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Melihan Unlu[1], Tugba Horzum[2]
[1]Aksaray University
[2]Necmettin Erbakan University

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Abstract

This study aims to analyze how the Mathematics Teacher Candidates (MTCs) define the "prism" and "pyramid" concepts in geometry. The research was carried out in 2017-2018 academic year with 92 MTCs. This test consists of 3 open-ended questions that MTCs can answer conceptually. The first open-ended question was as "Define a prism", the second was as "Define a pyramid" and the third was as "Explain the differences between prism and pyramid". The answers of MTCs were investigated according to correctness and generalization criteria. After data analysis, semi-structured interviews were held with 11 MTCs who had ineligible expressions in their definitions. The research revealed that MTCs were unsuccessful in defining prism and pyramid concepts and had difficulties in expressing the differences between these two concepts. Therefore, it was determined that they have insufficient knowledge.

Introduction

In Turkey, "General Proficiencies of Teaching Profession" is prepared by Ministry of National Education and according to it, teachers' proficiencies are handled through pedagogical content knowledge, subject matter knowledge and curriculum knowledge (Ministry of National Education [MoNE], 2017). The works in literature indicate that teacher's pedagogical content knowledge has an important influence on students' learning (Ball, 1988; Ball & McDiarmid, 1989). However, it cannot be denied that subject matter knowledge has an important place on the formation of pedagogical content knowledge. According to the studies there is a close relation between subject matter knowledge and pedagogical content knowledge (Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992; Capraro, Capraro, Parker, Kulm & Raulerson, 2005; Even, 1993; Ma, 1999; McDiarmid, Ball & Anderson, 1989; Türnüklü, 2005). So, a good level of subject matter knowledge both increases the quality of teaching and contributes the success of students as well (Ball, Thames & Phelps, 2008; Brown & Borko, 1992; Hill, Rowan & Ball, 2005; Ma, 1999). Subject matter knowledge of a teacher especially becomes more important in mathematics and geometry which are assumed as difficult by most of the students (National Council of Teachers of Mathematics [NCTM], 2000) and this knowledge has a great importance in teaching mathematics.

A deep subject matter knowledge is very important for a mathematics teacher (NCTM, 2000) and a teacher has to deeply know the subject s/he is going to teach (Türnüklü, 2005). Here what is actually meant by deep subject matter knowledge is what a mathematics teacher has to know about the subject matter. The first thing a mathematics teacher has to know is basic knowledge. This type of knowledge which is called as profound understanding of fundamental mathematics by Ma (1999) is the subject matter knowledge in a sense that a teacher must have. In the second place when thinking about what mathematics teacher has to know, the teaching process comes to mind. Many teaching activities during teaching process such as teacher's selecting proper teaching activities, asking productive questions, evaluating learning of her/his students depend on teachers' having sufficient content knowledge (Ball & McDiarmid, 1990). Besides, the teachers' deficiencies in subject matter knowledge affect students' learning negatively. For this reason, an effective teaching from a teacher with insufficient subject matter knowledge cannot be expected. Therefore, it is thought that MTCs who to possess sufficient subject matter knowledge.

According to Shulman (1986) mathematical concepts come in the third place for a mathematics teacher to know because the concepts are important components of content knowledge. Teachers have to know perfectly the fundamental characteristics, representations and alternative ways related to concepts on a subject matter. In this context Zazkis and Leikin (2008) indicates that definitions of mathematical concepts, the structures under these definitions and definition processes are the fundamental components of subject matter knowledge of mathematics teachers. This means that having this knowledge shapes the subject matter knowledge (Even, 1993). However, the researches show that teachers should understand these concepts or processes in order to form correct representations of the concept (Borko et al., 1992; Ma, 1999; McDiarmid et al., 1989). On the other hand, it is not enough to know the concept and carry on procedural processes. The main purpose is to build a balance between procedural and conceptual knowledge which is a part of subject matter knowledge. For that reason, conceptual processes should be handled and new approaches should be acquired. With this idea, researchers focused on knowing concept definitions, understanding and forming concepts (Leinhardt & Smith, 1985; Shulman, 1986; Vinner, 1991; Vinner & Dreyfus, 1989; Zazkis & Leikin, 2008). In the same scope, definitions of concepts are focused in the current study.

Mathematics has its own original concepts as all other disciplines have (Bozkurt & Koç, 2012). Even sub-learning area of mathematics such as algebra and geometry are structurally different from each other. For example, geometrical concepts are different from other mathematical concepts in three different ways. These are the images, definitions and peculiarities of a geometric figure (Fischbein, 1993). When the visual images of a geometric figure becoming more prominent than the concept definition (Türnüklü, Alaylı & Akkaş, 2013), students may experience difficulty in understanding the geometrical concept and so, required conceptualization cannot be provided (Kaplan & Hızarcı, 2005). In order to provide the required and consistent concept knowledge, richer experiences about the concepts can be provided for students; however, it is not so easy to form a consistent concept knowledge (Tall, 1988), because it is a known fact that same structures are not associated to a concept in different minds. As Dienes (1971) indicates, each individual may see and understand world in different ways and may exhibit different approaches. Therefore, individuals' knowing the concept definitions does not mean they understand the concept (Vinner, 1991). In this scope, it is known that there are disharmonies between the concepts formulized by mathematicians and the concepts individuals interpret (Tall, 1992). The structure of concept image-concept definition of Vinner and Hershkowitz (1980) can be mentioned here. According to this, the concept definition is defined as a whole of symbols and form of words which are accepted by mathematicians (Tall & Vinner, 1981).

