**Middle School Students’ Metacognitive Strategies in Problem Solving**

**Abstract:**

The purpose of this study is to investigate metacognitive strategies that middle school students used in the process of solving problems individually. The study group consisted of 37 middle school students at eighth grade. One non-routine word problem was asked to the students and written answers were collected. After solving the problem, a self-monitoring questionnaire based on reflecting retrospectively on their metacognitive strategies regarding the problem was required the students to complete. In order to obtain detailed data, semi-structured interviews were conducted with six of those students who three of them gave correct responses and the other three of them gave wrong answers to the problems. The data was analyzed with the help of Goos, Galbraith and Renshaw’s (2000) model of metacognitive activity during problem solving. The data revealed that metacognitive skills was related to problem solving and it influenced problem solving outcomes positively. Students who were able to use metacognitive skills gave correct answers to the problems and they were able to recognize problem requirement and objective, use various solution strategies and check the correctness of solution. Students who were not able to use metacognitive skills gave wrong answers to the problems and they had difficulty in understanding problems, determining solution strategies, recognizing their errors and correcting them. Most of the students also were not good at solving non-routine problem. Their responses to questionnaire and explanations about the solutions were not clear and sensible enough. It showed their lack of awareness of what and why they did while solving problems. They generally focused on obtaining an answer without considering its correctness. Since they were not able to establish a logical connection between their solutions and problem, they could not notice where they made mistakes to overcome. Thus, they mostly gave wrong answers to the problem.

**Keywords:** Problem solving, Metacognitive strategies, middle school students

**Introduction**

Metacognition is what one knows about his own thinking ways and how he monitor and regulate his thinking processes while engaging in a task (Goos, Galbraith & Renshaw, 2000). Flavell (1976) defines the metacognition as being aware of one's own cognitive processes or knowledge of any subject (p.232). This term is defined with two aspects which are knowledge about cognition and regulation of cognition. Knowledge about cognition includes declarative knowledge, procedural knowledge and conditional knowledge (Schraw, 1998). Declarative knowledge refers to knowledge about things such as himself/herself, strategies and the factors that influence his/her performance (Schraw & Moshman, 1995; Schraw, 1998). Procedural knowledge is awareness of how to do things. (Schraw & Moshman, 1995). It includes knowing which strategy and how it should be used (Schraw, 1998). Conditional knowledge refers to know why and when for cognitive activities (Schraw & Moshman, 1995). It allows students to adapt their knowledge to changing situations (Schraw, 1998). Regulation of cognition consists of metacognitive skills such as planning, monitoring, control and evaluation of cognitive processes (Ader, 2019; Whitebread et al 2009). Planning refers to determine appropriate strategies for the task, monitoring is to be aware of own performance while engaging in a task, control and evaluation indicates reviewing the process (Schraw & Moshman, 1995).

Educators emphasize the “active and constructive process of sense-making, understanding and problem solving in a community of learners” (De Corte et al. 2011, p.1 55). Various countries such as US (National Council of Teachers of Mathematics [NCTM], 2000) and Turkey (Ministry of National Education [MoNE], 2018) has introduced metacognitive skills as an important component of mathematics education (Ader, 2019). Based on the continuous changes in mathematics curricula, teachers should offer experiences for reasoning and problem solving (Ader, 2019). Metacognitive skills play critical role in monitoring and regulating cognitive processes (Chan & Mansoor, 2007) and help students to understand when, why, where and how to use their own knowledge to solve problems successfully (Carr & Jessup, 1995). On the other hand, students who are not able to notice the errors they made while solving problems, monitor what they did, use appropriate strategies and explain their solutions are not good at mathematics (Lucangeli & Cabrele, 2006; Carlson & Bloom, 2005). Therefore, it is one of the important issues in problem solving (Lester, 1994). Problem solving involves not only cognitive strategies but also metacognitive skills and more than implementing strategies to solve the problems. In fact, metacognitive skills are related to problem solving (Ader, 2013; Mayer, 1998) and students who have these skills can decide whether problem is sensible, analyze the consistency between strategies and solutions, determine correctness of the answer and recognize their errors. However, since analysis and observation of metacognitive skills is difficult, this issue has not been sufficiently investigated by mathematics educators.

