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Muhammet Doruk
Hakkari University

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Examination of Conceptual Knowledge of Freshmen Classroom Teacher Candidates on Function in the Context of Multiple Representations

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Abstract

The aim of this study is to question conceptual knowledge of freshmen classroom teacher candidates on function. In this context, after the teaching of the function, students' skills of defining the concept of function, interpreting the definition and multiple representations for the function were examined. In the study, qualitative research approach was adopted and was an example of a case study. The participants of the study constitute 15 students who were educated in the first class of the classroom teaching department of a state university in Turkey. The data of the study was collected in four consecutive stages. Each stage was arranged according to the information obtained from the previous stage. The students were applied to the understanding form (AF), Multiple representation form (MRF), algebraic and graphical representation form (AGRF) and semi-structured interview form (IF) for the function concept developed by the researcher respectively. As a result of the study, it emerged that more than half of the students have difficulty interpreting the definition, although the function was accurately defined. It was found that students were more successful in the schema and list representations than algebraic and graphical representations of function. Moreover, students had difficulty in transforming between algebraic and graphical representations. In general, sources of these difficulties were; lack of adequate understanding of the function definition, not taking into consideration domains of functions, complex structure of the concept of function and negativity of the students' mathematical backgrounds.

Introduction

The concept of function is among the important concepts of algebra. It is not only one of the key concepts of mathematics but also it plays a role in the definition of fundamental concepts of analysis, its specific significance, such as limit, derivative and integral. The teaching of the concepts of integral and derivative which are advanced mathematics subjects has been built on functions completely (Bayazit & Aksoy, 2013). It is on an important position in the development of mathematical thinking and is a concept associated with all branches of mathematics (Polat & Şahiner, 2007). In courses with a strong sequential structure such as mathematics, the winning concepts of new learning depends on the fact that the preliminary concepts have been actualized and having constructed a bond between them (Özüdoğru, 2016). In this sense, accurate understanding of all aspects of the function concept will facilitate the understanding of other fundamental concepts associated with the concept of function.

In general, the concept of function is based on a simple logic. The special kind of relation from A to B that each element of the A set, which is one of the two non-empty sets, that belongs to one and only one element of the B set, is called the function defined from A to B (Aydın, Camus & Kaya, 2018)". Two criteria are conspicuous in understanding this definition. The first one is "there is not an unpaired element of A set" and the second is "the mapping of any element of A set to just a single element of B". This definition is expressed in mathematical language as follows: "Let f be a relation from A to B. If for all $x \in A$, there exists $y \in B$ such that $(x, y) \in f$ and if $(x, y_1) \in f$ and $(x, y_2) \in f$ then $y_1 = y_2$, the relation f is called function from A to B (Öz et al., 2018)."

In order to relation to be described as a function, two conditions must be satisfied at the same time. Therefore, these two criteria can be called the conditions that relation be a function. In this study, the first condition is called "definiteness" and the latter is referred to as "unique value" condition. The vertical line test, which is used to determine whether a graphically given relation is a function, is an application of the unique value condition. In the studies most students often use vertical line testing without resorting to recognition (Özüdoğru, 2016) but they have difficulty understanding the logic of the vertical line test (Clement, 2001).

Researches conducted on fuctions emphasized that students had difficulty in understanding the concept of function in despite the simplicity of mathematics and the importance of thought (Akkoç, 2006; Clement, 2001; Tall & Bakar, 1992; Vinner, 1983; Vinner & Dreyfus, 1989). The difficulties of the students; in the understanding of the definition (Aydin & Köğce, 2008; Clement, 2001; Hatısaru & Erbaş, 2013; Tall & Bakar, 1992), in the development of the appropriate perception or concept image of recognition (Sierpinska, 1992; Süzer, 2011), multiple representations of the function and transitions between representations (Başturk, 2010; Elia & Spyrou, 2006; Hatısaru & Çetinkaya, 2010; Sierpinska, 1992; Şandır, 2006; Tall & Bakar, 1992; Tall & Vinner, 1981; Vinner, 1983; Yavuz & Hangul, 2014) is possible to collect in four groups as the difficulties experienced.

The concept of function is a rich concept in terms of multiple representations. Wilson (1991) emphasised that students should take advantage of multiple representations to better understand the concept of function. Because learning to use different representations provides a better and comprehensible understanding of the desired concept (Even, 1990). In general, schema, ordered pairs (list), algebraic (equation) and graphical representations are used during the representation of the function examples. In the Turkish mathematics teaching program, the schema, ordered pairs, algebraic and graphical representation of the function is emphasized (Tataroğlu-Taşdan & Çelik, 2015). Using different representations in teaching the concept of function to establish connections between representations help learning. (Tataroğlu-Taşdan & Çelik, 2015). As Sprinska (1992) emphasized in the use of representations, students should know every representation. In addition, intensity shouldn't be given to a single representation. No representation can fully represent the concept of function (Thompson, 1994) to be aware of the limitations of each representation is important in terms of understanding the concept of function (Sierpinska, 1992).

According to Clement (2001), teachers should determine the level of understanding the concept of function of the students, give more time to the meaning and definition of the concept of function, use non-typical examples as well as typical examples and function and to ensure that the definition is functionally used by discussing invalid examples with students. According to Sierpinska (1992), learning should not start from definition, students should know each representation, and one representation should not be concentrated. Polat & Şahiner (2007) determined that when the course content, course plan and method prepared by taking into consideration, common errors in the function are largely resolved. This study was designed as a product of this idea. At the end of the function instruction in accordance with the suggestions for teaching the function in the related literature, it was tried to find out whether the students have conceptual knowledge level about function concept.

