**VAN HIELE GEOMETRY TEST – SECTION A**

1. Which of these are squares?

1. K only
2. L only
3. M only
4. L and M only
5. All are squares K L M
6. Which of these are triangles?

U V W X

* 1. None of these are triangles.
  2. V only
  3. W only
  4. W and X only
  5. V and W only

1. Which of these are rectangles?

S T U

* 1. S only
  2. T only
  3. S and T only
  4. S and U only
  5. All are rectangles.

1. Which of these are squares?

F G H I

* 1. G and I only.
  2. G only
  3. F and G only
  4. All are squares
  5. None of these are squares.

1. Which of these are parallelograms?

J M L

* 1. J only
  2. L only
  3. J and M only
  4. All are parallelograms.
  5. None of these are parallelograms.

1. PQRS is a square.

Which relationship is true in all squares?

S

R

P

Q

* 1. PR and RS have the same length.
  2. QS and PR are perpendicular.
  3. PS and QR are perpendicular.
  4. PS and QS have the same length.
  5. Angle Q is larger than angle R.

1. In the rectangle GHJK, GJ and HK are the diagonals.

G H

K

J

Which of (A)-(D) is not true in every rectangle?

* 1. There are four right angles.
  2. There are four sides.
  3. The diagonals have the same length.
  4. The opposite sides have the same length.
  5. All of (A)-(D) are true in every rectangle.

1. A rhombus is a 4-sided figure with all sides of the same length.

Here are three examples.

Which of (A)-(D) is not true in every rhombus?

* 1. The two diagonals have the same length.
  2. Each diagonal bisects two angles of the rhombus.
  3. The two diagonals are perpendicular.
  4. The opposite angles have the same measure.
  5. All of (A)-(D) are true in every rhombus.

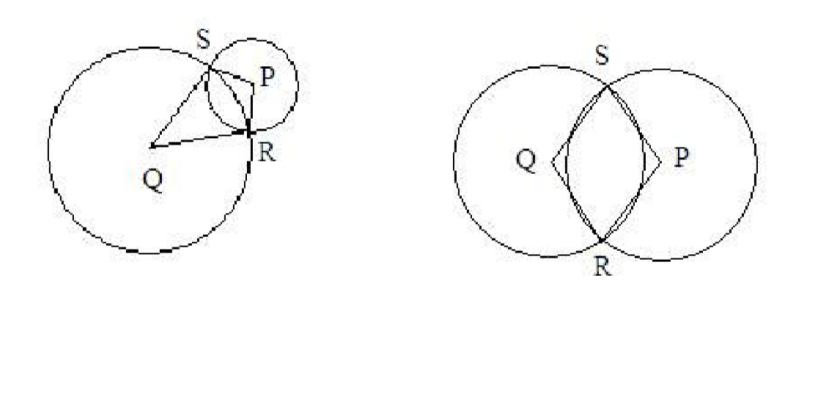
1. An isosceles triangle is a triangle with two sides of equal length.

Here are three examples.

Which of (A)-(D) is true in every isosceles triangle?

* 1. The three sides must have the same length.
  2. One side must have twice the length of another side.
  3. There must be at least two angles with the same measure.
  4. The three angles must have the same measure.
  5. None of (A)-(D) is true in every isosceles triangle.

1. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PRQS. Here are two examples.



Which of (A)-(D) is not always true?

* 1. PRQS will have two pairs of sides of equal length.
  2. PRQS will have at least two angles of equal measure.
  3. The lines PQ and RS will be perpendicular.
  4. Angles P and Q will have the same measure.
  5. All of (A)-(D) are true.

1. Here are two statements.

Statement 1: Figure F is a rectangle.

Statement 2: Figure F is a triangle.

Which is correct?

* 1. If 1 is true, then 2 is true.
  2. If 1 is false, then 2 is true.
  3. 1 and 2 cannot both be true.
  4. 1 and 2 cannot both be false.
  5. None of (A)-(D) is correct.

1. Here are two statements.

Statement S: ∆ABC has three sides of the same length

Statement T: In ∆ABC, ∠B and ∠C have the same measure.

