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Success in Introductory Calculus: The Role of High School and Pre-calculus Preparation

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Abstract

Calculus at the college level has significant potential to serve as a pump for increasing the number of students majoring in STEM fields. It is a foundation course for all STEM majors and, if mastered well, should provide students with a positive and successful first-year experience and gateway into more advanced courses. Studies have shown that a high percentage of students failing college calculus has caused a shortage of individuals entering fields that are heavily dependent on mathematics. Many students are entering college ill-prepared for the rigors of college-level mathematics. This mixed methods study examines the relationship among high school preparation, additional preparation in college, and introductory calculus success. Using a convenience sample of three calculus professors, seven calculus teaching assistants and two hundred and seventy one calculus students at a large public mid-west university in the United States, we investigated the relationship between the amount and level of high school mathematics and student performance in college calculus. Our results suggest that good preparations in high school pre-calculus form a springboard for future success in college calculus as well as subsequent advanced mathematics courses.

Introduction

Calculus is known as the introduction to the higher level mathematics needed by mathematicians, doctors, statisticians, engineers, and scientists. For many students, calculus is a deciding factor in whether or not they pursue a degree in one of these fields or switch to a career path that requires a lower level of math. It is important for students to fully grasp calculus if they plan to pursue a degree in one of these fields. Therefore, it is disheartening to see a large percentage of students failing introductory calculus or barely getting by with a C or D before moving on. However, if students decide to take math classes in college before tackling introductory calculus or if students end up taking introductory calculus multiple times to gain a better understanding of the material and better their grade in the class, they are almost guaranteed to take more than four years to earn a Bachelor's degree. The fear of having to spend more than four years working to earn a Bachelor's degree may possibly be a reason for so few students enrolling in mathematics intensive programs, while the number of careers in mathematics, statistics, and computer science are increasing.

Concerns have been raised about why such large percentages of students are failing calculus and how we can help these students succeed (Tall, 1993). Secondary schools have tried to determine the best ways to help their students succeed in college. Some secondary schools have started to offer calculus as an elective, while others have tried to improve their pre-calculus courses to better prepare their students for college calculus. Colleges have tried to determine ways to help their incoming students succeed, such as giving such students mathematics placement tests and looking at ACT (American College Testing) mathematics aptitude test scores to place students in appropriate mathematics courses and offering remedial mathematics classes. Each student is different and learns mathematics differently, but it is important to determine what outside sources are helping some students get ahead of their peers and how we can offer such opportunities to all students in hopes that they will all succeed.

This study observes the relationship among high school preparation, additional preparation in college, and introductory calculus success. With the voluntary involvement of three calculus professors, seven calculus teaching assistants and two hundred and seventy one calculus students at a large public mid-west university in the United States, we investigated the relationship between the amount and level of high school mathematics and student performance in college calculus.

One of the main goals of this study was to aid secondary school mathematics teachers in preparing their students for college-level mathematics. It is important for teachers to know what is expected of students entering college-level mathematics courses so that they are able to adequately prepare their students. Along with preparing students for college-level work, secondary school teachers are also in a position to help students see the importance of mathematics and its applications so that they encourage students to continue to study mathematics after high school.

Employment of mathematicians, statisticians, and computer scientists is projected to grow much faster than average for all occupations across the United States (Levine & Zimmerman, 1995). Since a high demand is expected in these fields, it is necessary that future generations see the importance of mathematics and are able to succeed in these fields. However, secondary school teachers are not the only ones who are concerned about the success of students in college calculus. Boards of education, school officials, school boards, and textbook companies are constantly trying to determine the best curriculum for secondary schools. College instructors are trying to gauge the level of background knowledge that their students hold upon entering their classes and what gaps they will need to fill. These are all questions on which we hope that this study gives insight.

Literature Review

One of the main goals of secondary schools is to increase the number of students who graduate ready for college (Conley, 2010). However, every year thousands of students enter college under-prepared for the difficulty of college level work (Tierney & Garcia, 2008). Conley claims that this is because secondary schools focus on making sure students are prepared for college entrance tests instead of making sure they are prepared for college. This leads to a large gap between what students learn in secondary education and the knowledge, study skills, and behaviors needed for success in college.