The most well-known theoretical framework used to determine the level of conceptual knowledge is the classification that was revealed by Benjamin Bloom and named as "Bloom taxonomy". This classification has been revised for the purpose of eliminating deficiencies and making it more modern (Anderson & Krathwohl, 2001). With the revised new taxonomy, the opportunity to evaluate learning or objectives not only in terms of information but also in terms of process (Bekdemir & Selim, 2008). According to this taxonomy, there is a significant indication of the level of conceptual information (arising from the level of knowledge), to be able to explain/interpret, to give examples, and to be able to convert into another form (Demirel, 2017). In this context, the skills of the freshmen in the department of elementary school teachers to define, interpret, give example and represent the function in different ways were examined and the answers to the following research questions were sought.

1. How do students define the concept of function?
2. How do students interpret the concept of function?
3. What kind of examples do students give for the function?
4. How is the students' ability to give examples of function in different representations?
5. How is the students' ability to transform between algebraic and graphical representations?

Method

Qualitative research approach was adopted in the study. The study was designed as a case study. The case study is an in-depth representation and examination of a limited system (Merriam, 2013).

Participants

The research group of the study consisted of students who were studying in the first class of the department of elementary school teachers at a public university in the Eastern Anatolia region of Turkey. The classroom consisted of 17 students. As two students did not want to participate in the study, the data collection process of the study was conducted with 15 voluntary students. The teaching of the function subject was conducted to the students under the Basic Mathematics course in the curriculum of the department of elementary school teachers. Students come across with the concept of function for the first time in the first year of high school. When students' mathematics curriculum at that period was examined (Ministry of National Education [MNE], 2013), it was seen that target acquisitions were involved such as "to explain concept of function, to do the graphical representation of the function, to draw graphs of exponential functions and to explain injective and surjective functions". The function, as an interest in this study, is treated as "a relation that maps each element of a set (domain) to one and only one element of another set (codomain)". The function is described as a machine that produces for some input values (x), to the output values ($f(x)$) in the framework of a specific rule. In this context, the f(x) in table or rule is given for an x value, and $f(1)$, $f(2)$, $f(a)$, $f(2x)$, $f(x+1)$, etc. values are found. The domain and image sets are shown on the graph of the function. In a graph of a function, the line that is plotted parallel to the y-axis from each point where the function is defined on the x-axis intersects the graph of the function in exactly one point (vertical line test). After the definition is given verbally in the program, it is visually described by the set mapping schema and examples of the function's schema, ordered pairs (list), algebraic, and graphical representations (Gökbaş & Erdoğan, 2016; Tataroğlu-Taşdan & Çelik, 2015). Türkelli (2016) determined that the ninth-grade high school students were more successful in the schema and list representations. In this program he also participated in the criticism of the previous program (Akkoç, 2006), he had opinion that there was more emphasis on the diagram and the list representation of the function.

Semi-structured interview was conducted with six students who volunteered among 15 students. The results obtained from the study were shared and the students' opinions were taken. In the study, the names of S1, S2..., S15 were used instead of the actual names of the students.

Function Instruction

In the function instruction, the studies on the function of the researchers was examined. A list of common mistakes and recommendations for effective teaching were compiled. The list was considered during the instruction. In this list; to stand on the points where students had difficulty, to benefit from multiple representations, to be equally involved in each representation, to be aware of the limitations of representations, not to start with a direct definition of function, to present real-life examples, describing, interpreting and expressing the function definition in different ways, using as many different relation examples as possible, making a definition-oriented assessment of function, emphasizing the relationship definition with rules and discussions with students in all events were prominent. The instruction of the function was completed in six weeks, two hours a week.

The teaching of function was started by giving examples of real-life. Examples given were discussed in the classroom environment. One of the examples given for the concept of function was "child-mother relationship". The relationship between children in a group and the women of children with their mothers was defined as function. The children's set was named domain and the set of women including the children's mothers was named as codomain. The function rule also stated that every child should have a mother and that a child should have only one mother. As a result of this relationship, it was stated that a set of mothers, where each child is paired, is called a set of images. It was emphasized that children in a set of definitions can be both in a finite number or an infinite number. The sample given in the classroom environment was discussed and the examples of functions offered by the students were evaluated in terms of function conditions.

After evaluating the function examples of the students, the formal definition of function was presented. The concept of relation in this definition was reminded again. Students were asked to make explanations of what they understood from this definition. At the end of the discussions, formal definition was explained to students. Later, examples for each representation functions or not function were evaluated with the students. It was paid attention not to weigh any representation, to evaluate an equal number of examples for each representation. Definition-oriented arguments were used in the evaluation of the samples. It was stated that the vertical correct test used for evaluating examples of graphical representations is an application of the single value condition in the function definition.

After the instruction of interpretation of the function definition, the function in multiple representations and the evaluation of the examples of non-function, function types (constant function, unit function, equal function, injective function, surjective function, permutation function, single/double function, number of functions) were educated by giving equal weight to multiple representations in a discussion environment. In the graphical representation of the injective function, the relationship between horizontal line test and injective function definition were highlighted. In the teaching of the "graph of a function", drawing of the graphs of the first and second-degree functions were investigated. Then, using the graph of the function of the $y = x^2$, by using the discovery approach, $y = x^2 \pm k$, $y = (x \pm k)^2$ and $y = x^2 + bx + c$ graphs of the functions were examined. The examples were plotted on the board at first and then the relationship of functions with the $y = x^2$ function was shown by means of dynamic graphic drawing programs. The instruction was ended with the teaching of operation in functions, inverse function and composition of functions.

Data Collection Tools

The data of the study was collected in four consecutive stages according to the results obtained from each section. The students were respectively applied to the Understanding Form for Function (UF), Multiple Representation Form (MRF), Algebraic and Graphical Representation Form (AGRF) and Semi-structured Interview Form (SSIF). UF focuses on the function definitions, insights and examples of students. With the MRF, the students were tried to expose their ability to make examples on multiple representations of the function and to determine whether the relations presented in multiple representations were functions. AGRF focuses exclusively on algebraic and graphical representations of relations. In the final stage, semi-structured interviews were conducted with the students in order to evaluate the results obtained. After the completion of the teaching of the function, the data were applied one week apart. The opinions and approval of an expert in mathematics education was taken during the development of data collection tools. Information about data collection tools is provided in Table 1.