There are various studies on the important role of metacognition in the success of problem solving (Artzt & Armour-Thomas, 1992; Pennequin, Sorel, & Mainguy, 2010; Swanson, 1990).Kuhn (2000) also emphasized the influence of the development of metacognition in early years on higher order thinking processes since it provided better cognitive skills. Students better performed the tasks in the process of learning mathematics and solving problem (Mevarech & Fridkin, 2006). Eggen and Kauchak (2001) stated that students who are aware of when they acted or did not act strategically are successful. Because learning become effective when is occurred consciously. Metacognition in problem solving help students to carry out the steps of problem solving and manage this process (Sevgi & Cagliköse, 2019). For solution of problems, it is not only the need for carrying out the stages of problem solving but also following, regulating and controlling these stages consciously. Therefore, metacognition is an essential element of problem solving process (Schoenfeld, 2005). Although various studies have revealed the positive relationship between metacognition and the success in mathematics (Kitsansas, 2002; Rysz, 2004), some have not supported these findings (Kuyper et al., 2000; Ader, 2004; Hong & Peng, 2008). This contradiction also demonstrates the need for more research on the relationship between metacognition and mathematics.

The exams such as PISA and TIMMS demonstrated that students were not good at solving mathematics problems (OECD 2004, 2007). As solving mathematics problems, students need to understand the given knowledge in problem, represent the problem using the knowledge appropriately, generate a plan to solve the problem and make the necessary calculations. Therefore, it is difficult to follow these steps for many students, especially students who do not have sufficient knowledge (Verschaffel et al., 2000). The poor performance in problem solving is not only due to the lack of mathematics knowledge but also the lack of students’ awareness of what, when and how to do, namely, metacognitive skills (Altun & Arslan, 2006; Garofalo & Lester, 1985; Mayer, 1998; Özsoy & Ataman, 2009; Pintrich, 2002; Schoenfeld, 1985). Many studies emphasize the important role of metacognition in problem solving and mathematics education (Desoete, Roeyers, & Buysse, 2001; Garcia, Rodríguez, González-Castro, González-Pienda, & Torrance, 2015; Jacobse & Harskamp, 2012; Mevarech, 1999; Mayer, 1998; Verschaffel, Greer & De Corte, 2000). However, there are deficiencies in understanding metacognition and its effects on being a good problem solver (Asik, 2015).

**Metacognition and Problem Solving**

There have been various definition about problem from past to today. Some have defined problem as the exercises that need to engage in using new mathematical knowledge and techniques, others have introduced it as complex tasks that the solver encounter obstacles (Schoenfeld, 1992). Problem solving is essential part of mathematics education and continues to carry its importance (Evans, 2012). Thus, one should know how to deal with a problem to understand mathematics (Van De Walle, Karp, & Bay-Williams, 2010). It is a process which necessitates comprehension, visualization, analysis, association, generalization and reasoning (Garafola & Lester, 1985). Polya (1985) defines problem solving in four phases which are (a) understanding the problem, (b) making a plan, (c) carrying out the plan and (d) checking out the solution. These phases are parallel with the planning, monitoring, control and evaluation skills in metacognition (Ader, 2019; Whitebread et al 2009).

Problem solving requires not only knowledge and cognitive skills but also metacognitive skills, when and how to use the knowledge and cognitive resources (Mayer, 1998). Cognitive skills provide to understand the task and use strategies for solution whereas metacognitive skills help to regulate the problem solving process and make decisions (Goos, Galbraith & Renshaw, 2000). Lucangeli and Cornoldi (1997) emphasized the important role of metacognition in mathematics education. For example, the early stages problem solving such as representation of problem and making a plan to solve the problem includes metacognition (Desoete & Veenman, 2006). After solution, the accuracy of it is checked. Therefore, problem solving requires using cognitive skills to know what and how to do and controlling the process. It refers to metacognition and increases the achievement in problem solving by enabling students to represent the problem mathematically and try various strategies on it (Davidson & Sternberg, 1998).