There are many factors that influence whether or not a student is ready for college. One of the biggest factors is the highest level of mathematics that the student took in high school. Adelman (2006) claims that students who take difficult mathematics classes in high school are more likely to go to college than those students who do not take difficult math classes. Adelman also state: “Of all the components of curriculum intensity and quality, none has such an obvious and powerful relationship to ultimate completion of degrees as the highest level of mathematics one studies in high school" (p.7). High school mathematics classes beyond Algebra II have a greater influence on whether or not students earn a college degree than factors that are more commonly thought of, such as socioeconomic status, ethnicity, or parents' educational backgrounds.

With an increasing number of students entering college unprepared, the number of students needing to take remedial courses is also increasing. A study in 2007 by California State University (CSU) shows that the average high school grade point average of the students in remedial college classes at CSU was nearly 3.15 on a 4 point scale, which is a B average (Howell, Kurlaender, & Grodsky, 2010). It is very frustrating for students and parents when their student who got good grades in high school is now in remedial courses in college because he or she is not prepared for college level material, even though his or her high school grades indicated that the student was ready. Roughly one third of high school graduates who enroll in college are required to take at least one remedial college course (Roderick, Nagaoka, & Coca, 2009). Many students who are placed in remedial courses begin at two year colleges. Figure 1 depicts the enrollment in math classes as percents of total math enrollment at two year colleges. Between 1980 and 2000, the percentage of students enrolling in remedial courses at 2 year colleges increased from 45 percent to 56 percent while the percentage of students enrolled in calculus did not exceed five percent.

Although many students who are required to take remedial courses begin at two year colleges, a large percent of university students are also placed in remedial courses. Table 1 outlines the enrollment in calculus and math classes that are commonly taken in place of or before calculus at North Dakota State University. Students end up spending thousands of dollars on remedial courses which do not count for college credit. The cost of remedial classes in public colleges is over two billion dollars (Merisotis & Phipps 2000).The lack of college readiness is seen most acutely in the areas of math and science. Students who complete math and science classes in high school that are beyond what is required to graduate are more prepared for college and careers than those students who only take what is required in those subjects.

A study conducted in 2009 indicated that only 42 percent of students who took the ACT were considered to be prepared for college-level algebra (Long, Iatarola, & Conger, 2009). If only 42 percent of students are ready for college-level algebra, that leaves us to wonder how many students are ready for college-level calculus, since algebra is typically a pre-requisite for calculus. A shocking 78 percent of high school graduates did not meet the readiness benchmarks for one or more entry level math courses. Meeting a readiness benchmark indicates that the student has a 75 percent chance of obtaining a grade of a C or higher in the entry level course and a 50 percent chance of obtaining a grade of a B or higher in the entry level course for the specific subject area.Many students gain the false impression that they are ready for college math courses because they pass their high school math courses and graduate from high school. However, college math courses require a much better understanding of mathematical concepts, principles, techniques, and the reasoning behind it all. Conley (2010) states:

> Students who exhibit college readiness possess more than a formulaic understanding of mathematics. They have the ability to apply conceptual understanding in order to extract a problem from a context, solve the problem, and interpret the solution back into the context. They know when and how to estimate to determine the reasonableness of answers and can use a calculator as a tool, not a crutch (p.54).

These are all skills that are expected of incoming college students. Therefore, it is important that students are learning these skills in their high school math classes. This leads us to wonder how we can better prepare students for college level mathematics. There is an obvious disconnect between what students learn in high school and what students are expected to know when they begin college. Are we failing to teach students what they should know when they graduate high school or are we expecting too much of students when they begin college? Until the beginning of the 1960's, calculus was considered a sophomore (second year in college) class.

The math course that was taken by freshman (first year in college) was an overview of many different areas of math which set up a basis to begin calculus for the students who continued to study math and a stopping point for students of other majors. In the 1960's, the freshman class was removed and calculus became a freshman class. In 1979, the failure rate of Calculus 1 students approached 50 percent, which led to a shortage of students pursuing degrees in mathematics, physics, and engineering (Clement, Lochhead, & Monk, 1981).

With this study, we hope to determine what is causing such a high failure rate among Calculus 1 students. We will look at what factors have helped students at a large public university at the most in preparing them for college-level calculus and what factors have held students back from succeeding in calculus.

