



ISSN: 2148-9955

International Journal of Research in Education and Science (IJRES)

www.ijres.net

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To cite this article:

Hong, D., Choi, K., Hwang, J., & Runnalls, C. (2017). Integral student's experiences: Measuring instructional quality and instructors' challenges in Calculus 1 lessons. *International Journal of Research in Education and Science (IJRES)*, 3(2), 424-437. DOI: 10.21890/ijres.327901

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Article Info

Article History

Received:
09 February 2017

Accepted:
01 April 2017

Keywords

Calculus
Cognitive demand

Abstract

In this study, we examined 10 integral lessons to understand students' opportunities to learn cognitively challenging tasks and maintain cognitive demand during integral lessons. Our findings reveal issues with implemented tasks as well as the way these tasks were presented to students. We also examined mathematicians' reasons behind their instructional practices, which show two common reasons of under-prepared students and time constraints. However, two mathematicians' in this study showed quite different instructional practices, which shows individualized faculty development might be critical in changing teaching and learning of calculus.

Introduction

Calculus 1 is a core and beginning course for students heading into disciplines in science, technology, engineering, and mathematics (STEM), as well as in the social sciences and business. Approximately 300,000 students were enrolled in mainstream Calculus 1 in the 2010 fall semester (Bressoud, 2015). There is increasing attention to learning mathematics in Calculus 1, with a large body of research documenting students' difficulties (e.g., Tall & Vinner, 1981).

In 2010, the Mathematical Association of America initiated a large-scale study known as *Characteristics of Successful Programmes of College Calculus* [CSPCC] to measure characteristics of successful calculus programs (Bressoud et al. 2013). The results revealed that calculus students can lose confidence, enjoyment, and motivation to continue studying mathematics after completion of their first-semester undergraduate calculus courses (Bressoud et al. 2013). Yet, when students experience both "good and ambitious teaching", they are more likely to continue on to Calculus 2 after completion of Calculus 1 and pursue STEM related majors (Bressoud et al., 2014). Good teaching includes availability to answer student questions and respond to students, needs, keeping reasonable pacing of the lecture to ensure all students are on the same page, and creating a positive atmosphere in which the instructors encourage students to ask questions. Ambitious teaching includes having challenging and unfamiliar problems, asking students to explain their thinking and less rely on lecture as the main mode of teaching. The fact that these findings are from institutions known for successful calculus programs shows the importance of examining the day-to-day instructional practices of calculus instructors and students in calculus classes.

Among the many topics in calculus, integral is often introduced at the end of the first calculus class, where students have opportunities to use their prior knowledge about limit and derivative to solve various integral tasks. Integral also serves as the basis for many real-world applications in science and engineering and is an essential topic in understanding differential equations and other advanced mathematics. Thus, it is important to examine students' learning experiences. In this study, we paid particular attention to two important factors in students' learning opportunities - implemented tasks and how those tasks are implemented.

In addition to what calculus instructors do in their classes, there are issues that influence and shape instructional decisions (Johnson, Caughman, Fredericks & Gibson, 2013; Weber, 2004). Accordingly, we interviewed mathematicians to understand what influenced their instructional decisions in teaching Calculus 1. Interviewing mathematicians can give insight into what matters to mathematicians when they teach Calculus 1. This study has two goals: 1) To examine current instructional practices in Calculus 1, focusing on integral lessons and cognitive demand, 2) To understand what factors matter to calculus instructors when they make instructional decisions. Here are the research questions that we attempted to answer.

1. How do mathematicians implement integral tasks during instruction?
2. How do the mathematicians' implementation of tasks influence cognitive demand during instruction?
3. What issues influenced mathematicians' instructional decisions?

Related Literature

Teaching Mathematics at the Undergraduate Level

It is well-known that lecture is the main mode of teaching in undergraduate mathematics; however, mathematics education researchers believe that greater learning occurs when students are actively involved in their learning process (Boaler, 2000; Nardi, 2008; Yoon et al., 2011). When students are challenged to provide mathematical reasoning and explanations, they will more likely to have conceptual understanding. Students' active involvement in their learning process is also important in undergraduate mathematics teaching (Blanton, Stylianou, & David, 2003; Rasmussen et al., 2006), illustrating the importance of students' experiences in their undergraduate mathematics classes.

