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Abstract

This study investigated whether strategy accuracy and flexibility on various types of complex multiplication problems could predict college GPA concurrently and longitudinally in 164 college engineering students. Additionally, it sought to answer whether low- and high-achieving students would show unique patterns of strategy flexibility, accuracy, and recognition of strategies on a complex multiplication task. Strategy recognition significantly predicted variance in GPA at Time 1 and Time 2; the higher performing students were better at recognizing and executing problems in a forced-strategy format, highlighting the importance of the ability to recognize strategies as an important math skill. Group differences accounted for variance on total recognition, total correct strategy associative property, and total correct strategy distributive property. Specifically, low- and average-achieving students were relatively similar in their ability to employ strategies, but high-achieving students were significantly better than the other two groups at executing the correct strategies. A significantly higher portion of students was more accurate on problems that incorporated a multiple of 10, which is consistent with the problem-difficulty effect. Moreover, students of all abilities were more likely to spontaneously use strategies in a non-forced strategy format, suggesting that students had at least some ability to understand the mechanisms behind strategies. Educational implications are discussed.

Introduction

The most recent Trends in International Mathematics and Science Study report (TIMSS; Mullis, Martin, Foy, & Aurora, 2012) revealed that 19% of students, in as early as the fourth grade, reported a pessimistic view of their ability to learn math. This feeling seems to persist among 18% of middle school children in the United States who do not meet the basic national benchmarks in math (National Assessment of Educational Progress [NEAP], 2009). Additionally, approximately one in four American students does not reach baseline competency for mathematics, a constant for American students since 2003 (PISA, 2012). The negative trend appears to span the lifetime, as McGlaughlin, Knoop, and Holliday (2005) reported that difficulty with college math is often cited as a likely reason for students with learning disabilities to drop out of college.

Math skills are not only predictive of overall math achievement, as well as STEM achievement, but also a powerful predictor of a student's overall academic success in college Bremer et al. (2013) examined the academic outcomes of students within the community college system and found that students who entered college with higher math placement scores were more likely to persist through college to completion even when compared to those with high reading and writing abilities. Additionally, poorer math achievement scores prior to entrance in college had a disproportionately negative relationship to overall grade point average (GPA), whereas the same was not true for students with weaker entering reading and writing abilities. This trend has prompted researchers to attempt to understand math learning processes across the lifespan so as to inform interventions, develop learning tools, and increase our national math performance (Zhang, Ding, Barrett, Xin, & Liu, 2013).

A solid foundation of research suggests that successful math strategy usage is the foundation of better math performance and achievement (Siegler, 2007). Furthermore, research is consistent in reporting strategy use as a tool for measuring arithmetic skill (Campbell & Austin, 2002). In the same context, low math achievement is often associated with a lack of conceptual understanding of math concepts, as is determined by the maturity of the strategy selection in solving problems (Siegler, 2007). Performance in math is at least moderately dependent upon the strategy selected, the efficiency of the strategy, and its execution (Campbell & Penner-Wilger, 2006;

Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Hecht, 1999; Imbo & Vandierendonck, 2008; Siegler, 2005, 2006, 2007). As such, understanding the process of STEM career advancement lies in understanding a student's mathematical ability, which may be assessed through the process of strategy selection and execution.

Adaptive Strategy Choice Model

Researchers attempting to understand patterns of behavior in problem solvers often refer to a concept known as the adaptive strategy choice model (ASCM; Siegler & Shipley, 1995), which is an overarching theory on the development and execution of strategy use among all individuals. It postulates that individuals have a repertoire of several strategies available for use across problems. When faced with various problems, individuals are likely to select and execute the most efficient strategy that they believe is not only the fastest, but also the most accurate (Siegler & Shipley, 1995). Siegler and Lemaire (1997) conducted research examining whether the ASCM could predict strategy choice and usage in adults. By assessing the strategy itself, calculating the speed and accuracy with which problems were completed, and cross-referencing them with subject reports of strategy usage, the researchers found that there were significant correlations between reports of strategy usage and the speed and accuracy with which problems were calculated. In other words, people chose the strategy that was fastest and most accurate. Additionally, the relative speed was the most accurate predictor of strategy use and accounted for 72% of the variance in choice of strategy.

Research in the area of strategy use among children has suggested that variability in strategy usage exists not only within the same grade, but also within individuals on different problems, confirming the ASCM (Cowan et al., 2011; Siegler & Shipley, 1995). In other words, strategy choice is often based on the efficiency of alternative strategies (Lindberg, Linkersdörfer, Lehmann, Hasselhorn, & Lonnemann 2013; Siegler & Shipley, 1995). Campbell and Austin (2002) reported that assessing an individual's strategy selection is a "useful, if imperfect, tool for analyzing strategic aspects of adults' simple arithmetic skills" (p. 993). However, there is scarce research on the arithmetic performance of the adult-aged population, despite evidence that performance in this population is correlated to math achievement and performance in STEM education and careers (Campbell & Xue, 2001).