Method

The method of research used in this study was the mixed method which involves the mixing of qualitative and quantitative research. Mixed method research includes the use of quantitative research for one phase of the research and qualitative research for another phase of the research. Mixed research was chosen as the method for this study because the quantitative data allows us to examine patterns and correlations among a large population while the qualitative date allows us to examine more complex patterns and why and where these patterns may occur (Creswell & Clark, 2007).

Permission

Since the research conducted involved human participants, certain precautions had to be taken to protect participants. Before beginning any research, the researchers received online training and permission from the Institutional Review Board of a large public Midwest University in the United States. The survey, interview questions, and letters of consent for the survey and interview participants were approved by the University's Institutional Review Board before any research was conducted.

Participants

Participants were 271 students enrolled in first year Calculus (Calculus 1) at a large public mid-west university in the United States in the Fall of 2014. Of the students 271 students, 6 did not answer all of the questions on the survey so their responses were excluded from the study. Of the 265 students, 181 (68.3 percent) were Freshman, 57 (21.5 percent) were Sophomores, 21 (7.9 percent) were Juniors, 4 (1.5 percent) were Seniors, and 2 (0.8 percent) claimed to be other.

Procedure

Surveys were conducted in every Calculus 1 recitation section in the Fall of 2014 at a large public Midwest University in the United States. Since recitation sections are smaller and more interactive than lectures, students are more likely to attend recitation than lecture. This improved results by surveying students of all levels instead of simply the students who attend lecture. Students who were willing to participate in the study answered a 17-item survey questionnaire including questions about their high school math classes, their ACT math aptitude score, previous math classes at the university, how often they attend calculus lecture, how prepared they feel they were for Calculus 1, and their current grade in Calculus 1.

Interviews

At the end of the survey, students were asked if they would be willing to meet for an interview. Of the students who indicated that they would be willing to be interviewed, seven students were selected and interviewed. Four of the students interviewed were women and three were men. Of the students interviewed, four were freshman, one was a sophomore, one was a junior, and one claimed to be other because he had switched schools and did not fit into any particular group. Of the students interviewed, four were receiving A's, one was receiving a C, and two were receiving D's in calculus.

Results and Discussion

Figure 1 displays the distribution of the highest level mathematics class that Calculus 1 students took in high school. Fifty eight percent of the students took calculus in high school while the highest class taken by others included subjects such as algebra, geometry, trigonometry, and precalculus. Over two thirds of the students that attended high schools in which calculus was offered took advantage of this opportunity by taking the calculus class in high school.