Although students' active involvement is important, it is very challenging to make changes from traditional lecture to more student involvement in undergraduate mathematics teaching (Kensington-Miller et al., 2013; Nardi, 2008). There are common issues and obstacles in reforming undergraduate mathematics teaching (Johnson et al., 2013; McDuffie & Graeber, 2003; Speer & Wagner, 2009; Walczyk, Ramsey, & Zha, 2007). Some common obstacles that prevent mathematicians and students from having more student-centered classes are time constraints, meaning covering a certain amount of materials in given time (McDuffie & Graeber, 2003), mathematicians being evaluated for their research rather than teaching (Walczyk et al., 2007), and lack of training and support to implement such teaching (Walczyk et al., 2007). While some mathematicians acknowledge the benefits of more student involvement and are willing to implement such teaching, training and support are not often provided (Walczyk et al., 2007). For example, Speer and Wagner (2009) illustrated one mathematician's struggle in providing analytic scaffolding and choosing students' responses to have productive class discussions. These are all important issues that can prevent mathematicians and students from engaging in productive mathematical discussions. Therefore, in this study, we gathered information about the mathematicians' instructional practices as well as issues and obstacles that impact their instructional decisions in teaching Calculus 1.

Mathematical Tasks Framework

In understanding instructional practices, mathematical tasks are important part because what students do in the classroom can be defined by the tasks teachers assign, and those tasks can determine how students understand the meanings of various concepts (Doyle, 1988). In this study, *mathematical tasks* are defined as a set of problems or a single complex problem that focuses students' attention on a particular mathematical idea (Stein et al., 1996). We defined implemented tasks as ones that calculus instructors and students actually worked on. Thus, instructional tasks that are implemented draw students' attention to particular concepts, and students have opportunities to be exposed to those concepts embedded in tasks they complete. While in the classroom, students spend most of their time working on various tasks. Therefore, providing worthwhile tasks is a critical part of class practices and has significant impact on students' learning and the type of knowledge that they attain (Boston, 2012; Franke et al., 2007; Munter, 2014). Worthwhile tasks make mathematics intriguing; offer students opportunities to use their prior knowledge; allow them to justify, conjecture, and interpret mathematical ideas; and engage in higher level thinking (Franke et al., 2007; Hiebert et al., 1997). Mathematical tasks can either limit or broaden students' thinking on mathematics that they are engaged in (Henningsen & Stein, 1997). Hence, it is important to examine mathematical tasks in various settings to understand instructional practices (Boston, 2012; Hiebert et al., 1997; Munter, 2014; Son & Senk, 2010; Hong & Choi, 2014; Hsu & Silver, 2014; White & Mesa, 2014; Wilhelm, 2014).

Stein and her colleagues conceptualized the Mathematical Tasks Framework (MTF), which shows the process of choosing and implementing mathematical tasks from their appearance in mathematics textbooks or other resources to their possibly revised form as teachers provide the tasks to their students and then finally to the tasks that are implemented by the teacher and students in the classroom. Each stage of the MTF can influence what students have an opportunity to learn. The MTF includes a four-tier rubric (Task Analysis Guide) to analyze the cognitive demand of mathematical tasks. With high-level cognitively demanding mathematical tasks, students engage in *doing mathematics* or make connections between concepts and procedures (Stein et al.,

1996). On the other hand, with low-level cognitively demanding mathematical tasks, students engage in simple *memorization* and *procedures without making connection*. For example, a simple integral task, $\int_2^5 x^3 + 1 dx$ is an example of a low-level task because students can simply use procedure to compute the integral. On the other hand, “Draw a region on the coordinate plane so that, when the region is rotated about a line, the resulting three-dimensional solid ‘looks like’ a donut,” is an example of a high-level task because there are no procedures or algorithms that students can follow mindlessly. Students need to figure out mentally how the shape will look if it is rotated about a line.

A limited number of studies have examined calculus tasks. White and Mesa (2014) analyzed 4,953 tasks that instructors gave to students. They underlined the potential cognitive demand of a task which indicates “the hypothetical operations used to produce the answer” (p. 676) in their definition of tasks. When they categorized the tasks into three groups (Simple Procedures, Complex Procedures, and Rich Tasks) across the instructors, 53% of tasks in all types of coursework were in Simple Procedures while 29% were Rich Tasks. In addition, White and Mesa (2014) suggested that instructors’ enactment of learning goals could vary even though they used a common textbook.

In particular, the findings in White and Mesa (2014) showed that more tasks with high cognitive demand appeared when fewer resources were available for a task, or tasks (namely exams) had greater weight in a course grade (49%). In contrast, Tallman and Carlson (2012) showed that 14.8% of test items in 150 Calculus I final exams required students to demonstrate their understanding. Using the task categories of White and Mesa (2014), at most, 15% of tasks in the final exams were classified as Rich Tasks. These findings reveal the necessity for more empirical evidence about tasks from low to high cognitive demand. However, these studies did not include how these calculus tasks were implemented in day-to-day lessons, which is one of the main areas we investigate in this study.