Problem solving provides to construct new mathematical knowledge which is named as doing mathematics (Van De Walle, Karp, & Bay-Williams, 2010). However, students have difficulty in problem solving which requires regulating their learning (Fuchs et al., 2003) and using certain strategies (Montegue, 2008). Desoete (2007) has suggested that teachers should give importance to metacognition for developing students’ problem solving skills. Students should be aware of cognitive resources and control how to use them (Lucangeli & Cornoldi, 1997). Metacognition helps students to use the knowledge and strategies effectively (Lucangeli et al., 1998) and overcome the challenges in mathematics (Carr & Jessup, 1995). The development of metacognition begins in early childhood and becomes more complicated form (Veenman, 2011). The frequency of using metacognitive skills and their effectiveness increase in time (Van der Stel & Veenman, 2010). Goos, Galbraith and Renshaw (2000) identified behaviors that should be observed during problem solving among students with metacognitive skills:

Table 1. A model of metacognitive activity during problem solving (Goos et. al., 2000)

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| **Problem Solving Behaviors** | **Metacognitive Skills (Monitoring/Regulation)** |
| *Before solving the problem:* Reading the problem more than once, trying to understand the problem, trying to restate the problem, checking the familiarity with a similar problem before, identifying the given information in the problem and considering which ways should be used to solve the problem. | Assess knowledge, understanding of problem and strategy appropriateness |
| *As solving the problem:* Checking the solution step by step, returning the beginning if a mistake was made, rereading the problem to control whether the solution steps was still true, determining whether got close to the solution, reviewing the solution and trying a different approach. | ***“Red Flag”:*** *Error detection*  Assess strategy execution  Correct errors  ***“Red Flag”:*** *Lack of progress*  Assess progress towards goal  Assess understanding of problem and strategy appropriateness  Change strategy |
| *After solving the problem:* Checking the correctness of the calculations, looking back the solution to control the consistency between what the problem asked and what was done, examining whether the answer was sensible and thinking on different solution ways. | ***“Red Flag”:*** *Anomalous result*  Assess strategy appropriateness and execution Assess result for accuracy and sense |

The notation of “red flags” is defined as warning signals of “the need for a pause or some backtracking while remedial action is taken” (Goos et al., 2000, p. 3). It alerts the lack of something in metacognitive process. Red flags are classified as “Error detection”, “Lack of progress” and “Anomalous result”. *Error detection* promotes to check and correct the errors as solving the problem. *Lack of progress* leads to reanalyze the problem in order to reassess strategy appropriateness and decide whether to use or change the strategy. Therefore, students reassess understanding of the problem to reach new information and to determine a new strategy. *Anomalous result* prompts to assess whether the solution is correct or meaningful. If the solution does not make sense or answer the problem, anomalous result leads students to check the calculation and strategy execution. In this study, this model was used to evaluate students’ metacognitive skills during problem solving process.

NCTM (2000) emphasize the importance of problem solving from elementary level to higher grades. Students fail in problem solving due to the lack of understanding problem, choosing appropriate strategy, regulating the solution, monitoring calculations and controlling whole process (Victor, 2004). These skills present the essential role of metacognition in problem solving. Problem solving is one of the complex processes that necessitates metacognitive skills in addition to cognitive resources. These skills are essential in problem solving because they provide identify the problem, determine the appropriate strategy, monitor whether the strategy is effective, control the accuracy of the solution and regulate the problem solving process (Sternberg & Hedlund, 2002). Researchers reveal that students who suffer from a lack of success in mathematics do not use cognitive and metacognitive strategies in an effective way as problem solving (Wilson & Clarke, 2004). Better problem solvers tend to represent the problem considering all the information whereas poor problem solvers mainly obtain mathematical operations by only focusing on the key words (Kramarski et al., 2002). The students who have high metacognitive skills are more successful in problem solving (Desoete, 2008; Hollingworth & McLoughlin, 2005; Schoenfeld, 1985) Thus, investigating students’ metacognitive skills is important to improve mathematics education (Livingston, 2006). Although the rising focus on the role of metacognition in problem solving, the known is limited about the nature of students’ metacognitive skills and the use of these skills in problem solving (Suriyon, Inprasitha & Sangaroon, 2013).