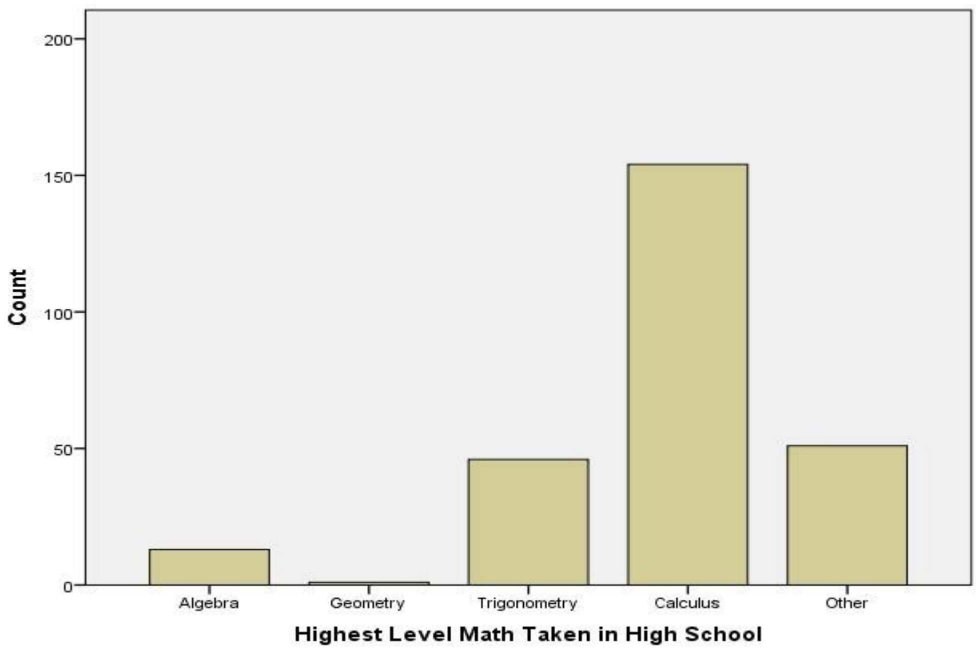

Figure 1. Highest level of math taken in high school

Pearson's chi square test is a common statistical test used to determine whether there is a relationship between two variables by comparing the frequencies observed in each category with the expected frequencies in each category. However, if the expected frequencies are less than five in a large percentage of categories, then chi square is not a reliable test. A test that can be used instead of chi square is the likelihood ratio. The likelihood ratio statistic is based on the creation of a model which maximizes the probability of obtaining the observed data. This model is then compared to the probability of obtaining the observed data if the variables are independent (Gall, Borg & Gall, 1996). We assume the null hypothesis that the variables are not correlated. If the likelihood ratio is greater than 0.05, we will accept the null hypothesis and conclude that there is no correlation between the variables. However, if the likelihood ratio is less than 0.05, we will reject the null hypothesis and conclude that there is a correlation between the variables.

Table 1 shows the highest level of mathematics taken in high school versus current grade in calculus cross tabulation using SPSS statistical software (Norusis, 2008). Since thirty five percent of cells have an expected count less than five, the likelihood ratio is used instead of the chi square test. There is a likelihood ratio of 0.154 which is greater than 0.05 so we accept the null hypothesis that there is no correlation between what classes students take in high school and what grade they receive in calculus.

Table 1. Highest level of math taken in high school vs. current grade

				Student Grade				
			A	B	C	D	F	Total
Highest Level Math Taken in High School	Algebra	Count	2	4	5	3	0	14
		Expected Count	3.3	3.9	4.5	1.6	8	14
		Standard Residual	-.7	1	2	11	-.9	
	Trigonometry		5	12	16	8	5	46
		Expected Count	10.8	12.7	14.8	5.2	2.6	46
		Standard Residual	-1.5	-.2	.3	1.2	1.5	
	Calculus		46	43	45	12	8	154
		Expected Count	36	42.4	49.4	17.4	8.7	154
		Standard Residual	1.7	1	-.6	-1.3	-.2	
	Other		9	14	19	7	2	51
		Expected Count	11.9	14	16.4	5.6	2.9	51
		Standard Residual	-.8	0	7	5	-.5	
Total		N	62	73	85	30	15	265

Table 2. Results of Chi-Square tests

	Value	*df*	**p**
Pearson Chi-Square	16.14	12	.185
Likelihood Ratio	16.89	12	.154
Linear-by-Linear Association	1.59	1	.207
N	265		

Although the data tells us that there is no correlation, we must look at possible reasons why students who have already taken calculus may not do well in college calculus including the transition to college life and the quality of high school calculus. Starting college can be a difficult time for many students. It can be hard to not only get used to the difficulties of college level work but also living on your own. One student interviewed stated that college is ``the first time you are out on your own and you get to do what you want, so classes really hurt." This feeling is shared by many college students. Another calculus student talked about how his calculus class is at a different time of day than his other classes and said that he is lazy and therefore does not go to calculus class and is now receiving a D in calculus.

Attendance in lecture plays a large role in how well students do in calculus. Table 1 displays the attendance of lecture vs. current grade in calculus cross tabulation. Since fifty five percent of cells have an expected count less than five, we look at the likelihood ratio statistic. Since the likelihood ratio 0.022 is less than 0.05, we reject the null hypothesis and conclude that there is a correlation between attendance of lecture and the grade that students receive in calculus. Getting used to having to study can be difficult for many students. One student said he didn't have to study for math in high school and claimed that his math teacher at his private school ``made sure that everyone, no matter where they were, were at the specific pace that she wanted." Another student stated that ``I feel like public school, it's not on the same level as the collegiate level because a lot of it's all spoon feeding."

The quality of high school calculus varies from one school to another. One student who had taken calculus in high school stated ``I'm surprised how much they didn't teach me in high school." There are many important concepts that are often not included in the high school calculus curriculum. Students indicated that subjects such as Newton's method for finding roots, linearization of functions, and having to memorize the derivatives of trigonometric functions were not included in their high school calculus classes.