Which is correct?

* 1. Statement S and T cannot both be true.
  2. If S is true, then T is true.
  3. If T is true, then S is true.
  4. If S is false, then T is false.
  5. None of (A)-(D) is correct.

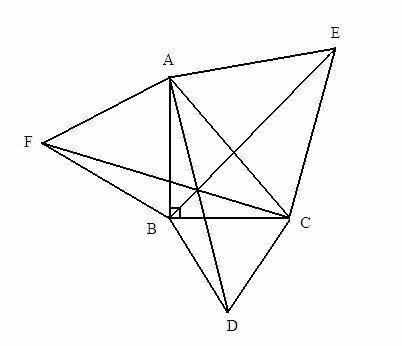
1. Which of these can be called rectangles?

P Q R

* 1. All can.
  2. Q only
  3. R only
  4. P and Q only
  5. Q and R only

1. Which is true?
   1. All properties of rectangles are properties of all squares.
   2. All properties of squares are properties of rectangles.
   3. All properties of rectangles are properties of all parallelograms.
   4. All properties of squares are properties of all parallelograms.
   5. None of (A)-(D) is true.

1. What do all rectangles have that some parallelograms do not have?
   1. Opposite sides equal
   2. Diagonals equal
   3. Opposite sides parallel
   4. Opposite angles equal
   5. None of (A)-(D)
2. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information, one can prove that AD, BE, and CF have a point in common. What would this proof tell you?

* 1. Only in this triangle drawn can we be sure that AD, BE and CF have a point in common.
  2. In some but not all right triangles, AD, BE and CF have a point in common.
  3. In any right triangle, AD, BE and CF have a point in common.
  4. In any triangle, AD, BE and CF have a point in common.
  5. In any equilateral triangle, AD, BE and CF have a point in common.

1. Here are three properties of a figure.

Property D: It has diagonals of equal length.

Property S: It is a square.

Property R: It is a rectangle.

Which is true?

* 1. D implies S which implies R.
  2. D implies R which implies S.
  3. S implies R which implies D.
  4. R implies D which implies S.
  5. R implies S which implies D.

1. Here are two statements.

I: If a figure is a rectangle, then its diagonals bisect each other.

II: If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

* 1. To prove I is true, it is enough to prove that II is true.
  2. To prove II is true, it is enough to prove that I is true.
  3. To prove II is true, it is enough to find one rectangle whose diagonal bisect each other.
  4. To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.
  5. None of (A)-(D) is correct.

1. In geometry:
   1. Every term can be defined and every true statement can be proved true.
   2. Every term can be defined but it is necessary to assume that certain statements are true.
   3. Some terms must be left undefined but every true statement can be proved true.
   4. Some terms must be left undefined and it is necessary to have some statements which are assumed true.
   5. None of (A)-(D) is correct.
2. Examine these three sentences.
   1. Two lines perpendicular to the same line are parallel.
   2. A line that is perpendicular to one of two parallel lines is perpendicular to the other
   3. If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?

p

* 1. (1) only
  2. (2) only

m

n

* 1. (3) only
  2. Either (1) or (2)
  3. Either (2) or (3)

1. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R and S, and the lines are {P,Q}, {P,R}, {P,S}, {Q,R}, {Q,S}, and {R,S}.

P

Q

R

S

Here are how the words “intersect” and “parallel” are used in F-geometry. The lines {P,Q} and {P,R} intersect at P because {P,Q} and {P,R} have P in common.

The lines {P,Q} and {R,S} are parallel because they have no points in common.

From this information, which is correct?

* 1. {P,R} and {Q,S} intersect.
  2. {P,R} and {Q,S} are parallel.
  3. {Q,R} and {R,S} are parallel.
  4. {P,S} and {Q,R} intersect.
  5. None of (A)-(D) is correct.