Table 1. Information about the data collection tools used in the study

Forms	Questions contained in the form
Understanding Form for Function (UF)	*Describe the concept of function.* *Describe what you understand from the function description with your own sentences.* *Please give an example of function.*
Multiple Representation Form (MRF)	*Give an example of function for schema, list, algebraic and graphical representations.* *Please indicate whether the relations represented in different structures are functions.* The form includes two relation examples for schema, list, algebraic, and graphical representation, respectively. Relations in schema, list and graphic form do not satisfy definiteness and unique value conditions, whereas relations in algebraic representation are in violation of both conditions. Algebraic examples are $y^2 = x$ and $\|y\| = x + 1$ relations from real numbers to real numbers. Graphical examples are $y^2 = x + 1$ relation from [-1, ∞) to real numbers and $y = x + 1$ from real numbers to real numbers relation plotted graph in the range of [-1, ∞).
Algebraic and Graphical Representation Form (AGRF)	*Indicate whether algebraic and graphically presented relations are function.* In this form, there are both algebraic and graphical representations for three relations. Algebraic representations of examples are presented below. $f(x) = 1, R \to R,$ $x^2 + y^2 = 1, R \to R$ $f(x) = \sqrt{x+1}, \quad R \to R$
Interview Form (IF)	*In the study, it was determined that students had difficulty in algebraic and graphical representations while there was no difficulty in determining whether giving examples about the schema and list representation of the function and whether the given examples are function or not. What do you think about the cause of this situation?*

** Activities used in MRF and AGRF were presented at the end of the study*

Data Analysis

In the analysis of the data, both descriptive analysis and content analysis were used. Content analysis was performed of the function descriptions, interpretations and function examples of the students. Descriptive analysis was used to analyze the data obtained in evaluating the performance of students for multiple representations. The students' function definitions, interpretations and examples of the students were resolved with content analysis and the examples given by the students in multiple representations were resolved with the help of descriptive analysis, depending on whether they were mathematically valid or invalid. In the formation of categories, the opinion of an expert teacher was taken who was a doctor in mathematics education. The categories were finalized by compromising with the expert teacher. In the study the students' expressions that allowed the formation of categories was presented without modification on. In this way, it was aimed to ensure the validity and reliability of the study

Results

Students' Definitions, Interpretations and Examples of Functions

In the first phase of the study, the general skills of the function concept of students were tried to uncover. Students were asked to define the concept of function, explain what they understood from the definition by their own sentences and give them an example of function. The data obtained from the students is summarized in Table 2.

Table 2. Students' definitions, interpretations and examples

Function description (f)	Function definition Interpretation (f)	Example (f)
Mapping all elements of domain to only one element of codomain (8)	Mapping (6)	The algebraic notation that domain and codomain sets were not identified (10)
Mapping each element in domain (4)	Definition-compatible Comments (3)	Example of a specified function with schema representation (4)
Single mapping from domain to codomain (1)	Linking to a rule (1)	Example of the function specified by the list method (1)
	domain and codomain set (2)	
Mapping one element of domain to one element of codomain (1)	No idle element (1)	
Injective match from domain to codomain (1)	Machine (1)	
	No answer (1)	

In Table 2, it was determined that eight students could accurately identify the function. This means that approximately half of the students could conduct at least the level of knowledge about the function concept. Examples of these students include the statements of S10 and S2.

S10: The function is called to match only one element in the codomain of each element in the domain set. In order to be a function, there will be no idle element in the domain and only one image.

S2: Function is a relation in which there is no idle element left in the domain and all the elements are mapped to only one elements of codomain. For example: Every kid has to go to a house to be protected from the cold, and if he doesn't, he freezes. So, it cannot be a function. This is how each element in the domain must match only one of the codomains.

Seven students could not be able to accurately identify the function. Four of these students defined "mapping each element in the domain" but they did not state that this mapping was only one. As an example for these students, the function definitions of S13 and S4 are as follows.

S13: The function is to map each element in the domain to the image set.

> *S4: f, when going from A to B, each element of A (with no idle element) matches any element of B, this connection is called a function.*

The student S15, defined the function "*f* (x) relation that matches only one of the defined image sets from A to B is called a function". This definition has been evaluated in the "single mapping from domain to codomain" category. S8 made "an elements in domain corresponds to an element in codomain and function is an expression defined from A to B" definitions and this definition provided "Mapping one element of domain to one element of codomain" category. S14 expressed "If the images for all elements of the real numbers are different from each other, we can call it a function". S14 specified the difference of each element in the domain as a function rule. It is possible to say that the concept of injective function was dominant within the function definition of this student.

Students were asked to interpret the concept of function in the way they understood. Students interpret this definition mostly as a mapping from the domain to the codomain. The following are sample student expressions that allow this category to occur.

> *S5: It is the elements in a set matches a correspond and a value in another set.*
> *S4: I think function is the thing with the entrants and subtracts.*

The three students correctly interpreted the definition using the features found in the formal definition. The statements of S6 and S9 from these students are presented as examples.

> *S6: There are two sets called domain and codomain. For example, if we say child as the domain, the codomain will be the mother. There will be no idle element in the domain. Every child has to have a mother and only a mother.*
> *S9: There will be no idle elements in the domain, and there will be only one image of the elements in the codomain.*

The function comments of six students were comments that were incompatible with the function definition, such as linking to a rule, domain and codomain, machine, no idle element. They could not explain their relationship with the function concept. Examples of students ' comments are presented below.