In the current study, it was aimed to examine students’ metacognitive skills in the problem solving process, the differences between students who use and do not use metacognitive skills and its effects on the success of problem solving. In this vein, the following research question guided this study with one non-routine problem: *How are middle school students’ metacognitive skills related to problem solving?*

**Methodology**

**Research Design**

Case study is a qualitative approach in which the researcher investigates a case or cases in detail through rich data collection involving multiple resource of information such as observation, interviews and documents (Creswell, 2007). In this study, case study design was used to reach a depth understanding of the students’ metacognitive skills in problem solving process.

**Participants**

The participants of the study consisted of 37 students at 8th grade in a middle school in the Eastern Anatolia Region of Turkey. The students were determined by typical sampling technical to represent eight classes. They also had average achievement scores in mathematics. Although metacognitive knowledge begins to develop in early ages, the appropriate use of metacognitive skills seems to start at the ages of 11-12 (Garcia et al., 2015). Besides, mathematics teaching program has introduced metacognitive skills as an important component of mathematics education (Ader, 2019). Thus, the focus was on 8th grade students’ metacognitive skills as solving problem since they are able to use these skills.

**Data Collection**

Most studies in the past did not much attend to non-routine problems in problem solving (Lee et al., 2012) and the aim of this study was to examine students’ metacognitive skills rather than evaluate their performance in problem solving. Therefore, one non-routine problem including complexity without depending on a specific mathematical knowledge was asked to the students and written answers were collected. The process took twenty to thirty minutes for solving the problem.

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| **Domino Problem:** Cem who is very curious about playing domino wants to find out how many domino stones there are. There are many options for this.  In the Domino game, each stone is divided into two and the numbers 0-6 in each half are indicated by dots. Each stone contains a pair of numbers, and there is only one of these pairs in the whole set. How many stones are there in the whole set? |

After solving the problem, the self-monitoring questionnaire based on reflecting retrospectively on their metacognitive strategies regarding the problem (Gooss et. al., 2000) was administered to the students (Appendix). It provides to examine how students deal metacognitive “red flags” indicating the need for regulation during monitoring. The original form of the questionnaire was developed by Fortunato et al. (1991) for use in 7th grade students. Goos et al. (2000) rearranged the questionnaire to be used in older ages. The questionnaire consisted of twenty-one statements under four titles that are “Before you started to solve the problem”, “As you worked on the problem”, “After you finished working on the problem” and “Ways of working on the problem”. The first title includes six items related to reading and understanding the problem and determining a solution strategy. The second title lists five items using solution strategy and monitoring the solution process. The third title refers to four items demonstrating the accuracy of the solution. The fourth title lists six solution ways to learn which ways were used by students. There were three options to mark *Yes, No* or *Unsure* for each item.

In order to obtain detailed data, semi-structured interviews were conducted with six of those students who three of them gave correct responses and the other three of them gave wrong answers to the problems. The interview questions were prepared based on problem solving behaviors and metacognitive skills that are expected as solving problems by the researchers. The recordings of the interviews approximately took twenty minutes for each student. The following questions were asked to the students to get more detailed information about their problem solving process and interpret these behaviors in terms of metacognitive skills.

1. Have you ever encountered a problem like this before?
2. What did you understand when you read the problem and can you express it in your own words?
3. What was given in the problem and what was asked to find out?
4. What kind of way did you choose to solve the problem?
5. Was the way that you chose to solve the problem helpful to reach the solution?
6. How did you check whether your solution was correct?
7. According to you, can the problem be solved in another way?

**Data Analysis**

In data analysis, content analysis technique was used to analyze the qualitative data. The students’ solutions, the questionnaire data and the transcripts of interviews were analyzed with the help of the model of metacognitive activity during problem solving (Goos et al., 2000) in Tablo 1. In order to provide the reliability and validity of the findings, direct quotations from the interviews, the students` written solutions and questionnaire responses were presented. In addition, two researchers (the authors conducting this study) separately analyzed the data and made their own coding by using the model. In consequence of comparing coding, the consistency between them was found as 88%. They discussed about the remaining 12% of the coding list in order to reach consensus.