1. To trisect an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel proved that, in general, it is impossible to trisect angles using only a compass and an unmarked ruler. From his proof, what can you conclude?
   1. In general, it is impossible to bisect angles using only a compass and an unmarked ruler.
   2. In general, it is impossible to trisect angles using only a compass and a marked ruler.
   3. In general, it is impossible to trisect angles using any drawing instruments.
   4. It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.
   5. No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

1. There is a geometry invented by a mathematician J in which the following is true:

The sum of the measures of the angles of a triangle is less than 180°.

Which is correct?

* 1. J made a mistake in measuring the angles of the triangle.
  2. J made a mistake in logical reasoning.
  3. J has a wrong idea of what is meant by “true”.
  4. J started with different assumptions than those in the usual geometry.
  5. None of (A)-(D) is correct.

1. The geometry books define the word rectangle in different ways. Which is true?
   1. One of the books has an error.
   2. One of the definitions is wrong. There cannot be two different definitions for rectangle.
   3. The rectangles in one of the books must have different properties from those in the other book.
   4. The rectangles in one of the books must have the same properties as those in the other book.
   5. The properties of rectangles in the two books might be different.

1. Suppose you have proved statements I and II.

I: If p, then q.

II: If s, then not q.

Which statement follows from statements I and II?

* 1. If p, then s.
  2. If not p, then not q.
  3. If p or q, then s.
  4. If s, then not p.
  5. If not s, then p.

**SECTION B**

**Answer all three (3) questions in this section by clearly showing working**

1. In the diagram below,  and = ∠*SRV* 27° and ∠*RTU* 55° . Find the value of *x*. You are to show your workings, giving a reason for each step.

°

°

1. Kwabena stores his toys in a container that has a cylindrical body and a conical lid, as shown below.

  
Find;

1. the surface area of the base of the container
2. the surface area of the cylindrical body (excluding the base and top)
3. the surface area of the conical lid
4. If Kwabena wants to cover the entire exterior portion of the container with paper. How much paper, in square feet, would he need?
5. Given that and bisects , show, giving reasons that and have equal corresponding angles and equal corresponding sides. What then can you say about the two triangles?

**MARKING SCHEME**

**Marking Scheme for the VHGT – Section A**

1. B

2. D

3. C

4. B

5. D

6. B

7. E

8. A

9. C

10. D

11. C

12. B

13. A

14. A

15. B

16. C

17. C

18. D

19. D

20. A

21. B

22. E

23. D

24. E

25. D

**SECTION B**

**QUESTION 1**

**METHOD 1**

∠ [Alternate angles, ]

∠ [Base angles of isosceles ]

∠ [Sum of angles in a triangle]................(1)

But ∠

Thus, from (1),

⇒

⇒

**METHOD 2 (ALTERNATIVE METHOD)**

∠ [Alternate angles, ]

∠ [Exterior angles of ]

∠ [Base angles of isosceles ]

∠ [Sum of angles in a triangle]

⇒

⇒

⇒

**QUESTION 2**

1. Surface area of the base of the container square of the radius of the base ()

=

1. Surface area of the cylindrical body = radius of the base () height of the cylinder ()

=

=

1. Surface area of the conical lid radius of the top (r) slant height of conical lid (l)

1. Amount of paper in square feet needed to cover the entire exterior portion of the container

**QUESTION 3**

|| = || [Given]

|| = || [Line AD bisects line BC]

|| = || [|AD| is common to both triangles]

∠ [Base angles of isosceles triangle]

∠ [Line bisects ∠]

∠ [Line is perpendicular to line ]

Therefore, the two triangles, and are congruent.

**LESSON PLANS**

**LESSON PLAN 1**

**Subject: Geometry**

**Topic: Properties of Angles Formed by Two Parallel Lines and Their Transversal**

**Duration of lesson: 120 minutes**

**Target group: CE Level 200**

**Tutor: Armah Robert Benjamin**

* **Relevant Previous Knowledge:**

PTs are familiar with concept of lines and how to measure angles. PTs have also learnt parallel lines during the traditional lessons. PTs can solve equations or expressions.

* **Teaching and Learning Materials:**

Mathematical sets, cut-outs of cardboards, pair of scissors, and masking tape.