Problem-Difficulty Effect

A more comprehensive and updated version of the notion of a problem size effect is the problem-difficulty effect. Problem-difficulty effect is an integral component in math strategy usage research and is the foundation on which most of the research rests. It suggests that response time will be commensurate with the strategy selected and the size of the problem. Research is consistent across both children and adults and substantiates the theoretical claim that the larger the problem and the more difficult the problem is, the more time and steps are required in solving it (Campbell & Xue, 2001; Imbo & Vandierendonck, 2008; LeFevre, Sadesky, & Bisanz, 1996). The problem difficulty effect is more nuanced, and research suggests that certain multiplicands (e.g., 2, 10) are easier to manipulate and analyze than others. Operation is another aspect of difficulty that works within the problem-difficulty effect, wherein quicker reaction time and higher accuracy are associated with addition over subtraction and multiplication over division (Imbo & Vandierendonck, 2008). Typically, the difficulty effect can be ameliorated by enlisting the help of specific strategies, namely, associative and distributive property strategies. The problem-difficulty effect has been able to predict rates of retrieval usage across various groups based on both the size and difficulty of the problem in relation to the participants' competence (Campbell & Xue, 2001; Imbo & Vandierendonck, 2008; LeFevre et al., 1996).

Perhaps just as important as the concept itself is the notion that self-reports of strategy use align with the problem-difficulty effect (LeFevre et al., 1996). Children and adults reported that strategy use is accurately correlated to response time and accuracy as would be expected by the reported strategy employed; this was found across operations and difficulty levels (Geary, Bow-Thomas, & Yao, 1992; LeFevre et al., 1996).

Strategy Selection

The general consensus among researchers in child arithmetic strategy use suggests that normally achieving children properly employ the most effective and efficient strategies for solving a given arithmetic problem, even if they display very little variety in strategy selection (Zhang et al., 2013). The most common conclusion for research examining the difference between children with math learning disabilities (MLD) and low-math achievers versus average-achieving peers is that they correctly retrieve fewer strategies, display incorrect counting, and display overall lower performance (Geary, Hamson, & Hoard, 2000; Geary & Hoard, 2005; Geary, Hoard, & Hamson, 1999; Russell & Ginsburg, 1984).

The research on adults' strategy selection is scarce. Like research with children, the ASCM is also applicable when discussing strategy selection among adults (Siegler & Lemaire, 1997). The slim research on adults suggests that there is a wide assortment of strategies utilized across various arithmetic problems and includes procedural and counting strategies even on simple arithmetic (Campbell & Xue, 2001; Hecht, 1999; LeFevre et al., 1996). Retrieval is considered as the predominant strategy for simple addition in adults regardless of math knowledge (Campbell & Alberts, 2009). In comparison to children, despite the wide variety of strategy usage, adults are not likely to make errors in simple arithmetic (Hecht, 2002). In regard to strategy usage, Campbell and Alberts (2009) found as problem size increases and thus procedural strategies are executed, adults are prone to the same accuracy errors as children (Campbell & Xue, 2001).

Research on strategy selection specific to multiplication is even more meager (Campbell & Alberts, 2009; Campbell & Xue, 2001; Hecht, 1999, 2002; LeFevre et al., 1996; Mauro, LeFevre, & Morris, 2003). However, research is abundantly clear that even simple multiplication elicits varied strategy usage (Hecht, 1999; LeFevre et al., 1996). Strategy choice and selection were similar between undergraduates and older adults suggesting these choices are solidified by adulthood and do not change over time (Siegler & Lemaire, 1997).

Research suggests that similar to strategy selection, strategy execution follows a pattern whereby children who are proficient not only select the best strategy, but also execute it properly. The other side of this theme is that those students who are low math achievers, or who have a math learning disability, not only poorly select strategies, but also exhibit difficulties with execution (Imbo & Vandierendonck, 2008; Lindberg et al., 2013; Liu, Ding, Gao, & Zhang, 2015; Siegler, 1988; Siegler & Lemaire, 1997; Zbrodoff & Logan, 2005; Zhang et al., 2013). A lack of ability to use strategies is typically found in conjunction with low conceptual understanding of math, which creates a lack of understanding of higher level math (Canobi, Reeve, & Pattison, 1998). Functionally, fluidity with easier math is thought to free up cognitive space to consume more difficult, higher order math (Royer, Tronsky, Chan, Jackson, & Marchant, 1999).

Similar to children, adults are more efficient when problems are solved using retrieval. In other words, problems based on direct retrieval are solved faster and more accurately than with any other strategy or combination of strategies (LeFevre et al., 1996). However, research varies substantially on the use of retrieval as the primary mode of problem solving, suggesting that at least some groups of adults are sacrificing working memory resources to solve problems even if and when those resources are necessary to carry out other tasks; this was reported across varying working memory conditions (Hecht, 2002). However, Hecht (2002) also noted that the reduced availability of attentional resources impairs the efficiency of retrieval. Campbell and Alberts' (2009) recent research found that larger problems resulted in substantially more procedural strategy selection because there is little to no memory strength in American adults. Hecht's study (2002) concluded that there is a need to explain strategy usage in adults, as the research is so varied and contradictory in regard to patterns of strategy selection.

Needed Research and the Current Study

This study investigated whether strategy accuracy and flexibility on various types of complex multiplication problems could predict college GPA concurrently and longitudinally, and explained unique variance in an engineering computation class comprised of learners of all ages and levels enrolled in college, when controlling for variables such as SAT scores and average grade in calculus. Additionally, it sought to answer whether low- and high-achieving students (average grades in calculus) would show unique patterns of strategy flexibility, accuracy, and recognition of strategies on a complex multiplication task. Furthermore, it contributed to our understanding of math development and what abilities are important for success in college and beyond. The following are four guiding research questions for this study**:** 1) What are the relative contributions of strategy accuracy and flexibility and prior performance variables on current GPA? 2) What are the relative contributions of strategy accuracy and flexibility and prior performance variables on future GPA? 3) What is the relationship between unique strategy accuracy, flexibility, and pattern of recognition when comparing students with differing levels of achievement? 4) What is the unique contribution of problem-difficulty effect on students' strategy selection, execution, and accuracy?