Importance of Maintaining Cognitive Demand

While implementing mathematical tasks that require high-level cognitive demand is important, these tasks do not guarantee the maintenance of cognitive demand during mathematics lessons (Boston & Smith, 2009; Henningsen & Stein, 1997). There are several factors that can alter cognitive level during the course of mathematics lessons. Use of student’s prior knowledge, scaffolding, appropriate wait time, sustained pressure for explanations, and student self-monitoring can help maintain cognitive demand (Boston & Smith, 2009; Henningsen & Stein, 1997). On the other hand, inappropriateness of the tasks, too much or too little time, a shift in focus to the correct answer, and routinized tasks by taking over the class discussions can decrease cognitive level (Boston & Smith, 2009; Henningsen & Stein, 1997). Therefore, simply selecting mathematical tasks with high-level cognitive demand may not be enough for students to be engaged in cognitively challenging mathematical activities. A task with high-level cognitive demand can be selected and set up by teachers, but during the implementation, it can be altered in such a way that students’ thinking level is only procedural with no conceptual connections. However, greater student learning gains occur in classrooms where high-level cognitively demanding mathematical tasks are consistently maintained throughout instruction (Hiebert & Wearne, 1993; Stigler & Hiebert, 2004). To assist teachers and students to sustain cognitive demand, it is important to examine instructional practices - what teachers and students do while mathematical tasks are being implemented. Many studies examined cognitive demands of mathematical tasks in elementary and secondary mathematics (Henningsen & Stein, 1997; Hong & Choi, 2014; Hsu & Silver, 2014; Wilhelm, 2014), but such investigation has not been often done in undergraduate mathematics.

Methods

Setting and Data

A Midwestern research university in the United States was the setting for this study. Two calculus instructors, Dr. A and Dr. B, have a PhD in mathematics with a specialization in topology and differential geometry, respectively. They have taught Calculus I several times and each class had 25 registered students. Each of Dr. A’s classes was 50 minutes long and Dr. B’s classes were 55 minutes long. Each mathematician taught five class sessions on integral concepts.

Sources of data came from class video and audio recordings. All calculus classes taught by the two mathematicians were videotaped. We positioned a camera at the back of the room to capture video and audio of

whole class discussion and a voice recorder was placed in front of the room to capture students' voices. In total, 33 and 24 video clips were collected from Dr. A and Dr. B, respectively. For this study, we examined 10 video and audio recordings (five from Dr. A and five from Dr. B) on integral lessons because both mathematicians spent five days covering integral lessons. To examine implemented tasks in regard to level of cognitive demand, we examined all 10 video clips and each implemented task in all 10 sessions on integral. There were 25 tasks from Dr. A's class and 10 tasks from Dr. B's class.

Cognitive Demand

Mathematical tasks are defined as a set of problems or a single complex problem that focuses students' attention on a particular mathematical idea. Thus, even if there are several problems on indefinite integral, we count them as one implemented task if those problems are about same integration skills (e.g. integration of polynomial functions). We defined implemented tasks as ones that calculus instructors and students actually worked on. As tasks are set up by the instructors, students begin to think about the content of the task which will influence what and how they learn (Stein & Lane, 1996). Low cognitive demands are memorization, algorithms, and procedures. High cognitive demands are procedures with concepts, requiring explaining, and reasoning (Stein et al., 1996). Table 1 shows examples of low and high level tasks.

Table 1. Cognitive demand examples

Cognitive Demand	Example
Low	$\int_2^5 x^3 + 1 dx$
High	Draw a region on the coordinate plane so that, when the region is rotated about a line, the resulting three-dimensional solid "looks like" a donut.

Class-based Influencing Factors

There are several class-based factors that influence cognitive level during the discussion of each implemented task.

Table 2. Class-based factors

Factors Associated with the Decline of Cognitive Demand	Factors Associated with the Maintenance of Cognitive Demand
Routinized Tasks: Teacher takes over the discussion and tells students what to do	Scaffolding of students' thinking and reasoning. Sustained pressure to provide explanations through teacher questioning
Only Seeking for Correct Answers: Emphasis shifted from meaning, reasoning, and concept to just correct answers	Use of students' prior knowledge Wait time
No Waiting Time: Students were not given time to think about each task	Students monitor their own thinking process

(Adopted from Henningsen & Stein, 1997)

For students to engage with cognitively challenging mathematical activities, selecting tasks with high level cognitive demand is not enough. The teacher needs to implement those tasks in ways that maintain cognitive demand. Consequently, we examined class discussion of each task to see these factors influence cognitive levels.

For example, a task with high level cognitive demand "Draw a region on the coordinate plane so that, when the region is rotated about a line, the resulting three-dimensional solid "looks like" a donut" can be routinized if instructor takes over the discussion and does all the work instead of giving students opportunities to explain their thinking or think about the task. In this case, cognitive demand won't be maintained.