> *S1: Binding an unknown to a rule with mathematical operations.*
> *S12: Function get any f (x) set. The domain is a set of values for a cluster that is f (x).*
> *S8: f, is a relation between A and B and which satisfy each element in the domain so that there is no idle element in the domain.*
> *S11: The function is a machine. You put the boards in the machine and want to have a table and other stuff. It is something like that.*

Students were asked to provide a function example in order to reveal the function example in their mind. Many of the students emerged that they offer formulas that do not specify a domain and codomain (Figure 1). Examples of valid functions are for schema and list representations (Figure 2-3). Below are examples of each category.

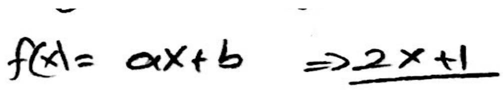

Figure 1. S10's Domain and codomain unspecified algebraic representation

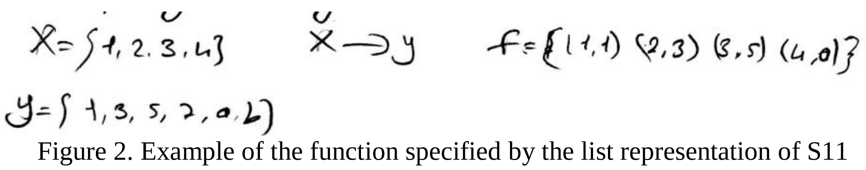

Figure 2. Example of the function specified by the list representation of S11

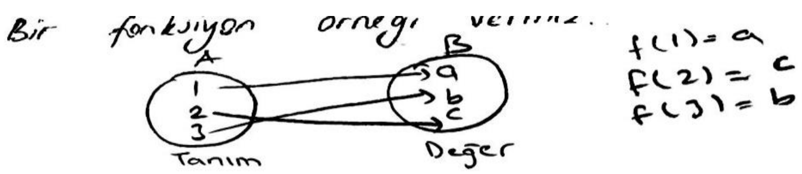

Figure 3. Example of the function specified by the schema representation of S3

Students' Skills on Multiple Representations of Functions

In the first part of the study, it was concluded that the students had the wrong conceptions, such as the function must have only the definiteness and the unique value condition, and that they had difficulty in giving examples in accordance with the multiple representations. It was also found that students did not specify a domain and codomain in function examples. In the second part of the study, in order to obtain more detailed information on these difficulties, students were asked to question the skills of the multiple representation of the function. In this respect, it was tried to find out the students' giving and evaluating example skills for multiple representations of the function.

First, it was questioned the students skills of giving function examples in multiple representations. Students were asked to give an example of the function's schema, list, algebraic (equation) and graphical representation. Students were specifically asked to specify domain and codomains in the function examples. The characteristics of the samples given by the students are presented in Table 4.

Table 4. Characteristics of the function examples that students provide in multiple representations

	Schema	List	Algebraic	Graphical
Valid example	14	13	5	4
Invalid example	1	1	8	9
No response		1	2	2

14 students gave valid examples with schema method. Similarly, 13 students did not have difficulty finding appropriate function examples with the list method. However, most of the students had difficulty in giving an example of functions suitable for graphical and algebraic form. Only four students gave graphical function and five students gave function examples in algebraic form. Students in these two representations had difficulty in specifying domain and codomain. In Figure 5, the valid and invalid example pairs given by the students for the schema, list, algebraic and graphical representation of the function are presented respectively.