Methods

Participants

Participants were 164 adults enrolled in the School of Engineering at a Northeastern university. Of the 164 participants in the study, 32 were female and 132 were male. All students were currently enrolled in the engineering computations class. Participants ranged in age from 18 years through the end of adulthood and were reflective of the greater community within the participating university, ranging in ethnicity, SES, and culture.

Measures

There were five measures used for this analysis, four of which were widely accepted, common measures of performance: GPA at Time 1 and Time 2, average GPA from all previous calculus courses, and SAT scores.

Two-digit multiplication. Based on Gao (2011), the researchers developed their own measure for assessing accuracy in complex multiplication and strategy flexibility using a multiplication worksheet of six 2-digit × 2-digit multiplication problems, where each one was presented three separate times for a total of 18 problems. In Gao (2011), the associative property strategy and distributive property strategy were most frequently chosen by their elementary school participants. Thus, in this the present study, we focused on these two number property strategies. There have been concerns regarding bias of strategy selection based on strategy efficiency (Siegler & Lemaire, 1997). For example, a strategy might appear to be efficient if it is frequently chosen by more advanced students, and more accurate strategies might be used more often for challenging problems and then appear to be less accurate.

In order to eliminate strategy selection and student characteristics (e.g., advanced students chose efficient strategy and low-achieving students chose inefficient strategy), we examined multiplication strategies within a forced-choice format (i.e., associative property strategy [18 × 5] × 3, distributive property strategy 18 × 10 + 18 × 5, and no specific strategy 18 × 15; Authors, 2015). Varying the presentation three times for each problem was based on previous research by Mauro et al. (2003), who found that manipulating the presentation of problems is an effective method for determining the mental processes behind math and how these constructs are represented in memory. Both multiplicands were drawn from two-digit numbers no greater than 35 and were based on research using and varying the presentation of multiplication problems described in Zhang et al. (2013).

Additionally, half of the six baseline problems included a multiple of 10 or 5 as one of the multiplicands (e.g., 12 × 35 versus 12 × 34). This was incorporated based on findings that for all ages, a problem being a multiplicand of 5 or 10 accounted for variation in strategy use; in particular, it was correlated with a higher instance of both retrieval and more efficient strategies because they are easier to manipulate (Lemaire & Siegler, 1995; Liu et al., 2015; Siegler & Lemaire, 1997).

Procedure

This research was conducted according to the American Psychological Association (APA, 2010) code of ethics. Institutional Review Board (IRB) approval was obtained. Participants were recruited from entry-level engineering classes. The researcher explained the purpose and procedure of the study. Participants signed forms of consent and FERPA consent forms to participate in the study. The FERPA forms explicitly stated that information regarding scores on the average grade in calculus, GPA at Time 1 and Time 2, and SAT would be accessed with consent to participate. Participants filled out all the consent forms and completed the 18 multiplication problems over three testing occasions.

Grading and Coding

Each individual's worksheet containing 18 problems was scored on a number of domains, including accuracy, strategy use, and recognition of the strategy. Accuracy was always determined to be the correct mathematical result of the problem, and students were scored either correct or incorrect on each number. A coding scheme was used to code each completed problem to determine the strategy used to solve that problem (see Table 1), adopted from the empirical literature on solving multiplication problems (Lemaire & Siegler, 1995; Siegler, 1988; Zhang et al., 2013). In the protocol, items 1 through 6 were non-forced (no strategy) problem formats.

Items 7, 8, 12, 15, 16, and 17 were distributive problems, and 9, 10, 11, 13, 14, 18 were associative problems. Protocols were scored for a variety of results that would be used in quantitatively solving for the accuracy and strategies were categorized into three major indexes.

Table 1. Explanation and examples of strategies discussed

Strategy Type	**Explanation**	**Example**
Counting Procedures		
Min	Starts with the larger number first, then counts upward toward the total using the smaller number	6 + 11 = ?, becomes, 6 + 1 = 7 + 1 = 8, and so on until adding 11 to the 6
Max	Starting with higher number first then counting upward toward the total using adding lower numbers to it	6 + 11 = ?, becomes11 + 1 = 12 + 1 = 13, and so on until adding the 6 to 11
Sum	Starting with 1 and counts upward using both numbers	6 + 11 = ?, becomes 1 + 1 =2 + 1 = 3, and so on until adding 6 above 11
Arithmetic Strategies		
Algorithm	Using full mathematical procedures, carrying out each operation individually	6 × 11, the child will multiply 6 × 1 = 6, then 6 × 1 = 6, finding 66 as the answer.
Retrieval	The knowledge of the answer	Inherent knowledge that 6 x 11 = 66
Associative	Understanding that the change in grouping of three or more factors does not change their product.	15 × 10 = 15 × (5 × 2) = (15 × 5) × 2
Distributive	Understanding that the product of a number and a sum is equal to the sum of the individual products of addends and the number	15 × 10 = 10 × (10 + 5) = 10 × 10 + 10 × 5

Accuracy in problem solving. Accuracy was defined as correctly or not correctly reporting the answer where one point was awarded for each correct answer; the maximum accuracy score was 18 (one for each problem). Accuracy was also separated by group so that accuracy on 5/10s versus non-5/10s problems could be calculated. Additionally, accuracy by strategy use was calculated; each time a participant utilized the provided strategy, the accuracy for that particular strategy was collected.