Mathematical Questions Asked and Students' Responses during Discussions of Implemented Tasks

In any classroom, numerous questions are asked by teachers. These questions are important parts of maintaining or declining cognitive demand because they can sustain pressure for explanations and promote higher level mathematical thinking or they can routinize the tasks and seek only correct answers (Boston & Smith, 2009;

Franke et al., 2007; Munter, 2014; Shoenfeld, 2013). We used two terms that previously used in literature, *novel* and *routine* questions. Mesa and her colleagues (2014) used the term *routine* for questions that students know how to procedurally figure out the answer using information given in class or in previous classes or courses and *novel* for questions that students are required to explain new and old connections between mathematical notions. Table 3 illustrates each example. “What should the substitution be?” was asked when they discussed an integration by substitution problem. The question only requires a simple short answer. It does not require explanation or reasoning. The second question was asked after the completion of an integral problem and asked students to connect and explain their knowledge of derivatives and integrals as “opposites.” The question also required students to think about whether their solution was correct or not, which requires more than a simple answer.

Table 3. Instructors’ questions by code

Question Type	Questions
Routine	What should the substitution be?
Novel	How can I make sure this problem I just did is correct?

In addition to these two categories, we also examined types of students’ responses, short or long, as well to see how students participated class discussions.

Instructional Quality Assessment

Using the MTF as the foundation, Boston (2012) developed ways to measure the quality of mathematical instruction called the instructional quality assessment (IQA) (Boston, 2012). IQA was designed to provide statistical and descriptive data about the nature of instruction and students’ opportunities to learn (Boston, 2012). There are important observable indicators of high quality mathematical instruction: 1) implementation of cognitively challenging instructional tasks, 2) opportunities for students to participate in high-level thinking and reasoning, and 3) opportunities for students to explain their mathematical thinking and reasoning (Boston, 2012). For this study, we used the IQA of three Academic Rigor rubrics (Boston et al., 2015): Potential of the Task, Task Implementation, and Rigor of Teachers’ Questions. Potential of the Task identifies the highest level of thinking and explanation that the written task has the potential to elicit from students. Task Implementation measures the highest level of thinking in which the majority of students actually engaged during the discussion of each task. Rigor of Teachers’ Questions assesses types of reasoning required by teachers’ questions during discussion of each task. Each of these three indicators is scored from 0 to 4 (with 0 indicating absence of the measure). The procedure for analysis of the discourse and tasks consisted of segmenting the transcript by each mathematical task discussed and conducting a line-by-line coding of the dialogue, as well as scoring all class discussions and implemented tasks by the level of engagement and cognitive demand as the rubric indicated (Boston, 2012). Measuring these three important aspects of mathematical instruction will help us identify areas in need of improvement and give specific feedback to calculus instructors to directly influence calculus students’ opportunities to learn.

Interviews with Mathematicians

To understand the factors and issues that matter to mathematicians and reasons behind their instructional decisions, we interviewed the two mathematicians during the semester. The first interviews were generally about their teaching philosophy and typical issues in teaching Calculus 1. During the semester, we watched sample video clips and asked them about reasons for their instructional decisions and what supported or prevented them from implementing their instructional practices. For example, if they were not able to maintain cognitive demand for some tasks, we asked them what influenced their instructional practices. If they implemented certain tasks and pedagogy, we asked them about their reasons to implement those tasks and pedagogy.

Coding Procedures and Reliability

From set up to implementation, all 35 mathematical tasks and class discussions for these tasks were coded by the authors and three graduate students. During weekly meetings, three graduate students were trained to understand task features, cognitive demand, and class-based influencing factors. Each coder examined the tasks

to determine the cognitive level of each task using the *Task Analysis Guide*. For task features, we examined how each task was set up, presented, and solved by either the instructor or students.

We also paid attention to factors that maintained cognitive demand during class discussions, such as using student's prior knowledge, scaffolding, appropriate wait time, sustained pressure for explanations, and student self-monitoring (Boston & Smith, 2009; Henningsen & Stein, 1997). Too much or too little time, a shift in focus to the correct answer, and tasks that become routinized can reduce cognitive level (Boston & Smith, 2009; Henningsen & Stein, 1997). Since some of these factors occurred at the same time during the discussion of a mathematical task, there are several cases where we used more than one code. Each coder coded tasks and segments independently and results were compared during research meetings. When the coders did not agree, we followed majority rule. In all, the percent agreement of the three raters for cognitive demand and task features was between 93% and 96%. For class-based influencing factors and mathematical questions, it was between 95% and 98%. Finally, for IQA rubrics, it was 93% to 98%.

Results

Cognitive demand

Table 4 shows the cognitive demands of implemented tasks by the two calculus instructors. Both Dr. A and Dr. B mostly implemented tasks with low-level cognitive demand. All tasks used by Dr. B required low cognitive demand while Dr. A employed high cognitively demanding tasks (12% of Dr. A's implemented tasks).