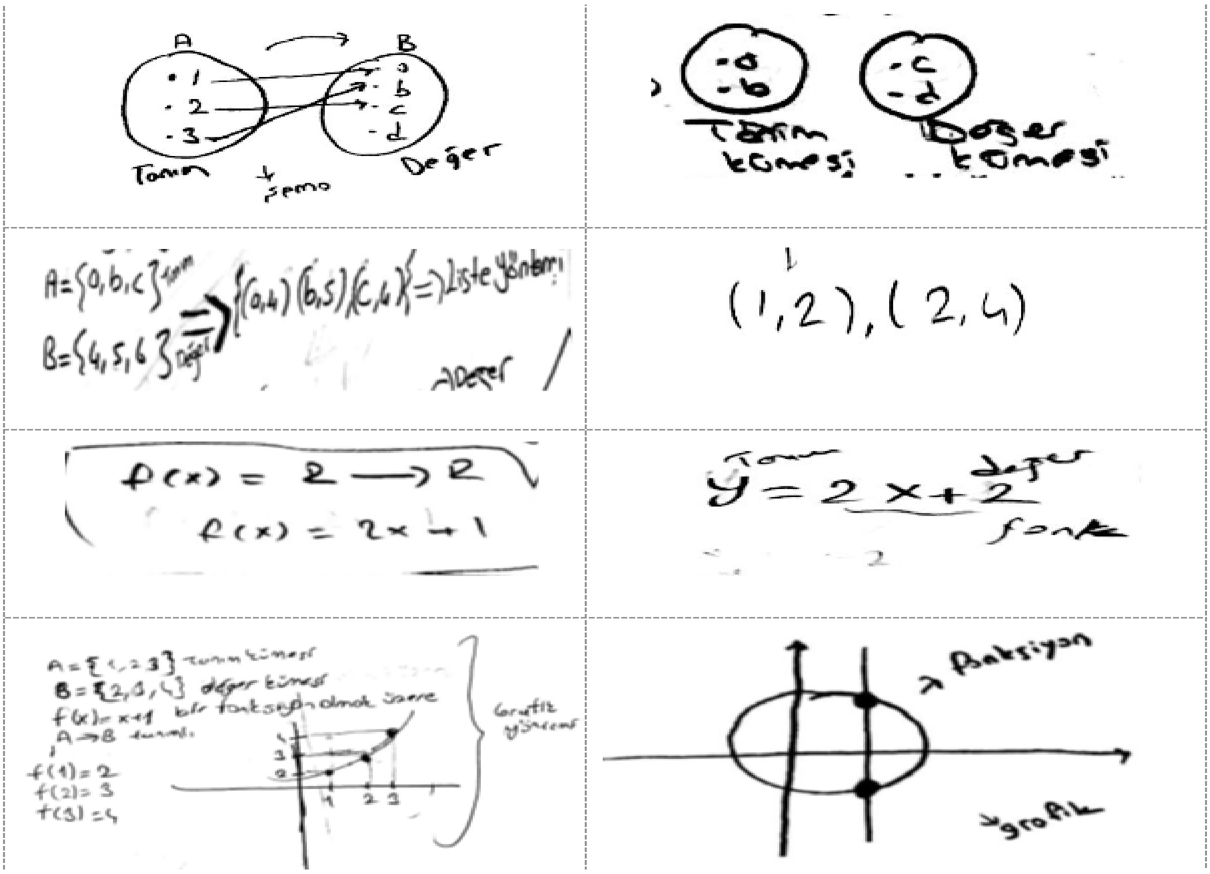

Fig. 5. Examples of valid and invalid functions that students provide for schema, list, algebraic and graphical representation respectively

In order to understand students' difficulties in multiple representations of functions in detail, examples of relations were presented in different representations. The students were presented with a schema, a list, algebraic and graphical forms, and the relation of the function that did not provide the definiteness and unique-value conditions. Students were asked to reach a reasoned decision about whether these relations were function or not. A total of eight relations were presented to students. Relations presented in algebraic form ($y^2 = x$ *and* $|y| = x + 1$ relations defined on the real numbers) were evaluated in both categories as they violated both conditions. In the graphical form, $y = x + 1$, which does not meet the definiteness condition, $y^2 = x + 1$ graphs which do not meet the unique value condition were used. In table 5, information about the skills of the students is presented.

Table 5. The right decisions given by the students in determining whether the relation of the same characteristics represented in different structures

Relation Properties	Schema	List	Algebraic	Graphical
No definiteness condition	12	*8*	*5-5*	*5*
No unique value condition	9	*13*	*5-5*	*9*

In Table 5, it was determined that the students were successful in the schema representation for the non-function relation which did not provide the definiteness condition (12) and that they demonstrated an above average success in the list representation (8). Most of the students were unsuccessful in evaluating algebraic and graphically represented relations. Most of the students were successful in the schema, list and graphical representation in determining the non-function relation that did not provide the unique value condition. In algebraic and graphical form, the students did not have the same success in detecting non-function relations. In the graphical representation, most of the students were successful in determining that the $y^2 = x + 1$ relation was not only a violation of the unique value condition. Students decided with the help of vertical line testing. This showed that the vertical line test was popular among students. When the average achievements were evaluated by taking into account both types of relations, it was found out that students were more successful in schema (average = 10.5) and list representation (average = 10.5), than algebraic (average = 5) and graphical representation (average = 7). With the effect of the vertical line test's popularity, it was seen that the students were more successful in the graphical representation than the algebraic representation.

When the findings were evaluated, it was found that even if they had the same characteristics, most of the students were successful in evaluating their functions with schema and list methods but failed to make evaluations in their algebraic and graphical representations. Examples of different decisions given in Table 6 are presented.

Table 6. Examples of decisions made by students for different representations of the same characteristics

Multiple representations (Unsatisfied condition)	*Sample Student Statements*
Schema (Unique value)	*S3 It's function. Because the domain has no idle element and is also combined with the codomain.* *S10: It is not a function. Because an element in a domain must match only one member of the codomain.*
List (Unique value)	*S6: It is not a function. An element in the domain matched two distinct elements from the codomain. A child can't have two mothers.* *S2: It's function. Because mothers match with their children.*
Algebraic (Definiteness-Unique value)	*S8: It's function. It can get different values for each y and x values.* *S11: It is not a function. Because y does not provide the for negative values of x.* *S6: It is not a function. x = 1 has two images as both – 1 and 1.*
Graphical (Definiteness)	*S4: It's function. Because a value in the definition has not been matched twice.* *S11: Is not the function, because the domain has an idle element.*

When Table 6 was examined, it was revealed that students misused the function definition when making wrong decisions. Even if the justifications were true, it was observed that the students had difficulty in applying these reasons to the examples. As it was seen in Table 6, it was determined that the students decided on the wrong decisions by taking into consideration only one of the conditions of being function. In this sense, it is possible to say that there are deficiencies in the idea that students should provide both conditions in the function definition for a relation to be a function. From the examples they offered to the wrong decisions of the students, it can be said that they were keeping any of the conditions of function more prominent. For example, S3 made a false decision by referencing the definiteness condition of being a function for the relation shown by the schema, in which a unique-valued condition is violated. Similarly, S4 made the wrong decision by accepting a reference to the unique value condition of being a function for the relation represented by the graph, in which the terms of the definiteness were violated. A source of the wrong decisions given by the students was the misinterpretation of the metaphors used for function definition. The deficiencies in understanding the “child-mother relationship” metaphor for the function definition was effective in making the students wrong decisions. Another thought was that the unique-value condition was the definition of injective function. This thought was effective in students’ misjudgment. This situation emerged both in the interpretation of the function definition of the students and in the process of evaluating the relations. Some students used horizontal lines instead of using vertical lines to determine whether the relations are function or not.

Students’ Transformation Skills between Algebraic and Graphical Representations

The data in Table 5 revealed that students had more difficulty in algebraic and graphical representations of functions. The third part of the study was conducted to uncover the ability of students to transform between these two representations. Both algebraic and graphical representations of the three relations, which were on real numbers, were presented to the students. Students were asked to determine whether these relations were function and to justify their decisions. In table 7, information about the different decisions given for the same relation with the correct answers is presented.