Besides, the definitions which compose the fundamentals of mathematical thinking are very important in formation of a mathematical concept, in differentiating a concept from other concepts and in expressing mathematical ideas (Çakıroğlu, 2013). However, concept image includes some structures associated with that concept in minds of individuals (Tall & Vinner, 1981; Vinner 1991). Although they are assumed as separate from each other, there are some relations between concept definition and concept image. For example, concept images are formed as the results of individuals' experiences with concept definitions and samples (Vinner & Dreyfus, 1989). So, these images may improve as cognitive structure improves. Here, formal definitions are structured by individuals, creating personal definitions that are the foreground of individual aspects of the formal definition. Definitions structured by individuals for a special concept (personal definition), their formal characteristic in minds, transmissions and relations among concepts are all important for conceptual understanding. Thus, teachers' knowledge about mathematical definitions are the main components of their content knowledge (Leikin & Zazkis, 2010). Because multifaced structures of geometric concepts and content knowledge of teachers may influence classroom activities, it is important to investigate the content knowledge of those who teach or will teach geometric concepts. Geometric content knowledge has two fundamental characteristics; the first is to know "what" a concept is and the second is to know "why and how" a concept is so (Tuluk, 2014). For this reason, it is thought that geometric figures and solids among the fundamental concept of geometry should be investigated in this scope, because it is a fact that students from preschool education till university at all levels have difficulties in defining, recognizing and expressing their characteristics of geometric concepts (Clements & Sarama, 2000; İncikabı & Kılıç, 2013; Tsamir, Trosh & Levenson, 2008; Türnüklü & Ergin, 2016). But unfortunately, it is also known that mathematics teachers and mathematics teacher candidates have difficulties in recognizing main mathematical and geometric concept, in illustrating the definitions and recognizing them, and even they have misconceptions (Bozkurt & Koç, 2012; Çakmak, Konyalıoğlu & Işık, 2014; Gökbulut & Ubuz, 2013; Gökkurt & Soylu, 2016; Horzum & Ertekin, 2018; Kaplan & Hızarcı, 2005; Leikin & Zaskis, 2010; Tsamir, Tirosh, Levenson, Barkai & Tabach, 2015; Türnüklü et al., 2013). It is well known that teachers (Gökkurt & Soylu, 2016; Tekin-Sitrava & Işıksal-Bostan, 2018; Tsamir et al., 2015) and teacher candidates (Alkış-Küçükaydın & Gökbulut, 2013; Altaylı, Konyalıoğlu, Hızarcı & Kaplan, 2014; Bozkurt & Koç, 2012; Çakmak et al., 2014; Çetin, Dane, Okur, Bekdemir, Baş, Kanbolat & Özturan Sağırlı, 2012; Gökbulut & Ubuz, 2013; Gökkurt, Şahin, Erdem, Başıbüyük & Soylu, 2015; Gökkurt, Şahin, Soylu & Doğan, 2015; Koçak, Gökkurt-Özdemir & Soylu, 2017; Marchis, 2012; Ubuz & Gökbulut, 2015) have difficulties with geometric solids especially prism and pyramid concepts.

Prism is a concept that individuals meet a lot in their daily lives, it has an important place among geometric concepts (Gökbulut & Ubuz, 2013). However, teacher candidates have difficulties in giving definitions (Bozkurt & Koç, 2012; Ubuz & Gökbulut, 2015; Gökkurt & Soylu, 2016), distinguishing critical characteristics (Alkış-Küçükaydın & Gökbulut, 2013; Ubuz & Gökbulut, 2015; Gökkurt & Soylu, 2016) and naming (Çetin et al. 2012; Horzum & Ertekin, 2018) according to the researches where their knowledge about prism is investigated. In their studies Bozkurt and Koç (2012) shows that MTCs make mistakes when they give definition for prism concept. Gökbulut and Ubuz (2013) have found that MTCs do not know necessary and enough logical principles for definition of prism, they cannot choose necessary and sufficient prism characteristics to form a definition and they cannot distinguish its critical characteristics. Horzum and Ertekin (2018) indicate that MTCs call a square prism which stands on its square surface on a desk as square prism. But they call the same prism a rectangular prism when it stands on its rectangle surface. Besides, MTCs have misconceptions about prism (Alkış-Küçükaydın & Gökbulut, 2013; Altaylı et al., 2014). Altaylı and her colleagues (2014) asked MTCs whether the new object is a prism or not when a prism is bended and the results show that 76.6% of MTCs found out that new figure was a prism even if the angles between the corners were right and 12% of MTCs suggested that the new figure was not a prism if the angle was not right. On the other hand, Çetin and his colleagues (2012) has investigated the conceptions of rectangular prisms of MTCs with the help of models and the results show that they have difficulty in naming the figures in skeleton shape and recognizing the characteristics about these models.

In addition, although pyramid is a concept with a definition and samples that individuals meet often in their daily lives, it is difficult to be perceived (Ubuz & Gökbulut, 2015). In related investigations teacher candidates have difficulty in giving definitions and distinguishing its critical characteristics. Ubuz and Gökbulut (2015) indicate that teacher candidates have difficulty in giving a mathematical definition about pyramid concept and their pyramid drawings are generally limited to prototype samples. Also Altaylı and her colleagues (2014) have found out that teacher candidates have misconceptions in definitions of prism, pyramid and cone concepts and they do not know the concepts of truncate cone, truncate pyramid at all. Çakmak and her colleagues (2014) have handled MTCs' subject matter knowledge of three dimensional solids (prism, pyramid, cylinder, cone, sphere) conceptually and have investigated three dimensional solids under drawing, defining, sampling, reorganizing and spatial thinking categories. The results show that MTCs have some difficulties in defining and recognizing problems in their subject matter knowledge about three dimensional solids. While defining them, MTCs made a general definition instead of giving a mathematical definition and they gave visual-oriented answer for recognizing solids' characteristics. Moreover Marchis (2012) stated that some students can't recognize basic geometrical shapes and solids.