**Findings**

Analyzes were made by considering the solution attempts of the students and the answers to the questionnaire. The strategies used by the students are classified in certain ways. The solution strategies the students used are grouped into three categories. The categories are linked to the outcomes of their problem solving activity – no answer, randomly writing the pairs, writing some of the pairs, with right strategy, wrong answer, or correct answer.

Table 2. Solution strategies and outcomes

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| --- | --- | --- | --- | --- | --- | --- |
| STRATEGY | Numbers of Students | | | | | |
| No Answer | Randomly writing the pairs | Writing some of the pairs | With right strategy, incorrect answer | Correct answer | Total |
| Random trial & error | 1 | 1 | - | - | - | **2** |
| Ordered listing | - | - | 3 | 7 | 3 | **13** |
| Systematic listing | - | - | 4 | 2 | 16 | **22** |
| **TOTAL** | **1** | **1** | **7** | **9** | **19** | **37** |

As seen in Table 2, one student using only the trial-and-error strategy could not answer the question. With the same strategy, one student wrote random pairs. The most preferred strategy of the students was making a systematic list (n = 22). Sixteen of these students reached the correct result. Although making an ordered list is preferred by many students (n = 13), only three of them could reach the correct result.

A high rate of Yes responses was showed Self-Monitoring Questionnaire statements referring to metacognitive strategies. For the four statements that might prompt initial recognition of the metacognitive “red flags” are investigated deeply. Response rates for these statements are shown in Table 3.

Table 3. Questionnaire responses to metacognitive “red flag” statements

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| --- | --- | --- | --- | --- | --- | --- | --- |
| “Red Flag” | Questionnaire Statement | Percentage of Students | | | | | |
| Responding Yes | | Responding No | | Responding Unsure | |
| **N** | **%** | **N** | **%** | **N** | **%** |
| **Error detection** | I checked my work step by step as I went through the problem. | 26 | **70** | 9 | 24 | 2 | 5 |
| **Lack of progress** | I asked myself whether I was getting any closer to a solution. | 15 | **40** | 15 | 40 | 7 | 19 |
| **Anomalous result** | I checked my calculations to make sure they were correct. | 28 | **76** | 7 | 19 | 2 | 5 |
| I asked myself whether my answer made sense. | 24 | **65** | 9 | 24 | 4 | 11 |

As seen in Table 2, the items with the most "yes" answers are error detection and anomalous result sentences. In the Lack of progress item, the “yes” answer is less than half of the students (40%). This situation reveals that although the students are sufficient to detect mistakes and anomalous results, they are lacking in making progress.

It is very difficult to speak of successful self-regulation by simply revealing that it accepts progress. The places where students have difficulty changing strategies in their written work or their mistakes are proof of self-control.

Metacognition can be described early, based on four items labeled "Red Flag". However, it would be unreasonable to reach a conclusion about the metacognitive activities of the students by considering the questionnaire answers only. Therefore semi-structured interviews were needed.

Semi-structured interviews were conducted with six students. Some of their answers were shown:

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| *Students were asked what they understand when they read the problem and whether they can express the problem in their own words*;  It was seen that all the students interviewed were able to express the problem with their own sentences. They also stated like;   * “I've read several times, I wanted to write the stones one by one“ * “I’ve realized that the dominoes were between 0 and 6, and there weren’t the opposite” |

All of the students stated that they understood the problem when they read it, and it was seen that they could express the problem with their own sentences. This situation shows that the students who cannot reach the correct solution of the problem do not have any problems in understanding the problem.

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| *Students were asked whether they have ever encountered such a problem;*  Most of the students stated that they had never encountered this kind of problem before. Only one student said;   * “I met problems like that before, but they were easier. Such as probability question about flipping the coin” |

None of the students stated that they have never encountered such a problem before. Therefore, it is possible to say that the domino question problem is a new and unusual situation for all students. This provides a good opportunity to examine students' metacognitive skills.