Table 7. Decisions made by students for algebraic and graphical representation of the same relation

Relations	Algebraic representation		Graphical representation		Different decision
	Function	Not function	Function	Not function	
First relation $(\boldsymbol{f(x) = 1, R \to R})$	13*	2	11*	4	6
Second relation $(\boldsymbol{x^2 + y^2 = 1, R \to R})$	3	12*	2	13*	5
Third relation $(\boldsymbol{f(x) = \sqrt{x+1}, R \to R})$	4	11*	6	9*	7

** Correct answers*

According to Table 7, it was revealed that most of the students were successful in determining whether the relations were function or not. Nevertheless, it was determined that some students made different decisions to different representations of the same relations. In the general of the study, it was revealed that 12 students made different decisions for algebraic and graphical representation of the same relation. This showed that most of the students were low on the ability to transform between algebraic and graphical representations.

Six students for the first relation, five for the second and seven for the third relation made different decisions to the different representations of the same relation. In order to understand the cause of this difficulty, it was focused on the students who had this kind of contradiction. Six students made different decisions for algebraic and graphical representation of the constant function. When examining the reasons that students used to make these decisions, it emerged that four students misinterpreted the unique value condition of the function. A student could not have been able to justify the wrong decision, and a student stated that there were no values that would be equal to the constant number, so that there wasn’t any idle element in domain. This situation revealed that the students misinterpreted the conditions of function. The following are examples of the decisions given by the algebraic and graphical representation of the constant function of S5.

> *S5 (Algebraic representation): It's function. Specifies a constant function. Specifies a function when x returns a value and if y can find the corresponding.*
>
> *S5(Graphical representation): It is not a function. Because an element in the domain cannot be equal to the two elements in the codomain.*

Five students have made different decisions for the algebraic and graphical representation of the unit circle. It was noteworthy that the students misinterpreted the conditions of function. Three students did not find a reason for their wrong decision. Following are examples of the decisions given by the algebraic and graphical representation of unit circle of S9.

> *S9 (Algebraic representation): It's function. It doesn't matter if the equation passes through the square. Because he says the function is from real numbers to real numbers.*

S9(Graphical representation): It is not a function. Because when we download one line, it intersects two different points.

Seven students made different decisions for algebraic and graphical forms of the third relation, in which the definiteness condition is not met, a student made the wrong decision for both cases. In making wrong decisions; it was effective that the idea of the fact that the three students would have an image of each element in the domain, that the two students needed to be the image of an element, and that the two students did not take into account the domain. One student did not find a reason for his wrong decision. The following are the reasons for the algebraic and graphical representation of S3, S8 and S7, who used different reasons.

S3 (Algebraic representation): It is not a function. Because square-rooted expressions do not indicate function.

S3(Graphical representation): It's function because it's cut at one point.

S8 (Algebraic representation): It's function. y is defined for each x.

S8(Graphical representation): It is not a function. It is defined in R, the roots cannot be negative.

S7 (Algebraic representation): $[-1, \infty]$ *is also defined. So, it is a function.*

S7(Graphical representation): It is not function because square-rooted expressions cannot be a function.

The Reasons of Difficulties in Algebraic and Graphical Representation According to Students

It emerged that students had difficulty in determining whether the relations presented in algebraic and graphical form were function, while they were successful in relations presented by the scheme and list representations. It was determined that this difficulty was profound enough to make different decisions about whether the algebraic and graphical representation of the same relation was function. It was determined that the ability of students to transform between algebraic representation and graphical representation was quite low. The thoughts of the students were wondered about the cause of this difficulty. The results of the study were shared, and interviews were conducted with six students who volunteered to express their ideas.

As a result of the negotiations with the students, the difficulties were cause of the deficiencies of the mathematical backgrounds of the students, the defects in the past teaching method, schema and list representation are just related to function concept and these representation are suitable for the metaphor (mother-child relationship) made about the concept of function, the fact that algebraic and graphical representations are more complex, function covers different subjects and skills. When they compered algebraic and graphical representations, they stated that algebraic representation was harder than graphical representation as vertical line test could not apply in algebraic representation. Below are the students' opinions on this subject.

S1: My teacher, the questions you asked me about functions was the part I couldn't do was about showing it as an equation. And the reason why I couldn't do it was not to study hard enough where I was lacking. But other than that, the graphic, list and schema are simpler to show, it can be determined by drawing with the pen, but I could not do algebraic because it requires more algebraic processing. There seems to be a distinct difficulty in algebraic expressions. I couldn't do because of that.

S2: I think the reason for this is that teachers who had been teaching lessons for a long time with rote-based logic. To be able to do the schema and list method of the functions and not be able to do the graphic and equation stems is because of the basis of the mathematics that were not done in time. I have no trouble identifying functions, but I know why my friends can't.

S3: Since the equation and graphic are more complex than the schema and list method, students are experiencing difficulties in determining whether such relations are functional.

S4: My teacher, I've been able to do questions about the list and the schema method. Because these two methods were directly related to the subject of the function, I could understand it, but I think I couldn't do it because the graphics and the equation covered some of the other math issues. So, we can call it a fundamental lack.

S5: I think make the topic of mathematics in the story allows the student to find a permanent solution on the subject. Of course, such as the schema and the list that can be displayed visually, keep the perception of the students high and provides benefits in the retrieval of desired return. For example, the mother-children story in the schema method. As the equation is thought of as complex structures It's difficult to comment on and build a solution in the mind. I think that this challenge is the inability to make a story for these methods.

S6: I think we can do it because the list and schema method are somewhat simplified and memorized. But the equation and graphics are a little more difficult for mathematical processing and technical wants, we can't. I think it's a bit of a lack of basis.