* **Learning Objectives:**

By the end of the lesson the PT should be able to:

* Identify some basic properties of parallel lines.
* Discover the relationships between the angles formed by two parallel lines cut by a transversal.

**Phase 1: Information/inquiry**

Researcher reviews PTs’ previous knowledge on angles and their properties. The researcher then asks PTs questions on the definition of parallel lines. The researcher further holds a conversation with the PTs concerning parallel lines and their properties, in well-known language symbols making the context clear. Researcher ensures that PTs have an understanding of parallel lines and can construct them.

**Phase 2: Guided/Directed Orientation**

**Activity: 1**

PTs in each group were given a rectangular cardboard cut-out, secured by taping them to a table and were guided to draw two diagrams of two parallel lines and a transversal forming eight angles each as can be seen below;

Names of the various pairs of angles were discussed with the PTs; the researcher gave PTs the standard definition of vertical opposite, corresponding, alternate and co-interior angles and pointed them out. For example, angles and , and , and , and and are vertical opposite, corresponding angles, co-interior angles and alternate angles respectively.

**Activity 2**

PTs were guided to trace and cut out the various angles, place some on each other to establish, for example, the congruency of vertical opposite, alternate and corresponding angles. They also placed some side by side for their vertices to meet to establish the relationship between co-interior angles as well as adjacent angles. These were done as follows;

* PTs trace and cut out the alternate angles and found in diagram 1. Researcher then asks PTs: “Do these angles form a particular shape?” Response should be they form a Z - shape.
* Repeat the above using diagram 2. Have PTs cut out angles and . Ask: “Do these form a particular shape?” Should resemble an N-shape.
* Next have PTs trace and cut out the co-interior angles and in diagram 1. Ask: “What type of shape do they form?” Should resemble a C shape. Repeat with angles and . Ask: “Do they form a similar shape?” These two form a backwards C shape.
* Have the PTs trace and cut out the corresponding angles and in diagram 1. Ask: “Do these angles form a particular shape?” Should resemble an F shape. Repeat with angles and . Ask: “Do these angles form the same type of shape?” They form an F shape that is turned backwards. Repeat with other examples of corresponding angles from diagram 1 and 2 and they all resemble an F shape that has been flipped or turned a certain way.

**Phase 3: Explicitation**

In this phase, the PTs were asked to describe what they have learned about the topic using their own language. PTs were also asked to come out with the discoveries they made from the hands-activities.

**Phase 4: Free Orientation**

Once all relationships were covered, the researcher incorporated measurements of the angles emphasizing which angles are congruent and which are supplementary. PTs at this point were asked to discover, independently, the values of the unknown angles from the diagram 3;

45°

55°

c

c

Researcher also asked PTs to discuss the relationships of angles in the diagram below;

**Phase 5: Integration**

Lesson overview

Diagram 5

Alternate Angles Shapes

Co-interior Angles Shapes

Corresponding Angles Shapes

**LESSON PLAN 2**

**Subject: Geometry**

**Topic: Properties of Quadrilaterals**

**Duration of lesson: 120 minutes**

**Target group: CE Level 200**

**Tutor: Armah Robert Benjamin**

* **Relevant Previous Knowledge:**

PTs are familiar with concept of triangles. PTs have also learnt Quadrilaterals in their traditional lessons.

* **Teaching and Learning Materials:**

Mathematical sets, papers, pair of scissors, computer and projector (for tutor) for displaying diagrams.

* **Learning Objectives:**

By the end of the lesson the PT should be able to:

* Discover properties of Quadrilaterals (squares, rectangles, rhombuses and parallelograms).
* Use relationships among sides and angles of Quadrilaterals (squares, rectangles, rhombuses and parallelograms).
* Use relationships among diagonals of Quadrilaterals.

**Phase 1: Information/inquiry**

The researcher reviewed PTs’ previous knowledge on types of triangles and their properties. The researcher further held a conversation with the PTs concerning triangles and their properties, in well-known language symbols making the context clear. This was to begin the activities with a sort of informal work that gives PTs an idea of the topic.