Table 4. Cognitive demand of implemented of tasks

Cognitive Demand	Low	High
Dr. A.	22 (88%)	3 (12%)
Dr. B.	10 (100%)	0 (0%)

Mathematical Questions Asked and Students' Responses during Discussion

Table 5 details the types of mathematical questions asked during task implementation. Questions were frequently routine mathematical questions, such as "What is the antiderivative of this function?" or "Is the area positive or negative?" However, occasionally novel questions were presented, pressing for further explanations from students (e.g. "How did you know that?").

Table 5. Types of mathematical questions asked

Mathematical Questions	Routine	Novel
Dr. A	131 (80 %)	32 (20 %)
Dr. B	16 (67 %)	8 (33 %)

We also looked at the types of students' responses. Tables 6 and 7 illustrate that many responses were short responses, such as " $\frac{1}{4}x^4 + C$," "Take the derivative," "0.44," and "6 times 9, divided by 2," rather than providing explanations or reasoning. In many cases, student answers to both routine and novel questions were very brief; discussion would end when a correct answer was provided.

Table 6. Dr. A – Types of student responses

	Short (0-3)	Medium (4-7)	Long (>7)	Totals
Integral Lesson 1	22 [85%]	4 [15%]	0	26
Integral Lesson 2	18 [69%]	7 [27%]	1 [4%]	26
Integral Lesson 3	29 [72.5%]	9 [22.5%]	2 [5%]	40
Integral Lesson 4	25 [68%]	9 [24%]	3 [8%]	37
Integral Lesson 5	27 [77%]	7 [20%]	1 [3%]	35
Totals	121 [74%]	36 [22%]	7 [4%]	164

Table 7. Dr. B – Types of student responses

	Short (0-3)	Medium (4-7)	Long (>7)	Total
Integral Lesson 1	1 [100%]	0	0	1
Integral Lesson 2	5 [100%]	0	0	5
Integral Lesson 3	3 [50%]	3 [50%]	0	6
Integral Lesson 4	1 [50%]	1 [50%]	0	2
Integral Lesson 5	2 [100%]	0	0	2
Totals	10 [71%]	4 [29%]	0	16

Influencing Factors

Along with selecting high cognitive tasks, it is crucial to maintain the cognitive demand of the task throughout its implementation. Table 8 lists different factors observed during the implementation of tasks that influenced cognitive demand. Routinized tasks where the teacher takes over the discussion and dictates to students what they are supposed to do decrease cognitive demand. In Dr. B's class, all of the tasks were routinized tasks whereas in Dr. A's class, half of the tasks were routinized. Tasks which only seek correct answers, where the emphasis shifts from meaning, reasoning, and concept to just correct answers, also affect the cognitive demand of implemented tasks in a negative way and were observed in about one-third of the tasks for both instructors. Another important factor which helps to maintain cognitive demand is providing enough wait time for students to think about each task. This was observed in about half of the tasks implemented by Dr. A compared to 10% of the tasks implemented by Dr. B. However, whether there was "wait time" or not, we can see from Tables 5 to 7 that because of types of questions being asked and types of students' responses, the impact of "wait time" on maintaining cognitive demand was minimal. Furthermore, factors such as scaffolding of students' thinking and reasoning, sustained pressure to provide explanations through teacher questioning, and use of students' prior knowledge did not occur during the 10 integral lessons.

Table 8. Influencing factors in integral lessons

Factors	Only Seeking Correct Answers	Routinized Tasks	Wait time
Dr. A	10 (40%)	12 (48%)	14 (56%)
Dr. B	3 (30%)	10 (100%)	1 (10%)

IQA Rubrics

We used the IQA rubrics to rate all 10 lessons. Tables 10 display the scores for each lesson using three IQA rubrics. The average for these rubric scores confirmed that the tasks were mostly about procedures and algorithms (a rating of 2 means that the tasks require procedures and algorithms), students are not engaged in high level thinking (a rating of 2 means that students engaged in using a procedure that was either specifically called for or its use was evident based on prior instruction, experience, or placement of the task), and questions required only short responses (a rating of 1 means the teacher asks procedural or factual questions that elicit mathematical facts or procedure or require brief, single word responses). This suggests the need to implement more cognitively demanding tasks and ask more novel questions so students can be pressured to explain their thinking. However, it is more important for us to know issues that influenced mathematicians' instructional decisions. Knowing why they were not able to maintain cognitive demand will be critical first step we need to take to making attempts to change instructional practices.