Spontaneous strategy. Spontaneous strategy was calculated by counting the number of times a participant used any strategy other than algorithm on a problem with no provided strategy (problems 1-6 of 18). Additionally, three problems were 5/10s problems, and three were non 5/10s and were also utilized toward calculating spontaneous strategy abilities.

Recognition of strategy. A student's recognition of strategy resulted in receiving one point. For example, 12 of the 18 problems were specifically designed to elicit a specific strategy; six were designed to elicit associative strategy usage, whereas the other six were designed to elicit the distributive strategy. Students were awarded points on their ability to simply recognize, which was defined as attempting to employ the specific strategy, regardless of accuracy in execution. Totals were collected for overall strategy recognition out of 12, and then for each of the two strategies (out of six), and finally, by both strategy and by being a multiplicand of 10 or not. This last measure was used to determine whether students were able to recognize strategies differently for easier problems where research suggested that these problems are easier to manipulate (Lemaire & Siegler, 1995; Siegler & Lemaire, 1997).

Inter-rater reliability. The principal investigator scored all protocols with the help of trained research assistants using an answer key and coding students' problem-solving accuracy, strategies, and recognition of strategies. Two junior graduate students were recruited and trained. To insure inter-rater reliability, 40% of the data were randomly selected and recoded to derive inter-rater reliability, which was computed by dividing the number of agreements by the number of both the agreements and the disagreements and then multiplying that outcome by 100. The inter-rater reliability for this sample was 91%.

Results

Tables 2 and 3 provide descriptions of the independent variables used in this study. Included are short descriptions of the variables themselves with examples. For example, distributive problems are described, and examples of the forced-choice format are provided. Descriptions are displayed for distributive, associative, and no strategy formats, as these were the variables for Research Questions 1 through 4.

Table 2. Examples of forced-choice multiplication strategies

Forced-Choice Multiplication Strategies	**Operational Definitions**	**Examples**
Distributive Property Strategy	Distributive property states that the product of a number and a sum is equal to the sum of the individual products of addends and the number.	18 × 15 = 18 × (10 + 5) = 18 × 10 + 18 × 5
Associative Property Strategy	Associative property states that the change in grouping of three or more factors does not change their product.	18 × 15 = 18 × (5 × 3) = (18 × 5) × 3
No Strategy	Students use algorithm to compute the product of two factors.	18 × 15 =

Table 3. Multiplication problems used during testing

Problem Difficulty	Grouping	Strategies Provided	Items	Cronbach's Alpha	Spearman-Brown Coefficient*
Products were multiples of 10	Group 1	No	25 × 12		
		Distributive	25 × (10 + 2)		
		Associative	25 × 4 × 3		
	Group 2	No	18 × 15		
		Distributive	18 × (10 + 5)		
		Associative	18 × 5 × 3		
	Group 3	No	14 × 35		
		Distributive	14 × (30 + 5)		
		Associative	14 × 5 × 7	.78	.78
Products were not multiples of 10	Group 1	No	17 × 21		
		Distributive	17 × (20 + 1)		
		Associative	17 × 3 × 7		
	Group 2	No	19 × 14		
		Distributive	19 × (10 + 4)		
		Associative	19 × 2 × 7		
	Group 3	No	13 × 28		
		Distributive	13 × (20 + 8)		
		Associative	13 × 4 × 7	.62	.58
Total				.84	.77

Note. Table reprinted from "The Relations Between Number Property Strategies, Working Memory, and Multiplication in Elementary Students," by R. D. Liu, Y. Ding, B. C. Gao, and D. Zhang, 2015, *The Journal of Experimental Education, 83*, p. 326. Copyright 2015 by Taylor & Francis Group, LLC. Adapted with Permission. *: Spearman-Brown Split-Half Coefficient (term abbreviated for table formatting purposes).

Table 4 provides descriptive statistics for all of the dependent and independent variables assessed in this research. On average, students performed better on the math portions of the SAT than on the verbal sections, which was to be expected given the use of only engineering majors. Additionally, the average total SAT score in this sample was slightly higher than the 2013 national average (College Board, 2013) of 1,497. GPAs, both cumulative and course specific, all centered around 3.00, which roughly equates to a B average.

In regard to the dependent variables, the average rate of accuracy across the 18 problems was 16.62 suggesting that a high number of individuals were almost completely accurate in their calculations. Average use of strategy was 2.11 suggesting that, on average, participants attempted to execute at least two different strategies across the 18 problems. Participants recognized on average 4.5 of the forced strategy formats of the total 12 problems and could generate only one strategy on average across the six no-strategy format problems.