As a result of literature review, prism and pyramid concepts are so important that it is necessary to investigate the difficulties about these concepts. The person who will teach these concepts has to learn them perfectly in order not to cause any misconception or contradiction in terms for learners (Çakmak et al., 2014). Therefore, investigating the differences between prism and pyramid concepts definitions, which is not mentioned in the literature, and how MTCs define them will reveal their content knowledge on prism and pyramid concepts. It is thought that it is important to know the difference between these two concepts and not to be confused. Because MTCs need to know the critical characteristics of the concepts and to think about the definitions so that they can determine the difference between these two concepts. Therefore trying to determine how the MTCs handle the difference between these two concepts and how they define these concepts offers a unique advantage to the field. Besides, knowing MTCs' ideas about prism and pyramid concepts and the differences between them can give clues about how they will teach in future.

Problem of the Research

This study analyses how the mathematics teacher candidates understand the "prism" and "pyramid" concepts in geometry. Therefore, the following questions are investigated:

1. How do MTCs define a prism?

2. How do MTCs define a pyramid?

3. How do MTCs define the differences between prism and pyramid?

Method

Research Design

In the current study, the case study method based on qualitative approach was used. A case study investigates a group or event deeply (Merriam, 2013). In this research, the case study method was chosen because MTCs' definitions of prism and pyramid was investigated by focusing in depth and the case was the selected group of MTCs to determine their definitions related to prism and pyramid.

Participants

Criterion sampling method, which is one of the purposeful sampling methods, was used in the research. In this method, researchers choose participants according to specific criterions (Patton, 2002). The acceptable criterion in this research is that MTCs should have taken "Geometry" and "Instructional Technologies and Material Design" courses which were taken for 14 weeks. According to this, MTCs took the Geometry course at their first academic year, and Instructional Technologies and Material Design course at their second academic year. Geometry course includes activities and theories about the line, plane and space in Euclid geometry, angles, triangles and quadrilaterals, circle, solid objects, and geometric place. Instructional Technologies and Material Design course includes activities and theories about learning and teaching technology, teaching tools and materials, concept maps, computer based training taking into account the mathematical concepts. The third criterion was that MTCs volunteered to participate the study and their names were not used, but instead S_1, S_2, S_3,….S_{92} pseudonyms were used. The reason of choosing this MTCs is the conjecture that they have enough knowledge as institutional and agreeable with geometry concepts investigated. In this sense, this research was carried out in 2017-2018 academic year with 92 MTCs, 47 of them were 3rd graders and 45 of them were 4th graders in mathematics teaching department of a university in Central Anatolian Region in Turkey.

Data Collection Tool

A test for data collection was developed by the researchers. Test was used to reveal definitions of MTCs about prism and pyramid concepts. This test consists of 3 open-ended questions which can be answered conceptually by the participants. The questions in this test were checked by an expert with PhD in mathematics education in terms of correctness of the language and pedagogical-academic accuracy and then, necessary corrections were made. So, the first open-ended question became as *"Define a prism"*, the second became as *"Define a pyramid"* and the third became as *"Explain the differences between prism and pyramid"*. After data analysis, semi-structured interviews were held with 11 MTCs who were randomly selected or who had ineligible expressions in their definitions. In the interviews, the question *"What did you want to say here? Can you explain?"* was asked to MTCs who had ineligible points in their definitions. Besides they were also asked to give examples and draw three different prisms and pyramids with the questions such as *"Can you draw 3 different prisms/pyramids? Which peculiarities make them prisms/pyramids?*"

In the current study open-ended questions were used as the main source of data to determine participants' definitions of prism and pyramid concepts. The documents obtained from the drawings related to the prism and pyramid concept and semi-structured interviews were used as supporting data. And through this data triangulation structural validity of the current study was obtained.

Data Collection and Analysis

During data collection process MTCs were told that this test was not an examination and they were asked to give detailed and clear answers. Besides, there was no time limitation when they were answering. MTCs were also asked to write their names as they would be interviewed in future. Besides, they were told to use their knowledge in their minds and they were not allowed to use any sources such as internet, book etc. Before the data analysis, documents, interview audio records and answers of open-ended questions were numbered separately for each participant. Thus, the relations between the data (obtained from open-ended questions, documents, interviews) could easily be seen and also reliability and validity of data analysis could be controlled. Besides, definitions of prism and pyramid in literature were investigated and in this direction, critical characteristics of prism and pyramid concepts were determined. The following prism and pyramid definitions were used in this study. A prism is defined as "*a polyhedron with two parallel, congruent polygon bases, and* *lateral faces that are parallelograms"* (Schwartzman, 1994, p.172). A pyramid is defined as *"a polyhedron obtained by connecting each vertex of a polygon to a single point outside the plane of the polygon; the sloping sides of a pyramid are therefore triangles"* (Schwartzman, 1994, p.177). According to this, the critical characteristics for prism were determined as follows; PR1. Identical (two) bases, PR2. Parallel bases, PR3. Polygon bases, PR4. Parallelogram lateral faces. The critical characteristics for pyramid were determined as follows; PY1. Polygon bases. PY2. Triangular lateral faces, PY3. It has an apex. And also the critical characteristics between prism and pyramid concepts were defined according to the critical characteristics of these two concepts. The criterias were determined as follows; a. The number of bases: Pyramid has 1 and prism has 2 (identical) bases; b. Bases are parallel: Prism has, but there is not for pyramid because it does not have two bases; c. Characteristics of lateral faces: Lateral faces are triangle on pyramid and parallelogram on prism; and d. Existence of an apex: Pyramid has an apex but prism does not have it.