Table 3. Attendance of lecture vs. current grade in calculus

			A	B	C	D	F	Total
Attendance of lecture	Early Day	Count	54	49	66	17	8	194
		Expected Count	45.4	53.4	62.2	22	11	194
		Std. Residual	1.3	-.6	.5	-1.1	-.9	
	Usually	Count	7	18	18	10	5	58
		Expected Count	13.6	16	18.6	6.6	3.3	58
		Std. Residual	-1.8	5	-.1	1.3	.9	
	Rarely	Count	1	4	-.1	3	1	10
		Expected Count	2.3	2.8	1	1.1	.6	10
		Std. Residual	-.9	.8	3.2	1.8	.6	
	Never	Count	0	2	-.12	0	1	3
		Expected Count	7	8	0	3	2	3
		Std. Residual	8	1.3	1.0	-.6	2.0	
Total		Count	82	73	85	30	15	265
		Expected Count	62	73	85	30	15	285

Table 4. Results of Chi-Square tests

	Value	*df*	*p*
Pearson Chi-Square	24.31	12	.018
Likelihood Ratio	23.73	12	.022
Linear-by-linear Association	7.72	1	.005
Number of cases	265		

One major difference between many high school calculus classes and college calculus classes is that college calculus professors do not allow you to use a calculator while many high school teachers do. Students who experienced this difference between high school and college calculus said that being able to use a calculator in high school calculus ``helped me to cut corners a lot", that they didn't have to show work so did eighty percent of it on a calculator in high school, and ``if you could remember how to punch it in to your calculator, you could basically get one hundred percent on the test." When asked whether not being able to use calculators in college calculus was a hard transition after being able to use them in high school, one student claimed that calculus was actually easier in college because the professor explained the rationale and proofs so she was able to understand the material.

Many high school calculus teachers simply give students formulas instead of giving proofs, definitions, or rationale behind the formulas. A student stated : “Usually when something ties into something else or you show how it works together, that really helps me understand something." This was a common feeling among many students who claimed to learn better when the reasoning behind the concepts are explained and when concepts are applied to real world situations. One student who took applied calculus in high school, however, was not allowed to use a calculator in applied calculus except for large computations. Applied calculus was the highest level of math offered in his high school. He claimed that ``it would have been nice to have Calculus 1 in high school as well but at the same time it was really nice just to kind of learn all the concepts first". He said that if his high school would have offered

Calculus 1 in high school, he probably still would have taken it at the University anyway because his high school math teacher stressed that each college has different notation that is good to learn before jumping into Calculus 2. Students were asked on a scale of one to five how prepared they were for Calculus 1 based on their high school math preparation. Table 3 displays the cross tabulation between how prepared students feel for Calculus 1 and their current grade in Calculus 1. Thirty six percent of the cells have an expected count less than five, therefore we use the likelihood ratio instead of the chi square test. We have a likelihood ratio of 0.00 which is less than 0.05 so we reject the null hypothesis that there is no correlation. Thus there is a very strong correlation between how prepared students feel for calculus and their grade in calculus. It is clear that students have a good understanding of how well they will do in calculus based on how well their high school prepared them for college level math.

The math preparation that students receive in high school is an important factor in how prepared they feel for college calculus. If they have never seen any calculus before beginning college calculus, it is often hard for them to comprehend what is going on because calculus is such a different area of math. One student stated:

> For some reason 102 (intermediate algebra), 103 (college algebra), 105 (trigonometry) all came to me just like that and I had never taken trig before but it came to me and then I understood it but calculus is something completely different for me for some reason. For me, calculus is a brand new concept so as fast as we are moving it's kind of hard to absorb everything.

Many students agree that calculus and the notation used in calculus is often hard to learn if you have never seen it before. One student claimed to have never seen summation notation before needing to be able to apply it to complex series in Calculus 1 even though she had taken calculus in high school. Notation such as this is one thing that should be taught in high school and pre-calculus college courses because it is like learning a new language which is needed before being required to apply concepts in that new language. Whether or not a high school offers calculus, there are many things that students wish would have been at least mentioned in their high school math classes such as notation, limits, and the definition of a derivative.