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| *Students were asked what was given in problem and what was wanted you to find;*  Students’ answers showed that almost all students understood what was given and what was wanted them in problem. They were able to express correctly in their own words. |

Most of the students were able to explain what was given and wanted in the problem in their own words. This situation is a positive indicator for students' metacognitive skills.

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| *Students were asked what pathway they chose to solve the problem, they stated like;*   * “I wrote the numbers on the stones one by one” * “I tried to draw it visually” * “I separated one stone from each stone and eliminated the others. I counted the remaining dominoes and found the result” * “I paid attention to whether the numbers were the same” |

While the students expressed the way they used to solve the problem, they explained what they did during the solution. Even the students who could not reach the correct conclusion made very reasonable explanations. This shows that although the students' metacognitive skills are high, they have mathematical deficiencies.

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| *When asked if this pathway moved it closer to the solution, whether the path they chose worked or not;*  The students stated that the path they chose worked and therefore they did not have to choose another way. |

Whether or not the students reach the correct result, they did not need to try a different way, thinking that the path they chose was correct. This situation is a negative indicator for metacognitive skills.

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| *Students were asked how the solution checked whether it was correct or not, they stated like;*   * I find them counting them all over again. * I look at the upper and lower numbers of domino pieces. It's correct if there is no other piece of the same stone   It was seen that the students did not need to check whether there was a missing stone or not, and in general they preferred to write all the pairs and eliminate the same ones. |

Since the students believed that their solution was correct, they did not doubt the result. While some of them checked the numbers they obtained cursory, some of them only checked the first and last numbers. Not even thinking about the possibility of another solution to the problem is a negative indicator for metacognitive skills.

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| *According to them, students were asked whether the problem could be solved in any other way;*  Most of the students stated that this problem could not be solved in any other way. This showed that students tended to believe that problems had a single solution. |

As it is similar to the previous questions, the students think that the problem is no other than their own solution. Thinking that the problem can only be the only solution shows that the students are insufficient in terms of metacognitive skills.

*Examples from students’answers*

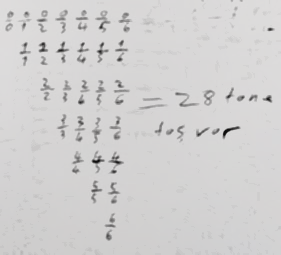
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Figure 1. First student’s answer

As seen above, the student solved the problem by using the method of systematic list making and reached the correct result (Figure 1).

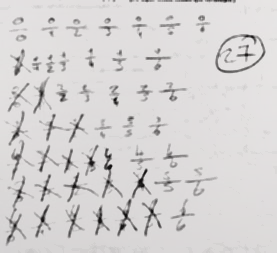
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Figure 2. Second student’s answer

The student tried to solve the problem by using the ordered listing method. He/she listed all the situations and eliminated the inappropriate situations. However, he/she made a mistake while listing the situations and reached the wrong conclusion (Figure 2).

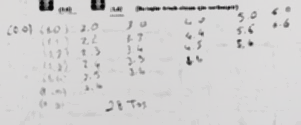
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Figure 3.Third student’s answer

As seen above, this student created stone pairs by using the method of systematic list making. Thus, he reached the correct result (Figure 3).

**Discussion and Conclusion**

The aim of the study was to examine students’ metacognitive skills in the problem solving process, the differences between students who use and do not use metacognitive skills and its effects on the success of problem solving. Various studies have shown that individuals with a high level of metacognitive awareness have high skills of understanding what they read and using what they understand in problem solving (Balci, 2007; Dogan, 2013; Johnson, 2002; Hassan, 2003; Gelen, 2003; Gumus, 1997; Kiremitci, 2011; Mayer, 1998; Ozbilgin, 1993; Vadhan & Stander, 1994; Van der Walt & Maree, 2007; Aydemi̇r & Kubanc, 2014). The written responses of the students to the questions, questionnaire responses and semi-structured interview records revealed that using metacognitive skills was associated with problem solving and positively affected problem solving results. These results are consistent with many studies in the literature (Balci, 2007; Kaplan, Duran, & Bas, 2016; Arsuk & Memnun, 2020).