The answers of MTCs were investigated according to correctness and generalization criteria (Gökbulut & Ubuz, 2013; Ubuz & Gökbulut, 2015; Zaskis & Leikin, 2008) and obtained data were arranged and interpreted in accordance with the research questions. The categories for defining prism and pyramid were investigated through both correctness and generalization criteria. However, the category of explaining the differences between prism and pyramid was investigated through only correctness criterion. Descriptive analysis was performed about definitions and explanations of MTCs. The data were classified as definition having one, a few or all critical characteristics. And then, they were categorized according to correctness and generalization criteria. The contents of these criteria are as the followings:

Correctness criterion: In the analysis performed with this criterion which belongs to Zazkis and Leikin (2008). When examining the correctness of the definitions provided by the MTCs, it was distinguished as *correct answer, insufficient answer* and *wrong answer* as in the study of researchers (Gökbulut & Ubuz, 2013; Ubuz & Gökbulut, 2015) for prism, pyramid and difference between these concepts. Then the definitions were classified as *correct answer* that had all critical characteristics and included necessary and sufficient conditions. In this study, the correct answer for the prism that include all critical characterics couldn't be found. On the other hand, the definition for the pyramid as "*three dimensional figure with a base in polygon shape, with triangle lateral faces whose upper corners unite in one point*" was evaluated as the correct answer.

Besides the definition handled by MTCs were classified as *insufficient answer* that had some missing critical characteristics or even non-critical characteristics and included logical structure of necessary but insufficient or sufficient but unnecessary conditions. For example, the definition for prism as "*a closed three dimensional geometric shape whose base and ceiling* (meaning the top base) *consist of similar geometric forms and lateral faces are rectangle*" has non-critical characteristics and some missing critical characteristics. So this definition was coded as insufficient answer.

Lastly in the correctness criterion, the definitions handled by MTCs were classified as *wrong answer* that had definition without any critical characteristics and included logical structure of *neither necessary nor sufficient* conditions. For example, the definition for pyramid as "*three dimensional object having corners*" does not include any necessary and sufficient conditions.

Generalization criterion: When examining the generality of the definitions provided by the MTCs, it was distinguished as in some researches like *special answer, general answer* (Gökbulut & Ubuz, 2013; Ubuz & Gökbulut, 2015; Zaskis & Leikin, 2008) and *quasi-special answer* (Gökbulut & Ubuz, 2013) for prism, pyramid concepts. Accordingly, this criterion was used to investigate whether a definition comprises all critical characteristics or it was a general definition comprising all other concepts. The definition with any critical characteristics were classified as "*general answer*". The following examples can be given for the general answer. For example, the definition for the prism as "*three dimensional geometric figure composed of width, depth and height*" and the definiton for the pyramid as "*three dimensional objects which have height, volume, side, corner*" can be any geometric object. Besides, the definitions which define prism/pyramid or comprise some critical characteristics were classified as "*quasi-special answer*". The following examples can be given for the quasi-special answer. For example, the definiton for prism as "*three dimensional geometric figures whose bottom and top bases are same polygons and four lateral faces are rectangle*" and the definition for pyramid as "*a structure whose bottom base consists of regular quadrilaterals and which has an apex*" have some critical characteristics. Lastly all definitions comprising all critical characteristics of prism/pyramid were classified as "*special answer*".

To achieve the reliability for data analysis coding processes were performed by two researchers separately. Then the points of "agreement" and "disagreement" were discussed and necessary changes were made. Then the first and the second coding results were united and consistency percentage through Miles and Huberman (1994) Realibility = Agreement/ (Agreement + Disagreement). At the end 88% agreement was achieved. The final categorizations were produced by resolving the disagreement through consulting with two researchers who have PhD in mathematics education. In order to increase external reliability of the study, all the processes of the study were detailed and supported with the quotations of the MTCs. In order to increase internal reliability of the study, findings were presented directly without any interpretation.

Findings

In this section, according to the data analysis, the definitions of MTCs were sampled according to determined categories and interpreted through frequency table.

Definitions of MTCs about Prism Concept

The definitions of MTCs about prism concept were investigated through both correctness and generalization criteria. For the correctness criterion, the definitions were classified taking into account the critical characteristics for prism defined as identical two bases (PR1), parallel bases (PR2), polygon bases (PR3), and lastly parallelogram lateral faces (PR4). In this context, prism definitions were classified under the titles including one or more of these critical characteristics (Table 1).

Table 1. Definitions about prism concept

	Criteria		Theme	f	%
Defining prism	Correctness	Correct answer	PR1 + PR2 + PR3 + PR4	-	-
		Insufficient answer	PR1	7	7.61
			PR2	2	2.17
			PR3	6	6.52
			PR4	3	3.26
			PR1 + PR2	7	7.61
			PR1 + PR3	11	11.96
			PR2 + PR3	1	1.09
			PR1 + PR4*	12	13.04
			PR1 + PR2 + PR3	1	1.09
		Wrong answer		42	45.65
	Generalization	Special answer		-	-
		Quasi-special answer		50	54.35
		General answer		42	45.65

*MTCs touch upon the point that lateral faces of a prism are quadrilateral or rectangle.

According to correctness criterion in Table 1, it can be seen that none of the MTCs could define prism correctly. However, 54.34% of MTCs mentioned one or a few critical characteristics of prism in their definitions that were accepted as insufficient answer. In these definitions, MTCs touched upon “identical bases and rectangular or quadrilateral lateral faces” (13.04%) criterion most. What attracts attention on this theme is that most of the MTCs touched upon lateral faces being rectangle and the rest emphasize the lateral faces being quadrilateral. For example, S_{55} defined prism as “*It is a closed shape whose base* (bottom) *and ceiling* (meaning the top base) *have the same polygon and lateral faces are quadrilateral”* and S_{52} defined prism as “*It is a closed three dimensional geometric shape whose base and ceiling consist of similar geometric forms and lateral faces are rectangle*”. In interview, S_{52} was asked to give example to represent her prism definition and she drew triangle, square and rectangular right prisms that stood on their bases as shown in Figure 1.