Table 9. Dr. A – IQA rubrics

	The Potential of Task	Task Implementation	Rigor of Teachers' Questioning
Integral Lesson 1	2	2	1
Integral Lesson 2	2	2	1
Integral Lesson 3	3	2	2
Integral Lesson 4	3	3	2
Integral Lesson 5	2	2	1
Average	2.4	2.2	1.4

Table 10. Dr. B – IQA rubrics

	The Potential of Task	Task Implementation	Rigor of Teachers' Questioning
Integral Lesson 1	2	2	1
Integral Lesson 2	2	1	1
Integral Lesson 3	2	2	1
Integral Lesson 4	2	2	1
Integral Lesson 5	2	2	1
Average	2	1.8	1

Interviewing Mathematicians

As we analyzed the video clips, we conducted several interviews with both mathematicians. We wanted to hear what factors and issues influenced their instructional decisions in teaching Calculus 1. The first interview was conducted immediately before the beginning of the semester.

Mathematicians' General Beliefs in Teaching Mathematics

In the first interview, both mathematicians expressed that their beliefs in general. They stated that they are more non-traditional teachers, meaning that lecture is not the only way to deliver content.

Dr. A: I am interaction-centered. I ask a lot of questions. I often give random extra credit problems. I tend to be funny so that students can relax. I promote class discussion and I will let them think it is ok to talk and ask questions. I also give group work to present.

Dr. B: I am trying to incorporate some of the mathematics education elements into the class. I will let students work on the problems right there. I pose some problems and talk about it. I will try to have class discussions at least once or twice a week.

They both claimed that they are non-traditional teachers and we were curious how they implemented non-traditional teaching. We then asked them influencing factors that shape their teaching.

Dr. A: The biggest issue is the kids who do not know algebra. People have trouble with factoring, when certain things can be canceled, and the laws of exponents. I am planning to go over some algebra concepts but I might not have time to go over them fully because I might run out of time at the end and what I do is dictated by the mathematics department. I think people should have empathy. They should understand where students are. You should teach students you have, not those students you wish you had.

Dr. B: There is always a struggle between covering what the syllabus says and continuing forward when I know some students are struggling with some of background tools, such as algebra, trigonometry. Trying to accommodate those needs. Time constraints, background skills needed. Good algebra skill is one determining factor for success.

Both mathematicians acknowledged that students' mathematical content knowledge and the pressure to cover topics in the course syllabus are common issues, which was also found in other studies (Johnson et al., 2013; Mesa et al., 2014). After the first interviews, we were interested to learn how they would handle these issues and whether they are able to implement non-traditional teaching.

Cognitive Demand of Tasks and Maintenance of Cognitive Demand during Discussion

In both classes, we found that low-level tasks and a decrease in cognitive demand were very common. As we watched sample class videos, we wanted to know reasons for their instructional decisions. The main question for our subsequent interviews was, "What influenced your instructional decisions?"

Dr. A: I think it will be a big disaster if calculus is more about concepts. There is only so much disaster that I can handle. They are used to plug in stuff and factor stuff. They do not want concepts. They want formula. They often want to know what is going to be on the exams and they complain if problems on the exams are different from homework problems. They want an example of each kind of question. With the amount of time that I have, I need to decide whether I want to go more deeply instead of showing multiple solution strategies. I rather do multiple problems because that is what my students want.

Dr. B: I am happy if my students are able to compute some limits, derivatives, and integrals and have a little understanding. Some of the motivation for me in asking the simpler questions is that many decades of teaching classes like calculus has given a clear picture based on asking the more complex questions and not getting any response even after allowing time in class for students to work on those, that most students are not able to do the more complex ones without a lot of help. If students can successfully describe some of the basic algebraic, arithmetic, or geometric ideas used in computing these objects, then, to a large extent, that is a success in that many students cannot even get to that level as evidenced by grading homework and tests over several decades.

It is obvious from these responses that the two mathematicians' expectations are not very high. They stated that including cognitively challenging tasks and asking demanding questions related to those tasks may not be the best instructional plans that they can have in Calculus 1 because of their students. In Dr. A's case, to meet her students' needs, she did many examples of different types, which led her to low-level tasks, examples and asking routine questions. As the results of those tasks and questions, they were not able to maintain cognitive demand during class discussion. Dr. B described similar situations based on his previous experiences. Indeed, shifting the meaning and seeking only correct answers and routinized tasks were the main factors that led to a drop in cognitive demand in both classes (Table 8).

Engaging Students and Promoting Productive Class Discussion

Another influencing factor that decreased cognitive demand was "routinized tasks," meaning mathematicians took over the discussion (Table 8 – 100 % for Dr. B). We watched sample clips with the mathematicians and made the suggestion to use students' responses and prior knowledge and ask more demanding questions, even if many tasks were at a low level, to possibly increase cognitive demand. Both mathematicians took our suggestion, but they were not able to implement our suggestions because time prevented them from having productive class discussions.

Dr. A: I think definitely building from where they are and sort of building up intuition is the only valuable way to teach calculus. If I have kids who want to understand and if I don't have pressure on how much material I need to cover, I would definitely figure out what the kid understands and start at the point where they stopped understanding. And just entirely building everything on understanding. If they understand, they won't be confused about which algebra rules to use when.