Students who could use metacognitive skills gave correct answers to the problems and used various solution strategies by being aware of the requirements and goals of the problem. They checked the correctness of the solution. It is important for students to follow and control their own processes consistently while solving problems in terms of metacognitive skills (De Bruin & Van Gog, 2012; Koriat, 2012; Pieschl, Stahl, Murray, & Bromme, 2012, Asik, 2015). Many studies in the literature have revealed that mathematics is related to reading underlying the individual's monitoring of the process and its controlled progress (Fite, 2002; Asik, 2015). Therefore, in this study, students' inability to follow the process and control their strategies may be based on their shallow reading and not trying to understand the question in depth. As expressed by Cakiroglu (2007), being aware of the problem is only possible when students have fully understood the problem.

Students who could not use metacognitive skills gave wrong answers to problems and had difficulties in understanding the problems, determining solution strategies, recognizing and correcting their mistakes. Most of the students were not successful in solving non-routine problems. As Schoenfeld (2006) stated, problem solving has a complex and relational structure that requires considering different variables. It is not correct to talk about certain solutions and strategies that will solve all problems. In order to make sense of and solve the complex structure in question, the individual should think in detail on the problem and participate in the context of metacognition (Lucangeli & Cornoldi, 1997; Asik, 2015). In this study, it was observed that their answers to the questionnaire and their solution suggestions were not clear and understandable enough. Kramarski, Mavarech, and Arami (2002), in their study with seventh-grade students to examine the effects of problem-solving activities on metacognitive skills, concluded that only a small part of the students who solved the problems controlled the answers they found. Vergnaud (1991), Brun et al. (1994) and Sandır, Ubuz, and Argun (2002) stated that students had difficulties because they could not yet give full meaning to the concepts that form the basis of the problems.

It has been determined that the students do not know what and why they are doing while solving the problems. According to Gourgey (1998), one of the characteristics of individuals with advanced metacognitive control is that they can transfer their previous experiences and knowledge to subsequent processes while solving problems. In this study, it is seen that the students lack this skill. Aksu (1984) (as cited in Sevgi & Caglikose, 2019) revealed that after the problem was solved and the result was found, the stage of deciding whether this result is meaningful or not was ignored by many students.

Generally, the students focused on getting an answer without considering its accuracy. Since they could not establish a logical connection between their solutions and problems, they could not detect where they made mistakes, and therefore they could not overcome their mistakes. According to Sengul and Yildiz (2013), one of the reason why students have difficulties in problem solving is that the student turns directly to the process without answering the why and why questions without questioning the problem.

Gelen (2003) states that students generally do not know how to use a strategy in solving a problem or why they choose that strategy after deciding on a strategy. On the other hand, if the student knows but cannot explain the learning strategy, this shows that the student's metacognitive skills are deficient, and the importance of teacher guidance in developing the student's metacognitive skills emerges (Desoete, 2007; Gelen, 2003; Ho, 2004; Mayer, 1998; Aydemi̇r & Kubanc, 2014). Supporting these studies, it can be said that these students also determined problem solving strategies according to unnecessary details in the problem. Ozsoy (2007) revealed that metacognitive skills have a significant effect on students' problem solving success, and students with high metacognitive levels are more successful. Young (2010), in his study examining the relationship between metacognition and academic achievement, concluded that there is a strong relationship between students' mathematics achievement and metacognition in the process of problem solving. Sevgi and Caglikose (2019) showed in their study with sixth grade students that students who use metacognitive skills are more successful in problem solving processes.

**Recommendations**

Based on the results of the study, the following recommendations have been developed:

* Taking into account the results of the study, various measures can be taken to develop metacognitive skills in secondary school students.
* Activities in the teaching process can be planned in a way that requires students to use metacognitive strategies, and students can be encouraged to use these skills.
* The teaching process can be designed in such a way that middle school students can encounter non-routine problems more.
* Long-term studies can be conducted to examine the development of students' metacognitive skills.

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