Table 5. Highest level of math taken in high school vs. prepared for college calculus

		Student Grades					
		A	B	C	D	F	Total
Prepared for Calculus	1 Count	1	1	5	6	4	17
	Expected Count	4.0	4.7	5.5	1.9	1.0	17
	Standard Residual	-1.5	-1.7	-.2	2.9	3.1	
	2 Count	4	10	18	11	5	48
	Expected Count	11.2	13.2	15.4	5.4	2.7	48
	Standard Residual	-2.2	-.9	.7	2.4	1.4	
	3 Count	10	21	42	12	3	88
	Expected Count	20	24.2	28.2	10	5	88
	Standard Residual	-2.3	-.7	2.6	6	-.9	
	4 Count	23	26	17	0	3	69
	Expected Count	16.1	19	22.1	7.8	3.9	69
	Standard Residual	1.7	1.6	-1.1	-2.8	-.5	
	5 Count	24	15	3	1	0	43
	Expected Count	10.1	11.8	13.8	4.9	2.4	43
	Standard Residual	4.4	9	-2.9	-1.8	-1.6	
Total	Count	62	73	85	30	15	265
	Expected Count	620	73	85	30	15	265

Table 6. Results of Chi-Square tests

	Value	*df*	*p*
Pearson Chi Square	99.49	16	.000
Likelihood Ratio	104.57	16	.000
Linear-by-linear Association	72.38	1	.000
Number of Cases	265		

Many of the Calculus 1 students interviewed had similar suggestions for improving high school math courses, and college pre-calculus and calculus courses. Every student learns differently but taking these suggestions into account at each level could help all students be more successful in their college level calculus courses. The biggest suggestion for high school teachers and pre-calculus professors in preparing their students for college calculus is to briefly mention concepts such as limits so that students will have at least heard them before. These teachers typically have a lot of content that they need to cover in their courses but simply planting a seed in their students' minds will give them a basis to fall back on when they begin learning more about these concepts in college calculus. Similarly, introducing mathematical notation while they are learning simpler concepts will be beneficial as they will already know the notation so they will be able to focus on the new calculus concepts instead of trying to learn the notation at the same time.

Other suggestions for high school and pre-calculus teachers include giving their students more homework to learn the concepts better and not allowing their students to use calculators so much. These strategies will help students prepare for the rigors of college level calculus. Students also noted the strategies that their Calculus 1 professors use or wish they would use which they find most beneficial to their understanding of calculus. One strategy includes telling the students where specific concepts are used and how they will help them succeed in their careers. This allows students to find purpose and take ownership of their learning. Similarly, students learn better when they are able to see the concepts used in real world problems. Just like the benefits of showing students where the concepts are used, it is also important to show proofs and give theory and rationale behind why the concepts are used. Finally, students find it helpful when professors review and check for student understanding so that if they are behind, they can be given a chance to catch up. It is our hope that these findings and suggestions will add to the body of scholarship that shed light on how mathematics teachers and professors can help increase student understanding of mathematical concepts in their classes. It is important to keep in mind that students learn in many different ways but by implementing these strategies in classrooms, we will likely be able to narrow the gap between high school and college mathematics courses.

Conclusion

Many of the Calculus 1 students interviewed had similar suggestions for improving high school math courses, and college pre-calculus and calculus courses. Every student learns differently but taking these suggestions into account at each level could help all students be more successful in their college level calculus courses. The biggest suggestion for high school teachers and pre-calculus professors in preparing their students for college calculus is to briefly mention concepts such as limits so that students will have at least heard them before. These teachers typically have a lot of content that they need to cover in their courses but simply planting a seed in their students' minds will give them a basis to fall back on when they begin learning more about these concepts in college calculus.

Similarly, introducing mathematical notation while they are learning simpler concepts will be beneficial as they will already know the notation so they will be able to focus on the new calculus concepts instead of trying to learn the notation at the same time. Other suggestions for high school and pre-calculus teachers include giving their students more homework to learn the concepts better and not allowing their students to use calculators so much. These strategies will help students prepare for the rigors of college level calculus. Students also noted the strategies that their Calculus 1 professors use or wish they would use which they find most beneficial to their understanding of calculus. One strategy includes telling the students where specific concepts are used and how they will help them succeed in their careers. This allows students to find purpose and take ownership of their learning. Similarly, students learn better when they are able to see the concepts used in real world problems. Just like the benefits of showing students where the concepts are used, it is also important to show proofs and give theory and rationale behind why the concepts are used.

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