Figure 1. Prism drawings of S_{52}

The second insufficient answer was the criterion “identical and polygon bases” (11.96%) that MTCs touched upon. Through this theme some MTCs emphasized that bases were generally polygon but some showed they had a limited prism knowledge specifying the bases as regular polygons. For example, S_{75} indicated that “*Prism is a three dimensional figure whose base and ceiling are identical regular polygon*”. Third, MTCs were handled equally (7.61%) the citeria “identical bases” and “identical and parallel bases” as the insufficient answer. For example, S_{82} defined prism as “*a three dimensional figure whose base and top base have the same shape*” and S_{72} defined it as “*objects whose bottom and top bases are parallel and in identical forms*”. Fourth, some MTCs (6.52%) emphasized in their prism definitions that only bases are polygon. One of the MTCs, S_{24}, emphasized that the bases were polygon on prism with her definition as “*an object having polygonal base and ceiling, and height*”. As the fifth insufficient answer, few MTCs (3.26%) mentioned in their definitions that only lateral faces were parallelogram (especially rectangle). For example, S_{32} saying that “*a three dimensional figure that is named according to their base shapes, whose lateral faces are rectangle*” indicated that lateral faces were rectangle but she did not mention about polygon bases. MTCs emphasized rectangle instead of parallelogram in their definitions, and this fact shows that they generally think right prism in their minds. As the sixth insufficient answer, only two of MTCs mentioned that only the bases were parallel. For example, S_1 defined prism as “*a figure whose base and ceiling are parallel*”. The last themes in insufficient answer category were “parallel and polygon bases” and “identical, parallel and polygon bases”. The answer of S_{77} could be given as an example: “*Prism is a three dimensional figure whose base area is extended as much as the height. The base can be any polygon such as triangle, quadrilateral etc.*” On the other hand, S_2 defined prism as “*an object that is formed by uniting the corners of parallel and identical planes*”. When S_2 was asked what she meant by this, she indicated that “*the plane is finite*”. Here, it is understood that S_2 actually uses the word plane for polygon and so, she defines prism as an object with identical polygon base formed by parallel planes.

The answers of 45.66% of MTCs which were evaluated under the wrong answer category, were not definitions but were general expressions which do not represent prism.This means that almost half of the MTCs did not mention any critical characteristics of prism. In their answers MTCs used geometric object, three dimensional figure, object, regular polyhedron, figure concepts often. For example, while S_{37} defined prism as *“three dimensional geometric figure composed of width, depth and height,* S_5 defined as “*three dimensional geometric objects having a certain height depth and width*”. The answer of S_{45} “*geometric figures which are called according to the base shape*” and the answer of S_{20} “*figures whose opposite sides are identical*” could be given as examples for wrong answer category. In order to understand what S_{20} meant by opposite side, he was asked to draw a prism. Then S_{20} drew square prism, rectangular prism and cylinder but he could not explain the relation between prism and cylinder because he gave the explanation as “*There is no relation between prism and cylinder. It came to my mind that the volume of prisms was three times as pyramids or cylinder was three times as the cone*”.

As it can be seen in Table 1, none of the MTCs could give any special definition about prism. Because 54.35% of MTCs gave definitions only to qualify the prism and include some critical characteristics, they were evaluated under quasi-special answer category. For example, S_4 who handled right prism in her drawing defined prism as “*three dimensional object with equal opposite sides*” and in interviews she was asked what she meant with saying side. Then after saying “*I meant top and bottom bases. Bases comprise of polygons. If bottom base is square, then top base is square, and if bottom base is triangle, top base is triangle as well*” she mentioned about polygon and identical bases. Similar to this, S_{62} said “*They are three dimensional geometric figures whose bottom and top bases are same polygons and four lateral faces are rectangle*”, he mentioned that bases were polygon and identical, lateral faces were rectangle, however, he did not say that bases had to be parallel. It can be seen here that S_4 and S_{62} made quasi-special answer which narrowed the concept and which comprised right prisms but no other prisms. And with the definition S_{91} “*Prism is a three dimensional figure formed by same polygons coming one overlap other*” mentioned that the bases were identical, parallel and polygon but did not use any expression about lateral faces’ being parallelogram. S_{91} makes here nearly a quasi-special answer not including all objects but including objects like cylinder with identical bases.

Definitions of 45.65% of MTCs were categorized as general answers. For example, S_{66} defined prism as “*Prism is a general name given to three dimensional figures which have width, depth and height*”; S_{76} defined it as “*Geometric figures which are shown as three dimensional with height, area and volume characteristics*”; S_{33} defined it as “*Geometric figures with a certain height and volume*”. Only S_{35} gave a different definition as “*The closed three dimensional shape which has edges and surfaces formed by the unification of regular geometric figures*”. If this definition is scrutinized, it can be deduced that she evaluates regular polyhedron as prisms. During interview this expression was reminded to her and she defined prism as “*It was a closed geometric figure having surface and edges and was given name according to the figure it formed. It has to have height.*

Formed by geometric figures but not regular ones" and thus showed cylinder as an example. But here again she has a misconception because prism is a special case of cylinder. As a result, these definitions of MTCs did not have any critical characteristics of prism concept. It was determined here that MTCs gave general answers that were based on "*having volume and height*" which was general characteristics of any object but these general characteristics was also a characteristics of prism without any critical characteristics and it comprised all objects.