Table 4. Descriptive statistics of dependent and independent variables

Variables	*N*	Min	Max	Mean	*SD*	Skewness
SAT Verbal	129	350.00	730.00	550.16	74.31	0.05
SAT Math	129	460.00	780.00	611.40	64.13	0.22
SAT Writing	129	320.00	800.00	551.78	79.44	0.19
SAT Total	129	1280.00	2270.00	1713.33	182.29	0.38
GPA Current Term	163	0.92	4.00	2.91	0.68	–0.54
GPA Cumulative	163	1.04	4.00	2.99	0.56	–0.36
Calculus 1	134	1.00	4.00	3.05	0.73	–0.46
Calculus 2	116	1.00	4.00	2.88	0.79	–0.09
Calculus 3	95	1.00	4.00	3.08	0.77	–0.56
Total Accuracy	156	0	18	16.62	2.05	–3.88
Total Strategy	156	0	4	2.11	0.79	–0.04
Total Recognition	156	0	30	4.52	4.45	1.41
Total Spontaneous	156	1	4	1.35	0.59	1.65

Note. Some students were transfer students, so they had missing SAT scores or calculus scores

Research Question 1

To address the first research question, a hierarchical regression was completed to assess the relative contributions of Strategy Recognition and Total Strategy on GPA Time 1 (Table 5). For the regression, GPA Time 1 was the dependent variable and SAT total scores were the control variable and entered in the equation as step 1. Based on preliminary correlational analysis, student's total recognition, total accuracy, total strategy use, and total spontaneous strategy use were entered in the equation as step 2, step 3, step 4, and step 5, respectively.

Finally, a hierarchical regression analysis was conducted to determine whether SAT Total, Total Recognition, Total Accuracy, Total Strategy, and Total Spontaneous Strategy explained unique variance of GPA at current time. The results are presented in Table 5. SAT Total explained 15.8% of the variance in students' GPA Time 1. After taking into account the SAT Total, Total Strategy Recognition significantly explained 4.4% unique variance in students' GPA Time 1. Total Accuracy, Total Strategy, and Total Spontaneous Strategy did not significantly explain unique variance in the student's GPA Time 1. In the ANOVA model summary, the Total Strategy also emerged as significant at the p < .05 level, contributing to 2.7% of the variance, but in the hierarchical regression, with all other variables considered, it did not meet significance levels (t = –1.729; p = .086). Strategy Recognition explained unique variance (4.4%) in GPA at Time 1. While Total Strategy approached significance, the explained differences did not reach statistical significance. Therefore, the null hypothesis was confirmed that Total Strategy did not explain variance in GPA above and beyond accuracy in GPA Time 1 after controlling for SAT scores.

Table 5. Hierarchical regression analysis for strategy use measures on GPA Time 1

Steps	Variables	Dependent Variables: GPA Time 1				
		β	R^2	R^2 Change	t	p
1	SAT Total	.359	.158	.158	4.251	<.001***
2	Total Recognition	.288	.201	.044	3.096	.002**
3	Total Accuracy	.080	.207	.006	0.990	.324
4	Total Strategy	–.187	.234	.027	–1.729	.086
5	Total Spontaneous	.012	.235	.000	0.116	.908

*p < .05, **p < .01, ***p < .001

Research Question 2

GPA Time 2 was the dependent variable and SAT Total scores were the control variable and entered in the equation as step 1. Based on preliminary correlational analysis, students' Total Recognition, Total Accuracy, Total Strategy use, and Total Spontaneous Strategy use were entered in the equation as step 2, step 3, step 4, and step 5, respectively. The results are presented in Table 6. In this regression, SAT Total explained 21.1% of the variance, while Total Recognition explained 5.5% of the variance in total GPA. Aside from SAT Total, Total Recognition was the only other variable that significantly contributed uniquely to the outcome variable of GPA Time 2 (p < .005). Total Accuracy, Total Strategy, and Total Spontaneous Strategy did not significantly provide unique contributions to predicting the students' GPA Time 2. In the final model, SAT Total (t = 4.856; p < .001) and Total Recognition (t = 3.155; p < .005) significantly contributed.

Table 6. Hierarchical regression analysis for strategy use measures on GPA Time 2

Steps	Variables	Dependent Variables: GPA Time 2				
		β	R^2	R^2 Change	t	p
1	SAT Total	.407	.211	.211	4.856	<.001***
2	Total Recognition	.288	.266	.055	3.155	.002**
3	Total Accuracy	.001	.266	.000	0.007	.994
4	Total Strategy	–.128	.280	.014	–1.209	.229
5	Total Spontaneous	.003	.280	.000	–0.003	.978

*p < .05, **p < .01, ***p < .001

As predicted, Strategy Recognition significantly predicted variations in GPA at Time 2, accounting for 5.5% of unique variance in GPA at Time 2. Total Strategy did not significantly explain variance in GPA Time 2. Therefore, the null hypothesis was confirmed that Total Strategy did not explain unique variance in GPA Time 2 above and beyond accuracy in current GPA after controlling for SAT scores.

Research Question 3

To assess the relationship between Strategy Recognition, Total Strategy, and Accuracy for students with differing levels of achievement, a series of ANOVAs were run (see Table 7). Students were grouped based on level of math achievement and were identified as either Group 1, Group 2, or Group 3. Level of achievement was defined based on the average of all final calculus course grades. High-achieving students were Group 3, with calculus GPAs from 4.00+ through 3.67, roughly equating to A through A+ averages; students in achievement Group 2 had calculus GPAs that ranged from 3.67 > x > 2.67 or A- through B- averages. Group 1 was identified as those with calculus GPAs between 2.67 > x > 0.00, or anything less than a B- average.