Discussion and Conclusions

The study revealed that students could often define the concept of function but had difficulty in interpreting the concept of function. When the definitions of the students who had difficulty in defining the function were examined, it was seen that the students failed to evaluate the two conditions of being a function jointly. In their definitions, these students either stated only the definiteness condition or only the unique value condition. Some definitions of students did not include the two conditions. Moreover, some students thought that the unique value condition was equivalent to injective function definition. In the earlier research, students were asked to define the concept of function. In some researches, students never expressed the formal definition (Hatısaru & Erbaş, 2013; Türkelli, 2016). In some studies, very few students could give a satisfactory formal definition (Clement, 2001; Ural, 2012; Vinner, 1983). Polat & Şahiner (2007) reported that 40% of the students in the department of elementary school teachers study had a misconception that the function had to be injective and surjective. It was found that students in the study gave more formal definitions and had less difficulty in using injective function concept instead of function than the relevant literature.

When the students' interpretations on definition of function, only three students were able to interpret the definition correctly. Students frequently interpreted the definition as matching but could not make a proper explanation. Other students had the meaning of machine, linking to rule, no idle element, domain and codomain sets. In the previously conducted researches, students also had the concept of function; any mapping (Ural, 2012; Vinner & Dryfus, 1989), mapping rule (Vinner, 1983), dependency relationship (Tall & Bakar, 1992; Türkelli, 2016; Vinner & Dryfus, 1989), formula, equation or equality (Türkelli, 2016; Vinner, 1983; Vinner & Dryfus, 1989), machine (Clement, 2001) matched the statements. Doruk and Kaplan (2018) found that the comments of the function definition of primary mathematics teachers were in the form of conceptual comprehension, misunderstanding and concept complexity. Similarly, in this study, there were interpretations that are not compatible with the definition. As students could not interpret this definition in their own words although they were successful in making the definition of function, there might be problems in their conceptual understanding.

When the examples given by the students and the reasons used in evaluating the relations were examined, it was seen that there were some cognitive difficulties that prevent the fact that students could not have conceptual knowledge about the definition of function. These difficulties are listed below:

- Failure to consider functions as common.
- Seeing only one condition adequate for function.
- The function was considered only as a mapping, regardless of the conditions of the functions.
- Usage of non-mathematical referenced warrants and prototype examples.
- Perceiving unique-value condition as a definition of injective function.
- Applying vertical line used to test the unique value condition of function as horizontal due to the perception of injective function.
- Focus only on rule or shape of relations, regardless of domain and codomain of functions.
- Misinterpretation of metaphors used for function definition.

When the difficulties identified were examined, it could be said that the most common reasons for the difficulties in students were that they did not have a conceptual knowledge about the definition of function and the concept images that were not compatible with the definition. These reasons were mentioned in some studies in the literature (Akkoc, 2003; Hatisaru & Erbas, 2010; Tall & Bakar, 1992). The difficulties listed above; use the concept of injective function for the concept of function (Bayazıt, 2010; Doruk & Kaplan, 2018; Polat & Sahiner, 2007; Vinner, 1983), seeing function only as a mapping (Ural, 2012), not paying attention to the domain and codomain in function (Hatısaru & Erbaş, 2013; Özüdoğru, 2016), the use of function as a rule only (Dreyfus, 1989) and the use of examples without mathematical reference in evaluation (Özüdoğru, 2016; Tall & Bakar, 1992) can be related to the previously identified difficulties. Difficulties in understanding the conditions of function and misinterpretation of the metaphors used for function were identified in the study for the first time.

Some students could not use the function definition correctly when evaluating the relations in the study. Similarly, Hatısaru & Erbaş (2013) reported that students of vocational high schools could not use the mathematical definition of the function efficiently when evaluating whether relations given with list, algebraic and graphical representations were functions. The results from this study presented detailed information on this difficulty. Students' deficiencies in comprehension of definiteness and unique value conditions got effected to efficiently usage of formal definition.

When the students' performances of giving examples and evaluating the examples with multiple representations were examined, it was determined that the students had no difficulty in giving and evaluating examples with schema and list representations. However, it was revealed that the students had difficulty in giving and evaluating examples with algebraic and graphical representations. In earlier studies, it was determined that Turkish students were more successful in the schema and list representations (Akkoç, 2006; Türkelli, 2016). According to the students, the reasons of these difficulties were: deficiencies in their mathematical background, past teaching methods, schema and list representations only related to the function concept, these representations are suitable for the metaphor (the mother-child relationship) related to the concept of function, algebraic and graphical representations are more complex as they cover different topics and skills. It is possible to say that these views of the students are supported by the relevant literature. The researchers stated that in the Turkish curriculum, they were given weight to the schema and the list method associated with it (Akkoç, 2006; Turkelli 2016; Yavuz & Başturk, 2011). In the studies conducted in connection with this situation, it was determined that schema and list representations were evoked definitional properties in the students' minds, and other representations evoked examples (Tataroğlu-Taşdan & Çelik, 2015). Researchers indicated that a learning based on memorization of rules and definition was realized (Özüdoğru, 2016; Polat & Şahiner, 2007). Algebraic awareness is required at the structural level to understand the concept of function (Sierpinska, 1992). Wilson (1991) also stated one of the cause of difficulties is that functions concept includes many sub-concepts. As schema and list representations are most commonly used methods in introduction of function concept, the students could mostly establish relationship these representations with the concept of function. In this context, it can be said that the students saw these representations as part of the function concept. Conversely, it can be said that giving less place in the introduction function concept and requiring skills such as graphic reading and equation interpretation make algebraic and graphical representations difficult.

It was found that the students were successful in determining whether the algebraic and graphical representations of the same relations were functions. Most of the students had comprehension of the constant function and the circle. In study of Tall & Bakar (1992), most of the students did not accept the constant function in algebraic form as a function but accepted the graphical form. Most of their students accepted the circle as function. Accordingly, it can be said that the students in the study were more successful in terms of constant function and circle. When the decisions of the students about the different representations of the same relations were examined, it was revealed that most of the students fell into contradiction. Most of the students had difficulty in transforming between algebraic and graphical representation. This result is consistent with the study results in which students had low ability to transform between representations (Baştürk, 2010; Şandır, 2006; Tall & Bakar, 1992; Yavuz & Hangul, 2014).