Dr. B: I do try to engage students at times in tasks I hope will access their prior knowledge, but as I would guess you have heard from others, the time available becomes an issue. There always seems to be a struggle between what is the best use of time in the classroom and is the expectation for the list of topics students will 'see' in the class. The main challenge is what "seeing" means - if a professor talks about a topic but no one understands what he/she said, was that a good use of time? Or if a professor spends a good deal of time having students work in lecture time on table or numerical or graphical based developments of ideas (e.g. the fundamental theorem of calculus, or computations of definite integrals as limits of Riemann sums) but only gets through 1/2 to 2/3 of what was on the syllabus, are students prepared for the next math or science class they are supposed to take? These are hard questions and there are huge differences of opinions from different people on many of these questions. Many mathematicians think that their job is to tell correct ways to solve problems and students' job is to do it themselves.

Again, the amount of material to cover during the semester is one of the main issues. This is commonly found in other studies (Johnson et al., 2013; Mesa et al., 2014). The fact that calculus classes are offered in sequence also pressures them to cover all the topics in the syllabus. Two common issues that matter to mathematicians are underprepared students and time constraints, but what is notable here is the way these two mathematicians handled these two issues. Dr. A did many procedural tasks and asked many routine questions. She thought that

implementing more procedural tasks is necessary at this level to promote interactions between her students and herself. She believed that doing many procedural tasks is what the students wanted and needed. On the other hand, Dr. B lectured most of the time, believing that it is his job to show them correct ways to solve problems. That is why we only found a few tasks and questions during his lessons compared to Dr. A. In addition to these comments, Dr. A expressed the following, which also described challenges she have.

Dr. A: I can't take out doing many examples because that is what the students want. I am fully aware that I am not making the choices that are not necessarily always good. But I still feel like they are the only choices that I can make under the constraints. I've had lots of opportunities to reevaluate my choices and I still believe that it is sad that I am making them. But I feel that those are the choices that I have to make.

We can see the dilemma and challenges that she have and it appears that what both mathematicians can do is limited and it will be very challenging for them to try to change their instruction. It appears that challenges that mathematicians have are very difficult to overcome.

Summary and Discussion

Underprepared Students, Maintenance of Cognitive Demand, and Mathematicians' Beliefs

In this study, we examined 10 integral lessons to understand students' opportunities to learn cognitively challenging tasks and maintain cognitive demand during these lessons. We also interviewed both mathematicians several times to understand factors that influenced their instructional decisions.

Although they believe that there are benefits of implementing student-centered teaching, there are two main issues that prevented them from implementing such practice. Two main issues influenced their instructional decisions - underprepared students and time constraints. Both mathematicians mentioned that implementing high-level tasks and asking demanding questions may not be the best way to teach Calculus 1 because of students' lack of preparation. As a result, they mostly did procedural tasks or lectured. Dr. A commented several times during the interview that her students were most concerned about formulas, rules, and what items would be on the exam. Dr. B stated that most mathematicians think that it is their job to demonstrate correct ways of doing mathematics. It appears that his instructional practices also resembled his fellow mathematicians, resulting in many procedural tasks and lecture being the dominant method. One suggestion that we made was to use students' prior knowledge and responses to have more productive discussions and to eventually maintain cognitive demand. If students are not ready to understand high-level concepts and tasks, it gives more reason to use their prior knowledge and it is ideal to use their prior knowledge and responses to maintain cognitive demand, even if the tasks are at a low level.

Time Constraints

Not being able to implement our suggestions led to another important issue of time constraints, which was raised by both mathematicians. Both of them either knew or agreed that wait time and using students' prior knowledge and responses can be beneficial. Thus, we suggested for them to practice these; however, they did not or were not able to because of time constraints. How can they be convinced to change their class practices? In a previous study, one calculus instructor was able to think about his class practices and reset his goals and prioritize what he needed to accomplish in an attempt to resolve the issue of time constraints (Thomas & Yoon, 2014). Some mathematicians believed that covering less topics in depth is more beneficial than covering all the topics in the syllabus (Johnson et al., 2013). Also, a recent report from CSPCC shows that when calculus instructors and students agreed that there was enough time to cover calculus topics, students are more likely to continue with Calculus 2 rather than dropping out (Johnson, Ellis & Rasmussen, 2014). On the other hand, when calculus instructors and students are concerned with time, students are likely to drop from the calculus sequence and not pursue STEM degrees (Johnson et al., 2014). What is interesting is that calculus instructors displayed more student-centered instructional practices (opportunities to explain their thinking, less relying on lectures and more questions were being asked to students) had enough time to teach and understand difficult calculus concepts while instructors who stated time issues exhibited more traditional lecture-based instructional practices and did not have enough time to understand difficult calculus concepts (Johnson, Ellis & Rasmussen, 2014). Contrary to what mathematicians think, the CSPCC report suggests that it may not be an issue of time that prevents the implementation of student-centered instructional practices and the coverage of a set amount of