Table 7. Descriptive statistics of independent variables and one-way ANOVA

Variables	Group	N	Min.	Max.	Mean	SD	F	p
Total Accuracy	1	50	12	18	16.80	1.49		
	2	65	0	18	16.37	2.61		
	3	41	13	18	16.78	1.61	0.80	.451
Total Strategy	1	50	1	3	2.06	0.71		
	2	65	0	4	2.12	0.89		
	3	41	1	3	2.15	0.73	0.15	.861
Total Recognition	1	50	0	11	3.66	3.65		
	2	65	0	13	3.43	3.77		
	3	41	0	30	7.29	5.19	12.43	<.001***
Total Spontaneous	1	50	1	3	1.36	.60		
(No strategy imposed)	2	65	1	4	1.42	.64		
	3	41	1	3	1.24	.49	1.08	.344
Recognition-Associative	1	50	0	6	2.00	2.33		
	2	65	0	6	1.55	1.93		
	3	41	0	6	2.88	2.15	4.92	.009**
Recognition-Distributive	1	50	0	6	1.52	2.44		
	2	65	0	6	1.88	2.57		
	3	41	0	6	3.56	2.68	8.07	<.001***

Note. Group 1=low-achieving students in math. Group 2=average students in math. Group 3=high-achieving students in math. *p < .05, **p < .01, ***p < .001

Descriptive statistics and measures of strategy use by group membership (high-achieving [3], average [2], and low-achieving [1]) are displayed in Table 7. As the table indicates, the average rate of accuracy was consistent across ability groups suggesting that individuals of all ability levels could properly execute complex multiplication problems by hand. Rates of Total Recognition (i.e., the number of times the participants used the strategy we anticipated them to use) suggested that the high-achieving group was much more likely to recognize strategies overall than the average and low-achieving groups (M = 7.29 for high-achieving, while average and low-achieving groups performed similarly [M = 3.66 and M = 3.43, respectively]). Participants were consistent in that the number of strategies attempted (i.e., the count of different types of strategies, where each strategy counts only once), along with number of spontaneous strategies (i.e., the number of times that students generated their own strategy in the non-forced problems) remained equivalent. Employing the correct strategy for both distributive and associative problems suggested that the high-achieving group in both conditions performed better. On the associative problems, the average mean for the high-achieving group was M = 2.88, while the average and low-achieving groups were M = 1.55 and 2.00, respectively. The distributive problems produced an even larger gap between high-achieving individuals and their average and low-achieving counterparts, with M = 3.56 for the high-achieving participants and M = 1.88 and M = 1.52 for the average and low-achieving individuals.

For the 12 items that utilized a forced number property strategy, six distributive and six associative, we measured the number of times participants recognized the strategy. High-achieving students outperformed the other two groups to a significant degree (F = 12.43, p < .001) on the measure of Strategy Recognition. Post-hoc scheme multiple comparisons using a Scheffe's analysis suggested that although average and low-achieving students did not differ significantly (p = .958), high-achieving students were significantly different from both average and low-achieving students (p < .001 for both). To better understand this comparison, we broke it down into associative versus distributive problems to see whether there were group differences within the distributive and associative problem formats. Similarly, there was a significant difference for performance on both the associative (p < .01) and distributive problem (p < .001) formats, although the significance was higher for the distributive problems. Post-hoc scheme multiple comparisons suggested that the high-achieving group outperformed the average-achieving group to a significant degree (p < .01). For the distributive problem format, the high-achieving group outperformed both the average (p < .01) and low-achieving (p < .01) groups to a significant degree. Group differences accounted for variation on Total Recognition, Total Correct Strategy Associative, and Total Correct Strategy Distributive.

Research Question 4

To further understand the unique contribution of problem-difficulty effect on students' strategy selection, execution, and accuracy, four paired-sample t-tests were calculated to see if problem-difficulty effect impacted student performance. The first t-test was total accuracy on 10s versus non-10s. The second was total spontaneous strategy on 10s versus non-10s. The third and fourth t-tests compared distributive 10s versus non-10s and associative 10s versus non-10s. The results are presented in Table 8.

Table 8. Descriptive and paired sample t-tests of 10s and non-10s problems

Variables	*N*	Mean	*SD*	*t*	*p*
Total Accuracy (10s)	156	8.40	1.08	1.72	.087
Total Accuracy (non-10s)	156	8.22	1.34		
Total Spontaneous Strategy (10s)	156	1.54	1.07	6.74	<.001***
Total Spontaneous Strategy (non-10s)	156	1.06	0.93		
Total Accuracy-Associative (10s)	156	2.78	0.55	0.12	.905
Total Accuracy-Associative (non-10s)	156	2.77	0.56		
Total Accuracy-Distributive (10s)	156	2.75	0.54	0.76	.448
Total Accuracy-Distributive (non-10s)	156	2.71	0.61		

Note. Group 1=low-achieving students in math. Group 2=average students in math. Group 3=high-achieving students in math. *p < .05, **p < .01, ***p < .001

On the initial statistics, the easier problems (i.e., products were multiples of 10) elicited higher accuracy rates overall. Across all problems, the accuracy was slightly higher for problems that were a multiple of 10 versus those that were not (M = 8.40 versus M = 8.22). Similarly, the most drastic difference emerged when comparing the average spontaneous use of strategy on the 10s (M = 1.54) versus the non-10s (M = 1.06) problems. Accuracy on the 10s versus non-10s distributive and associative resulted in only slightly higher accuracy rates for the 10s problems. Specifically, the means for the distributive problems were 10s (M = 2.78) while the non-10s resulted in M = 2.77. On the two-tailed paired samples test, only the spontaneous strategy resulted in significance, p < .000. This suggests that the spontaneous generation of strategies was significantly more likely to occur for problems that incorporated an imbedded strategy of being a traditionally easier problem composition (t = 6.74, p < .001). The total accuracy on the 10s versus non-10s overall approached significance (t = 1.721, p = .087), but did not reach a significant level. In other words, students were significantly more likely to spontaneously generate a strategy on a non-forced problems format when the problem was a multiple of 10 (easier problem).