In the study that function teaching implemented, some positive results were obtained when compared to the difficulties detected in the literature. It emerged that students had achieved more success in understanding of formal function definition, performing definition-based evaluation without resorting to prototype examples, evaluation of relations presented multiple representations. This result obtained from the study is coincide with the previous studies of emphasizing the importance of the definition information in multiple representations (Ural, 2012), expressing that students using the definition features would be more successful in the transformation between multiple representations (Akkoç, 2005) and the teaching of function using multiple representations would be more successful (Akçakın, 2015; Can, 2014). Polat & Şahiner (2007) determined that the errors were eliminated by the course content, lesson plan and method which were prepared by taking into consideration the common mistakes in function. In this study, it was illustrated that positive results were obtained with small changes in course content and teaching method. According to the above suggestions, the teaching was successful in the students' definition-centered thinking, but it revealed two different difficulties:

The first one is the misconception of the students in the conditions of function, and the second is the misinterpretation of the daily life examples used in the introduction of the function.

Recommendations

For the effective function instruction, it is important to conceptually understand the definition of function by the students. Accordingly, the meaning of the two conditions in the definition of function, underlying mathematical ideas and that two conditions are indispensable for the function and differences from other concepts should be emphasized. The metaphors given for the function definition should be carefully selected and the relationship with the function concept should be explained in detail. In the evaluation of the course, the examples that students can conceptually use the function definition can be discussed. The discussions about the conditions which the non-functional examples might be a function are useful.

The students had difficulty in transforming between algebraic and graphical representations. Considering that this difficulty can be caused by the deficiencies in the mathematical background, studies on equation interpretation and graphic reading skills of students can be done. By taking into consideration the results obtained from these studies, necessary implementations can be done in order to improve the students' skills. In this way, students' difficulties in algebraic and graphical representation and negative judgments can be overcome. It is considered that such studies will be useful for the conceptual learning of the subject of function.

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Author Information

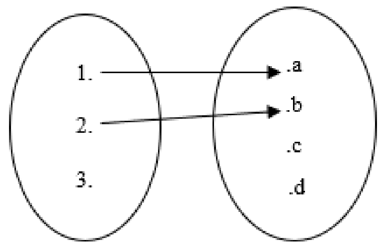
Muhammet Doruk
Hakkari University
Turkey
Contact e-mail: *mdoruk20@gmail.com*

Appendix. Activity Forms

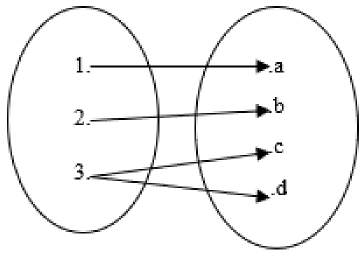
Activities in Multiple Representation Form (MRF)

- Give an example of functions for schema, list, algebraic and graphical representations.
- $A = \{1,2,3\}$, $B = \{a, b, c, d\}$. Please indicate whether the following relations are functions from A to B. Why?

a)



b)



c) $f = \{(1, a), (2, c)\}$

d) $g = \{(1, a), (1, b), (2, c), (3, d)\}$

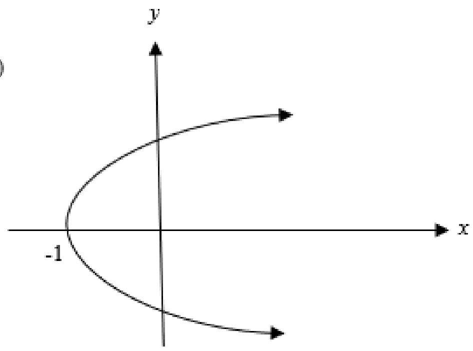
- Please indicate whether the following relations are functions from R to R. Why?

a) $y^2 = x$

b) $|y| = x + 1$

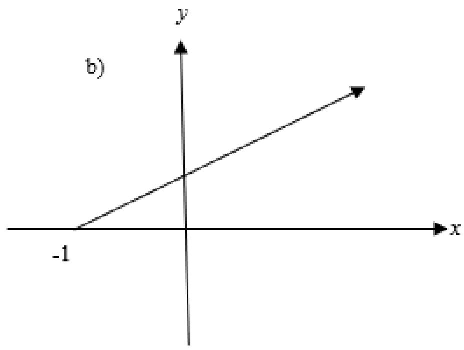
- Please indicate whether the following h and k functions represented graphically are functions. Why?

a)



$h: [-1, \infty] \to R$

b)

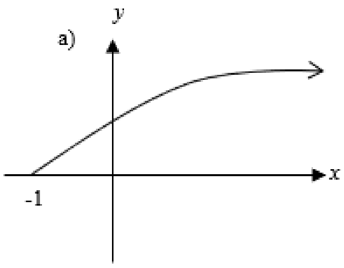


$k: R \to R$

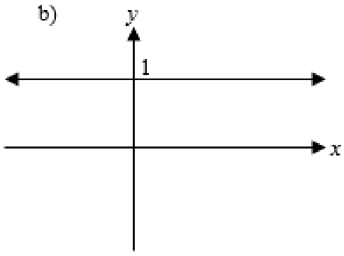
Activities in Algebraic and Graphical Representation Form (AGRF)

- Please indicate whether the following algebraic and graphically represented relations are functions from R to R. Why?

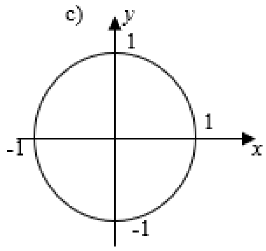
a)



b)



c)



d) $x^2 + y^2 = 1$

e) $f(x) = 1$

f) $f(x) = \sqrt{x+1}$