**Phase 2: Guided/Directed Orientation**

**Activities:**

PTs in their various groups were given a work sheet showing diagrams of the various quadrilaterals (square, rectangle, parallelogram and rhombus) and guided through activities as follows;

Draw a diagonal to the parallelogram by connecting to form and as shown below;

Guide PTs to use properties of parallel lines to indicate all angles on the diagram. With and, guide PTs to show that and , and as follows;

[Alternate angles, ]

[Alternate angles, ]

is a common side

and

Thus, it has been shown that, and , therefore

, Also

Now guide PTs to summarize the properties of a parallelogram;

* Both pairs of opposite sides are parallel.
* Both pairs of opposite sides are equal in length.
* Both pairs of opposite angles are equal.
* Both diagonals bisect each other.

In a similar way, guide PTs in their various groups to discover the special properties of squares, rectangles and rhombuses.

**Phase 3: Explicitation**

In this phase, the PTs were asked to describe what they have learned about the topic using their own language. PTs were asked to come out with the discoveries they made from the hands-activities (that is the properties of special Quadrilaterals).

**Phase 4: Free Orientation**

Now, PTs independently solve the example below;

2

2

1

2

2

1

1

1

2

4

3

 is a rhombus. Show that:

1. the diagonals bisect each other perpendicularly;
2. the diagonals bisect the interior angles.

**Phase 5: Integration**

Lesson overview.

**LESSON PLAN 3**

**Subject: Geometry**

**Topic: Relationships between Properties of Quadrilaterals**

**Duration of lesson: 120 minutes**

**Target group: CE Level 200**

**Tutor: Armah Robert Benjamin**

* **Relevant Previous Knowledge:**

PTs are familiar with concept of triangles. PTs can discover the properties of quadrilaterals.

* **Teaching and Learning Materials:**

Mathematical sets, papers, pair of scissors, computer and projector (for tutor) for displaying diagrams.

* **Learning Objectives:**

By the end of the lesson the PT should be able to:

* Define and classify special types of Quadrilaterals.

**Phase 1: Information/inquiry**

Tutor reviews PTs’ previous knowledge on properties of Quadrilaterals (squares, rectangles, rhombuses and parallelograms). The tutor further holds a conversation with the PTs concerning Quadrilaterals (squares, rectangles, rhombuses and parallelograms) and their properties, in well-known language symbols making the context clear.

**Phase 2: Guided/Directed Orientation**

**Activities**

PTs in their various groups are guided to examine the special Quadrilaterals very carefully and use a Venn diagram to show the relationships that exist among them. The researcher guided PTs to generally establish the following using the appropriate terminologies as follows:

* Parallelograms, rhombuses, rectangles and squares all have two pairs of parallel sides, so parallelograms are the largest set.
* Rhombuses have four congruent sides, so they are equilateral and rectangles have four congruent angles, so they are equiangular.
* Squares are both equilateral and equiangular, so they have the characteristics of rhombuses and rectangles and hence belong to both groups.

Relationship among Special Parallelograms

**Phase 3: Explicitation**

In this phase, the PTs were asked to express in their own words what they have discovered in the previous phase. PTs were asked to come out with the discoveries they made from the hands-activities.

**Phase 4: Free Orientation**

Now, PTs are asked to answer the following questions independently;

1. Which of these are true and which are false? Explain why in each case.
2. All squares are rectangles
3. All squares are rhombuses
4. All squares are parallelograms
5. No rectangles are rhombuses
6. No rectangles are parallelograms
7. No rectangles are squares
8. Some parallelograms are rectangles.
9. Draw a shape using these conditions. If the task is impossible, say why.
10. A rectangle that is *not* a rhombus.
11. A rectangle that is a rhombus.
12. Draw Venn diagrams to show the relationships between the following sets (in some cases you may need to include an extra set):
13. Squares and rectangles.
14. Parallelograms, rhombuses, and squares.
15. All quadrilaterals, rhombuses and parallelograms.

**Phase 5: Integration**

Lesson overview.