Discussion

Previous research suggested that there is an important relationship between childhood math skills, specifically the ability to use strategies, and math performance (Siegler, 2007). Other research reported that these differences in the math abilities of young people extend into adulthood, affecting career paths and later academic performance (Campbell & Austin, 2002; Campbell & Xue, 2008). This study aimed to extend the research to understand the implications of math strategy usage in adult learners and whether strategy and accuracy remain as powerful predictors of their academic achievement in college.

Strategy Accuracy and Flexibility in Relation to Concurrent GPA

The first research question was developed to explore the relative contributions of strategy accuracy and flexibility and prior performance variables on current GPA. There were two hypotheses associated with this question. First, the researcher expected to find that the ability to recognize strategies would explain unique variance in a student's GPA. Previous research with children and young people suggested that the ability to use strategies is predictive of math achievement later in life (Siegler, 2005, 2006, 2007; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Jordan et al., 2007). The second hypothesis was that the total number of strategies used by the participants would explain unique variance in GPA above and beyond accuracy after controlling for SAT scores. This was based on the ASCM (Siegler & Shipley, 1995) that suggests a larger repertoire of available strategies is associated with a richer understanding of mathematics and numbers (Canobi et al., 1998; Imbo & Vandierendonck, 2008). Statistical analyses confirmed the first hypothesis, whereby strategy recognition explained unique variance in GPA. The ability to recognize strategies in a forced problem layout was associated with higher GPA; the higher performing students were better at recognizing and executing problems in a forced-strategy format, highlighting the ability to recognize strategies as an important math skill. The findings suggest that the ability to maintain previously learned math strategies and recognize effective strategies might be a unique learning characteristic associated with high-achieving students. In later stages of learning, effective learners should not randomly utilize different strategies. Instead, once they become familiar with effective strategies corresponding to specific problems, they should work to become proficient with such strategies and should be able to retrieve such strategies when they encounter similar problems. The second hypothesis was not confirmed by the research, suggesting that the number of different strategies an individual uses to execute problems does not contribute to GPA. In retrospect, adults are likely to be more set in their ways, so the same research that applies to children may not apply here. Additionally, there are only a few effective strategies in regard to multiplication: retrieval, which is the most effective, then distributive and associative number properties (Fürst & Hitch, 2000; LeFevre et al., 1996; Logie, Gilhooly, & Wynn, 1994). Therefore, it is likely that many inefficient problem solvers were trying many different strategies, seemingly at random, rather than efficiently choosing strategies. Finally, the slim amount of research available suggests that when there is no forced-format for strategy, adults will use a varied set of strategies with a pattern that has yet to be identified (Hecht, 1999; LeFevre et al., 1996). Thus, it makes sense that the number of strategies was inconsistent across groups.

Strategy Accuracy and Flexibility in Relation to Future GPA

The second research question examined the same hypotheses above, with the addition of determining if unique variances persisted into the future. Similar to research discussed earlier, pointing to the predictive ability of math strategies for future performance, the unique variance in GPA was found to persist one full year after the initial data collection, suggesting that the ability to recognize strategies was associated with general academic performance over time. Since there was no unique variance to the number of strategies associated with current GPA, it was unlikely that this would be found for future GPA. The research confirmed that total number of strategies used did not explain unique variance in future GPA.

Strategy Accuracy, Flexibility, and Patterns of Recognition among Students with Differing Levels of Achievement

The third research question aimed to explain whether high-, average-, and low-achieving students would be distinguishable on measures of strategy recognition, total number of strategies, and accuracy patterns. Although there were no differences in accuracy or number of strategies across ability groups, rates of total recognition properly distinguished between the ability groups. Specifically, low- and average-achieving students were relatively similar in their ability to employ strategies, but high-achieving students were significantly better than both groups at executing the correct strategies. This continues to point to a specific mechanism that is either present or not present in students who are good at math, and highlights that this may not be a spectrum of good at math or poor at math, but rather that there is some mechanism at work in the high-achieving group that helps them consistently outperform all the other students (Zhang et al., 2013). This may be the presence of number sense as suggested by other researchers (Geary et al., 2004; Mulligan & Mitchelmore, 1997).

Problem-Difficulty Effect

This question explored the presence of a problem-difficulty effect in adult learners. Although there is considerable research available that suggests the presence of a problem-difficulty effect in producing quicker, more accurate answers (Campbell & Xue, 2001; Imbo & Vandierendonck, 2008; LeFevre et al., 1996), most of this is limited to our understanding of addition and subtraction. Furthermore, our understanding of the implications of the problem-difficulty effect as it relates to ability to employ strategies is confined to the child population (Siegler & Lemaire, 1997). As such, this research question was designed to examine the unique contribution of problem-difficulty effect in adult learners' strategy selection. As discussed, half of the administered problems incorporated multiples of 10, which are thought to be some of the easiest multiplication problems to solve (Siegler & Lemaire, 1997). Research here confirmed the presence of the problem-difficulty effect as assessed through both accuracy and spontaneous strategy usage. The accuracy measure did not distinguish between ability groups, which might be due to the fact that college students at all levels of math performance have basic calculation skills in two-digit multiplication. Thus, the differences between ability groups were not considerable. A significantly higher number of students were more accurate on problems that incorporated a multiple of 10, which is consistent with problem-difficulty effect. Moreover, students of all abilities were more likely to spontaneously use strategies in a non-forced strategy format suggesting that students had at least some ability to understand the mechanisms behind strategies. The researcher here suggests that the average and low-performing students were not efficient enough to apply strategies to arithmetic problems when the mechanism of problem solving solution was less clear and obvious.