topics. It could be more about their belief that “there is not enough time to cover everything” that keep them from implementing student-centered teaching. Mathematicians in Johnson et al’s (2013) study were willing to implement inquiry-based curriculum and the calculus instructor in Thomas and Yoon’s study (2014) was willing to implement student-centered teaching. These studies reveal to us that although it is challenging and mathematicians are limited by several factors, having student-centered teaching is possible in undergraduate mathematics teaching. We acknowledge that time is an important issue for many calculus instructors because calculus classes are normally offered in sequence. That said, attempting to resolve time constraints by resetting and prioritizing goals is something that calculus instructors should consider.

Conclusion

Changing instructional practices in calculus is very challenging. The two mathematicians in our study knew about the benefits of asking demanding questions and getting more responses from students; however, there are dilemma and challenges that limit what they can do. Subsequently, we can ask, “What can we suggest so mathematicians can try to overcome challenges?” Pedagogical issues in STEM areas are identified by many researchers (Henderson et al., 2011). More than 60% of studies recommend changes in the curriculum, pedagogy, and teaching while others recommend changes in policy and shared vision (Henderson et al., 2011). To change mathematicians’ pedagogy, they need to have opportunities to rethink about their current practices. Similar to our results, when their current practices conflict with what is recommended, mathematicians will be less willing to make any changes (Weber, 2004, 2012). In our study, both Dr. A and B were not able to implement our suggestions because of the conflict between our suggestions and issues they have. At elementary and secondary levels, professional development is very common and gives teachers opportunities to reconsider and reset their goals and teaching; however, these opportunities are not often seen at the undergraduate level (Speer, Smith, & Horvath, 2010).

Although we believe providing professional development/collaboration opportunities to mathematicians is important, previous attempts at collaboration and professional development in undergraduate STEM areas have not always been successful (Henderson et al., 2011). Thus, it is desirable to develop collaboration using models from previous studies (Barton, 2011; Barton et al., 2015). Reflecting on one’s own teaching and changing the concept of one’s teaching are keys to successful professional development (Hannah, Stewart & Thomas, 2011; Henderson et al., 2011). Through this process, mathematicians rethink their practices and attempt to revise their ways to teach undergraduate mathematics (Hannah et al., 2011; Paterson et al., 2011).

When collaborating with mathematicians, we suggest providing them with opportunities to think about the Mathematical Tasks Framework, maintenance of cognitive demand, and results from previous studies (CSPCC) so they can think about ways to resolve issues of students and time. These suggestions may sound obvious, but mathematicians are not often provided with training and support for their pedagogy (Walczyk et al., 2007), so it is beneficial to collaborate with them and provide professional development/collaboration opportunities. For example, had two mathematicians in our study had opportunities to be informed about CSPCC’s results about time constraints, they might be more convinced to rethink about their instructional decisions on many procedural tasks and lectures. However, it is too naïve to say that just informing them CSPCC’s results can change their instructional practices. Since time constraint is an important issue, it is ideal to change their practices in a few lessons at a time. If they are successful in changing their instructional practices for those lessons without a conflict with time, they will be more convinced to implement more reform-oriented teaching and reset and prioritize goals for a few more lessons.

In CSPCC, a large public institution successfully implemented “ambitious teaching” in calculus. The study highlighted challenges of implementing “ambitious teaching” but also suggested that with systematic support from the department and training of calculus instructors, this institution was able to successfully implement ambitious teaching for about two decades (Larsen et al., 2015). Ambitious teaching at this institution focused on having challenging tasks and asking students to explain their thinking, but, a more important component was training their calculus instructors to be familiar with such teaching (Larsen et al., 2015). The trained calculus instructors were told that implementing “ambitious teaching” is the norm in teaching Calculus 1, which helped them think about the way they teach (Larsen et al., 2015). Speer (2008) showed that one calculus instructor was able to implement reform – oriented teaching, using students’ responses, providing feedback and asking more demanding questions (Speer, 2008). The calculus instructor in Speer’s (2008) study also had a training and orientation sessions where he was informed about recommended teaching strategies. In our context, we can think of the training sessions as professional development for mathematicians, which allows them to reflect on and rethink their instructional practices. Through the process, calculus instructors can think about their current

practices to identify possible issues that they have. Then, they might be able to find rooms to change their instructions as calculus instructors did in other studies (Johnson et al., 2012; Speer, 2008). It is challenging to make changes in undergraduate mathematics teaching, but collaborating with instructors and providing professional development opportunities to reflect on their teaching and rethink their beliefs and goals are possible ways towards reforming undergraduate teaching.

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