Definitions of MTCs about Pyramid Concept

The definitions of MTCs about pyramid concept were investigated through both correctness and generalization criteria. For the correctness criterion, the definitions were classified taking into account the critical characteristics for pyramid defined as polygon base (PY1), triangular lateral faces (PY2), and lastly having an apex. (PY3). In this context, pyramid definitions were classified under the titles including only one or more of these critical characteristics (Table 2).

Table 2. Definitions about pyramid concept

	Criteria		Theme	f	%
Defining pyramid	Correctness	Correct answer	PY1 + PY2 + PY3	13	14.13
		Insufficient Answer	PY1	2	2.17
			PY2	1	1.09
			PY3	10	10.87
			PY1 + PY2	6	6.52
			PY1 + PY3	30	32.61
			PY2+ PY3	10	10.87
		Wrong answer		20	21.74
	Generalization	Special answer		13	14.13
		Quasi-special answer		59	64.13
		General answer		20	21.74

According to Table 2, 14.13% of MTCs defined pyramid correctly. Those who gave correct answers used all three critical characteristics (bases are polygon, lateral faces are triangle and it has an apex). For example, S_{69} defined pyramid as "*object with a base comprised of polygon, with lateral faces comprised of triangles and with an apex"* whereas S_{92} defined it as "*a three dimensional figure with a base in polygon shape, with triangle lateral faces whose upper corners unite in one point*".

However, the definitions of 64.13% of MTCs were in insufficient answer category because these definitions had one or a few critical characteristics of pyramid. Accordingly, MTCs mostly touched upon the criterion "polygon bases and having an apex" (32.61%). The answer of S_{90} could be given as an example for this, "*Pyramid is a figure whose base consists of a certain polygon which gives its name and whose lateral faces unite on same apex*". The other insufficient answers were the criterions "having an apex" (10.87%) and "triangular lateral faces and having an apex" (10.87%) that MTCs touched upon. S_{66} emphasized the apex through her definition "*It is a geometric figure whose lateral faces intersect on a point and where the line descends from these points is called height*". S_{31} defined as "*They are closed three dimensional figures whose lateral faces are triangle and united on an apex and they have a certain height and volume*" and touched upon some critical characteristics such as apex and triangle lateral faces, but does not say anything about the base. As the third insufficient answer, only six of MTCs (6.52%) touched upon the criteria "polygon bases" and "triangular lateral faces". For example, S_{50} defined pyramid as "*a geometric object whose bottom base is polygon and lateral faces are triangle*". And fourth only two of MTCs (2.17%) touched upon only the criterion "polygon bases". For example, "*Three dimensional figure whose base consists of any polygon*" definition of S_{20} could be given as an example for this category. And lastly, only one of the MTCs referred "triangular lateral faces" criterion. For example, S_7 handled this situation with expression "*They are three dimensional closed figures with triangle lateral faces*". S_7 was asked to explain what this closed figure was in interviews. S_7 answered as "*Pyramid's base consisted of different polygons like triangle, square etc. or any kind of polygon"* and touched upon lateral faces qualifications and polygon base, but still this was an insufficient answer.

The answers of 21.74% of MTCs which were evaluated under the wrong answer category, were not definitions but were general expressions which do not represent pyramid. That is to say, these MTCs did not mention about any critical characteristics of pyramid. Examples for this could be the definitions of S_{23} as "*a figure formed through uniting the corners of a plane on one point'* and S_{47} as "*three dimensional objects which have height, volume, side, corner*". Here, S_{23}'s expression "*uniting the corners of a plane*" and S_{47}'s expression "*three dimensional object having corners*" were not very clear and they were problematic daily life expressions instead of mathematical language.

According to generalization criterion on Table 2, 14.13% of MTCs could make special definitions for pyramid. All critical characteristics existed in these definitions. For example, with her definition as "*The base is a geometric figure. Their lateral faces are triangle and they unite on an apex*" S_{93} gave a special definition comprising the cone as well. For pyramid S_{68} said "*A figure formed by a base and lateral faces uniting on a point. Base is any polygon and side faces are triangle*" and S_{85} said "*A pyramid is a three dimensional figure whose all lateral faces are triangle and with an apex where all lines from the corners of a polygon are united*", and these were special definitions comprising all pyramids.

Definitions of 64.13% of MTCs were evaluated as quasi-special answers because of qualifying only pyramid and including some critical characteristics. For example, S_{61} did not mention about the polygon base in her definition as "*A pyramid is a geometric object that is named according to its base, its lateral faces are triangle and they unite on an apex*" and S_{44} did not mention about the apex in his definition as "*Pyramids are three dimensional objects which are named according to the polygon on the base and have triangle lateral faces*". Therefore, these definitions were classified as quasi-special answers. On the other hand, S_{30} defined prism as "*a structure whose bottom base consists of regular quadrilaterals and which has an apex*"; S_{41} defined it as "*a figure with a volume whose base is square, rectangular etc. and whose ceiling is just an apex*"; and S_{43} defined pyramid as "*a closed figure with a square base and identical four triangles on four sides of the square*". Here, definition of S_{30} comprises the pyramids that have bases with regular geometric figures, but do not comprise the other pyramids; the definition of S_{41} comprises the quadrilateral pyramids with quadrilateral bases formed by square or rectangular but does not comprise other pyramids; and the definition of S_{43} comprises only square pyramid but not other pyramids and so, all these are narrowing quasi-special definitions. In her interview S_{43} was asked to make drawings on pyramids and she continued her narrow understanding of square pyramid in the drawings on Figure 2.

