, W. O., Smith, A. A., & Thomas, S. M. (2011, November). *A priority research agenda for understanding the influence of the Common Core State Standards for Mathematics: Technical report*. Chapel Hill, NC: Horizon Research, Inc.

Hirsch, C. R. (Ed.). (2007). *Perspectives on the design and development of school mathematics curricula*. Reston, VA: National Council of Teachers of Mathematics.

Kane, T. J., Owens, A. M., Marinell, W. H., Thal, D. R. C., Staiger, D. O. (February, 2016). *Teaching higher: Educator's perspectives on Common Core implementation*. Retrieved from http://cepr.harvard.edu/files/cepr/files/teaching-higher-report.pdf.

Keiser, J. M., & Lambdin, D. V. (1996). The clock is ticking: Time constraint issues in mathematics teaching reform. *Journal of Educational Research, 90*(1), 23-30.

Koestler, C., Felton-Koestler, M., Bieda, K., & Otten, S. (2013). *Connecting the NCTM process standards & the CCSSM practices*. Reston, VA: National Council of Teachers of Mathematics.

Le, V., Stecher, B. M., Lockwood, J. R., Hamilton, L. S., Robyn, A., Williams, V. L., Ryan, G., et al. (2006). *Improving mathematics and science education: A longitudinal investigation of the relationship between reform-oriented instruction and student achievement*. Rand Education. Accessed on August 9, 2016 from http://www.rand.org/content/dam/rand/pubs/monographs/2006/RAND_MG480.pdf

Lloyd, G. M. (1999). Two teachers' conceptions of a reform-oriented curriculum: Implications for mathematics teacher development. *Journal of Mathematics Teacher Education, 2*(3), 227-252.

Lloyd, G. M., & Wilson, M. (1998). Supporting innovation: The impact of a teacher's conceptions of functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education, 29*(3), 248-274.

Martin, W. G., & Hart, E. W. (2012). Standards for high school mathematics in the Common Core State Standards era. In C. R. Hirsch, G. Lappan, & B. Reys (Eds.), *Curriculum issues in an era of Common Core State Standards for Mathematics* (pp. 47-60). Reston, VA: National Council of Teachers of Mathematics.

McCallum, W. (July, 2012). *The Common Core State Standards in Mathematics*. Paper presented at the 12th International Congress of Mathematical Education, Seoul, South Korea.

Miles, M., Huberman, A., & Saldana, J. (2014). *Qualitative data analysis: A methods sourcebook*. Thousand Oaks, CA: Sage.

National Council for Curriculum and Assessment. (2012). *Mathematical syllabus: Foundation, ordinary, and higher level: Leaving certificate*. Government of Ireland. Retrieved from https://curriculumonline.ie/getmedia/fd79ce76-9a07-42fb-9dd7-ac6036861816/SCSEC25_Maths_syllabus_examination_in_2014_eng.pdf

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2001). *Principles and standards for school mathematics*. Reston, VA: Author.

Opfer, V. D., Kaufman, J. H., & Thompson, L. E. (2016). *Implementation of K-12 state standards for mathematics and English language arts and literacy: Findings from the American teacher panel*. Santa Monica, CA: Rand Corporation. Retrieved from http://www.rand.org/content/dam/rand/pubs/research_reports/RR1500/RR1529/RAND_RR1529.pdf.

Perry, R. R., Finkelstein, N. D., Seago, N., Heredia, A., Sobolew-Shubin, S., & Carroll, C. (July, 2015). *Taking stock of Common Core math implementation: Supporting teachers to shift instruction: Insights from the math in common 2015 baseline survey of teachers and administrators*. San Francisco, CA: WestEd.

Remillard, J. T. (1999). Curriculum materials in mathematics education reform: A framework for examining teachers' curriculum development. *Curriculum Inquiry, 100*(4), 315-341.

Remillard, J. T., & Bryans, M. (2004). Teachers' orientations toward mathematics curriculum materials: Implications for teacher learning. *Journal for Research in Mathematics Education, 35*(5), 352-388.

Romberg, T. A. (1997). Mathematics in context: Impact on teachers. In E. Fennema & B. S. Nelson (Eds.), *Mathematics teachers in transition* (pp. 357-380). Mahwah, NJ: Erlbaum.

Schoen, H. L., Cebulla, K. J., Finn, K. F., & Fi, C. (2003). Teacher variables that relate to student achievement when using a standards-based curriculum. *Journal for Research in Mathematics Education, 34*(3), 228-259.

Schoenfeld, A. H. (2009). The soul of mathematics. In D. A. Stylianou, M. L. Blanton, & E. J. Knuth (Eds.), *Teaching and learning proof across the grades: A K-16 perspective* (pp. xii-xvi). New York: Routledge.

Selling, S. K. (2016). Making mathematical practices explicit in urban middle and high school mathematics classrooms. *Journal for Research in Mathematics Education, 47*(5), 505-551.

Senk, S. L., & Thompson, D. R. (Eds.). (2007). *Standards-based school mathematics curricula: What are they? What do students learn?* Mahwah, NJ: Erlbaum.

Shaughnessy, M., Ball, D. L., Mann, L., & Garcia, N. (April, 2015). (How) can explicitness about mathematical practices support equitable instruction? Presentation at the Annual Conference of the National Council of Supervisors of Mathematics, Boston, MA.

Spillane, J. P. (2004). *Standards deviation: How schools misunderstand education policy*. Cambridge, MA: Harvard University Press.

Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 319-369). Charlotte, NC: Information Age Publishing.

Wilson, M. R., & Lloyd, G. M. (2000). The challenge to share mathematical authority with students: High school teachers' experiences reforming classroom roles and activities through curriculum implementation. *Journal of Curriculum and Supervision*, *15*, 146-169.

Author Information

Jon D. Davis
Western Michigan University
1903 W. Michigan Avenue
Kalamazoo, MI 49008-5248
Contact e-mail: *jon.davis@wmich.edu*

Jeffrey Choppin
University of Rochester
RC Box 270425
LeChase Hall 448
Rochester, NY 14627

Corey Drake
Department of Teacher Education
Michigan State University
620 Farm Lane, Room 116M
East Lansing, MI 48824

Amy Roth McDuffie
Washington State University Tri-Cities
2700 Crimson Way
Richland, WA 99354

Cynthia Carson
University of Rochester
RC Box 270425
LeChase Hall 448
Rochester, NY 14627