Educational Implications

This research supports the idea that math strategy usage is important in the learning process and remains essential even in college students. Specifically, this research suggests that the ability to recognize efficient strategies in forced problem formats remains a good predictor of math achievement. Furthermore, the ability to recognize and execute efficient strategies appears to be integral in higher level math performance, and it might be above and beyond the ability to simply solve problems using strategies. This research expanded the work with children (Imbo & Vandierendonck, 2008; Lindberg et al., 2013; Liu et al., 2015; Siegler, 1988; Siegler & Lemaire, 1997; Zbrodoff & Logan, 2005; Zhang et al., 2013) to suggest that issues with execution accuracy might not be the best predictor in adults. The accuracy in this research was fairly high, particularly compared to accuracy rates in children found across the studies mentioned above.

In regard to the importance of the number of strategies, this research resulted in some divergence when compared with earlier research particularly that executed using children. Our findings concur with Zhang et al. (2013), who found that high variability was relatively meaningless in explaining unique variances in achievement. If the strategy use was random, the ability to use effective strategies described unique variance. In further line with Zhang et al. (2013), higher achieving adults were able to utilize effective and efficient strategies for solving a given arithmetic problem. Notably, the number of strategies, which was supposed to reveal the ability to be flexible, was also not related to math achievement. This indicates that random use of a wide range of strategies might not be an effective way to problem solve. Especially for older populations, who learned basic arithmetic strategies in their elementary years, the ability to recognize effective strategies previously learned appears to be particularly critical.

This research confirmed that high achievers were using memory-based problem solving and number property strategies as our high achievers were not only much more likely to use the number property strategy in the forced choice problem format, but also they were more likely than their counterparts (average and low achievers) to impose their own efficient strategy (e.g., direct retrieval, associative number property, distributive number property). Furthermore, our research identified that algorithm was more frequently selected as a strategy for complex multiplication in comparison to other strategy choices. While this research did not quantify the incidence of strategy use that varied per problem format, problem size did seem to help students identify and execute multiplication, using strategies.

The extremely low incidence of retrieval was perhaps expected, but is less understood. Not only was low incidence of retrieval confirmed through this research, but also the notion that larger problems result in substantially more procedural strategy selection because there is little to no memory strength in American adults. This research also sought to add more evidence for the existence of the problem-difficulty effect. In our final hypothesis focused on examining the presence of the problem-difficulty effect, it seems to be that multiplicands of 10 are in fact easier to solve as evidenced by the increased usage of strategies across ability levels (Siegler & Lemaire, 1997). The above research provides more evidence for the ASCM as purported by Siegler and Shipley (1995). Moreover, it continues to extend their research into the adult population enhancing the theory behind the ASCM and confirming the presence of this model into adulthood and across learners of all ability types.

There is practical application of this research to our education system. Today, children are using cumbersome strategies to obtain correct arithmetic answers. This research suggests that those students who are inherently good at math and can use retrieval and number sense perform better than those who rely on traditional algorithm or lengthy strategies such as the lattice method. These strategies may be doing more harm than good when they are not placed in the context of developing basic math fluency by establishing automaticity. While the expectation of doing multiplication without a calculator ends in early elementary school, the mechanisms behind this ability persist into adulthood and produce disparities not only in math performance on isolated tasks but also in general attainment. Further research could contribute more profoundly to determine the direction of the math curriculum, but it seems that many students are exiting the most educationally formative years of their lives without the fluency on basic arithmetic skills to sustain a career in STEM fields.

Limitations and Conclusions

There are a number of limitations to this study. Most integrally, this was a sample of convenience taken from a local college, which restricts the generalizability of the results. However, researchers still found a wide range of students performing at various achievement levels, and thus believed the sample was representative of all ability levels. Gender may also be another limitation of this research. The skewed presentation of men in this sample may be due, in part, to the low enrollment percentages of women in engineering majors; as such, they are underrepresented in this sample. This consistent finding hinders the generalizability of the results as they may be true for only half the population (in this case, men), as our sample population was 80% male. Additionally, literature and collection methods did not support the ability to assess whether the measures collected for this research, such as strategy selection, execution, and efficiency, have better predictive ability than other measures. For the purposes of this research, it is only possible to determine that they were or were not predictive, generally.

Future research should continue to explore the *how* of solving multiplication problems in adults to better understand the learning process and what mechanism is contributing to the high-achieving students' strategy recognition. Understanding this process will help more children advance their math skills allowing them entrance into STEM fields. This future research should explore the impact of instruction into adulthood, comparing students of all ability levels to determine the best methods for producing math success in all learners. Additionally, those interested in furthering this research should seek computer-based assessments that calculate response time. Another interesting angle of this research would be to analyze the differences between men and women discussed in the limitations section, and the cultural differences between groups. The next few studies should begin to determine the trajectory of math achievement and strategy usage in a cross-sectional or longitudinal study. Beyond understanding arithmetic, exploring the comparison between calculus and arithmetic could offer important information to this field. Conducting this research but comparing strategy use to measures of calculus abilities could help distinguish which abilities are more at play in adult math achievement.

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