Figure 2. Pyramid drawings of S_{43}

Definitions of 21.74% of MTCs were categorized as general answers, because of not including any critical characteristics of pyramid. For example, S_{42} defined pyramid as "*An object consisting of bottom base and lateral faces*" and S_{13} defined it as "*They are three dimensional objects with bases, volume and height*". S_{13} mentioned about the existence of base and when she was asked to define the pyramid again in interview, she explained as *"three dimensional objects having a base and an apex, volume and height"*. When she was asked to draw pyramid, she drew square pyramid, rectangular pyramid and cone in Figure 3. Although S_{13} knows the criterion that a pyramid should have an apex, she also has a misconception of assuming a cone as a pyramid.

Figure 3. Pyramid drawings of S_{13}

It can be seen here that MTCs make general definitions to comprise all objects, depending on the general characteristics "*having volume and height*" and "*formed by a base and lateral faces*" of any object but these were not critical characteristics.

Explanations of MTCs about the Differences between Prism and Pyramid Concepts

The categories for explaining the differences between prism and pyramid were investigated through correctness criteria. The explanations were classified taking into account the critical characteristics of prism and pyramid concepts as the number of bases (a), parallelism of bases (b), characteristics of lateral faces (c), and existence of an apex (d). In this context, the explations were classified under the titles including only one or more of these critical characteristics (Table 3).

Table 3. Definitions about the differences between prism and pyramid concepts

	Criteria		Theme	f	%
Explaining the difference between prism and pyramid	Correctness	Correct answer	a+ b + c + d	1	1.09
		Insufficient answer	a	16	17.39
			c	3	3.26
			d	10	10.87
			a + b	2	2.17
			a + c	5	5.44
			a + d	16	17.39
			c + d	3	3.26
			a + b + c	1	1.09
			a + c + d	17	18.48
			a + b + d	6	6.52
			a + b + c + d	2	2.17
		Wrong answer		10	10.87

According to Table 3, only one MTC could explain the differences between prism and pyramid perfectly. S_{52} explained as "*Prism has opposite, parallel and identical faces. Pyramid does not have. While the lateral faces of prism square or rectangle, they are triangles on pyramid. Pyramid has an apex, but prism does not have. All sections are identical on prism. But sections are different on pyramid and they are not identical*"; this explanation was evaluated in correct answer category because it mentions about all critical characteristics such as number of the base, parallelism of bases, quality of lateral faces and apex. S_{52} answered correctly. However, she indicated that lateral faces on prism were square or rectangular. This proves that she has a right prism understanding.

Explanations were accepted as insufficient answers because most of the MTCs (88.04%) mentioned one or a few critical characteristics of the differences between prism and pyramid in their explanations. MTCs touched upon mostly here "number of the bases, quality of lateral faces and existence of apex" (18.48%). S_8 explained the differences between prism and pyramid as "*While lateral faces of pyramid are triangle, lateral faces of prism are rectangle. Pyramid has one base and one apex*", but he did not mention about the parallelism of the bases. Another point which was attracting attention in this explanation was that MTC indicated the lateral faces were rectangle on prism. It is understood here that MTC limits the prism with right prism.

Other differences touched upon most among critical characteristics (17.39%) were "number of bases" and "number of bases and existence of the apex" as the insufficient answer. For example, S_{10} explained as *"Pyramids have only bases, but prism should have both ceiling and base"*, and so she emphasized that prism and pyramid differed from each other according to the number of bases. S_{51} explained as "*Both bottom and top bases of prisms consist of polygons, however, pyramids' bottom bases have polygon and unite on an apex, that's to say, there is no top base*", and so she indicated that prism and pyramid differ from each other through number of bases and existence of an apex.

According to Table 3, it is seen that some MTCs (10.87%) only mentioned about apex in their explanations. For example, one of the teacher candidates, S_{25}, explained this difference as *"Pyramids unite on one point, but prisms do not*". The next theme which was explained by only six of MTCs (6.52%) as the the differences between prism and pyramid were handled according to "base numbers, parallelism of bases andexistence of an apex" criteria. For example, S_{77} explained as *"The base in a prism is displaced as much as the height. During displacement the size of base polygon does not change. In pyramids while base area is displaced as much as the height, the size of the base polygon becomes smaller. At last it unites on a point."* However, five of MTCs suggested that both base number and the characterictic of lateral faces would change for prism and pyramid concepts. As an example for this the explanation of S_{83} could be given; *"On prism there are two bases, bottom and top, but pyramid has only one base. On prism side faces are rectangle but on pyramid they are triangles.*"

Only three MTCs mentioned about the characteristic of lateral faces and the existence of an apex in their explanations. S_{72} mentioned about these two critical characteristics in his explanation as "*The lateral faces of prism are rectangle but on pyramid they are in triangle, prism does not have an apex pyramid has one*". Again only three MTCs tried to explain the differences between prism and pyramid through mentioning only *the characterictic of lateral faces*. For example, S_{19} explained as "*Lateral faces of a pyramid are triangle but they are rectangle on prism*". Here what takes attention is that the prism definitions of MTCs cannot pass beyond right prism. Besides, two MTCs emphasized both base number and parallelism of bases. S_{35} explained these differences as "*While a prism has two base surfaces, pyramid has one. On prism opposite bases are in same geometric shapes. But pyramid has one base type*". Only two of MTCs mentioned about all four critical characteristics but with deficiencies. For example, S_{60} explained as *"While two bases are necessary to complete the figure after uniting lateral faces on prism, one base is enough for a pyramid. While lateral faces are triangles on pyramid, they may be square, rectangle triangle on prism. We can find the height on a pyramid through a perpendicular from apex to base, but we can find height from any area between two bases on prisms*" Here S_{60} has a misconception of triangle lateral faces for prisms but she has a right prism conception as she indicates square and rectangular lateral faces. And finally, only one MTC mentioned about base number, base parallelism and characteristic of lateral faces. For example, S_{71} said that *"Pyramid does not have a top base, and its lateral faces are triangle, but lateral faces of prism are quadrilateral. On pyramid, parallelism is not important but on prism bases must be parallel*". With this explanation he indicated three criteria and quadrilateral lateral faces of prism. Ten of the MTCs did not mention about any critical characteristics between prism and pyramid in their explanations, and so, these explanations were evaluated in wrong answer category. The following explanation of S_{43} could be an example for this theme *"Prisms are named according to their bases, but pyramids are not named so".*

Discussion

In the current study, definitions of MTCs about prism and pyramid concepts were investigated and their explanations about the differences between these two concepts were handled. Definitions of MTCs were investigated through both correctness and generalization criteria for prism and pyramid concepts. According to the correctness criterion, results showed that none of the MTCs could define prism concept correctly. Hence it can be asserted that MTCs were inadequate in defining the prism concept and determining the critical characteristics for prism as in the literature (Bozkurt & Koç, 2012).

In other words, it could be seen that MTCs' understanding of prism has emerged by moving away from formal definition and towards personal definitions. In literature, it is found that personal definitions are made more than academic definitions (Tall & Vinner, 1981; Türnüklü et al., 2013; Türnüklü & Ergin, 2016). Moving away from the academic definitions showed that critical characteristics of prism were not paid enough attention. Prism definitions of more than half of MTCs comprised one or a few critical characteristics. Therefore, these definitions were insufficient because they were necessary but insufficient structures. In the same way, Gökbulut and Ubuz (2013) also revealed that teacher candidates do not know necessary and sufficient logical principles, they cannot choose necessary sufficient characteristics of prism to define it and they cannot distinguish its critical characteristics. The most handled definition with insufficient answers for prism given by MTCs was based on the criterion "identical bases and quadrilateral or rectangular lateral faces". It is noteworthy that the vast majority of MTCs refer to the fact that the lateral faces of the prism are rectangles instead of parallelograms. This finding indicated that MTCs generally had right prism in their minds for prism concept. This right prism concept was in line with other researches with different levels of participants (Gökkurt & Soylu, 2016; Horzum & Ertekin, 2018; Türnüklü & Ergin, 2016). On the other hand, this result was in contradiction with the results of Altaylı and her colleagues' (2014) research where MTCs were asked whether the new object is a prism or not when a prism is bended. In their research 76.6% of MTCs indicated that new figure was a prism even if the angles between the corners were right and 12% suggested that the new figure was not a prism if the angle was not right. According to the findings, it is one of the original results of this study that MCTs expressed the fact that the lateral faces were quadrilaterals instead of parallograms. It may seem correct that the lateral faces are quadrilaterals. But, if these quadrilaterals are trapezoids with only two parallel sides or a deltoids, then the prism will not occur. Because then, parallelism on the bases of identicality criterion cannot be provided. For that reason, the participants contradicted themselves with their own expression of identicality of the bases. The second definition MTCs most mention was based on "identical and polygon bases" criterion. It was another finding of this research that MTCs emphasized that the base of a prism must be "regular polygon". The reason for this may be that MTCs have been studying with concrete prism models having regular polygon bases since primary school. For example, showing the models like right prism whose base is regular polygon to students as example may be a reason for the conceptions of regular polygon base and rectangular lateral faces.

Therefore, it can be suggested that various samples and models should be used in geometry education to emphasize critical characteristics (Hershkowitz, 1989) and besides, the samples for geometric object, non-sample situations should also be given (Oberdorf & Taylor-Cox, 1999). Also it can be effective that students should study with skeleton models related to geometric objects (Çetin et al., 2012). As a result, definitions of MTCs for prism were formed from most to the least by "identical bases", "polygon bases", "quadrilateral or rectangular lateral faces" and "parallel bases" critical characteristics respectively. This proves that MTCs can decide whether an object is a prism or not by looking at its bases. This situation may be a reflection of the expression "objects are given names according to their bases" which is used in geometry lessons by teachers. Nearly half of MTCs tried to define prism without referring to any critical characteristics of prism and so they were placed in wrong answer category. As a result of this category, it is suggested that MTCs have misconceptions or they express general characteristics belonging to many geometric objects. For example, misconceptions such as "three dimensional geometric figures consisted of width, depth and height", "closed, three dimensional figure having surface, side and formed through united plane geometric figures" were often met in the answers of MTCs. These expressions can be valid for many three dimensional objects. But as Bozkurt and Koç (2012) asserts when prism is defined as a three-dimensional object, a very wide cluster is emphasized and beside of the prism, other objects like pyramid, cone are also included. The expression "every three-dimensional object having volume is prism' of MTCs which can be found in literature (Alkış-Küçükaydın & Gökbulut, 2013; Çakmak et al., 2014; Gökbulut & Ubuz, 2013; Gökkurt & Soylu, 2016) and does not have any critical characteristics is also a misconception, because this definition is valid for other three-dimensional objects as well. Also some general expressions like "geometric figures being named according to their base shapes" are found. Actually, prisms and pyramids are named according to their bases. However, this expression does not have any critical characteristics for definition of prism. It can be asserted here that the participants have not conceptually acquired the prism concept. So, as Linchevsky, Vinner and Karsenty's (1992) suggests, this situation may prevent participants from making correct definitions. Within this context, MTCs can be given education about how to build environments where they can teach geometric concepts and definitions through using different materials.

According to the generalization criterion of prism definitions, it was seen that none of MTCs could give a special definition of prism. However, more than half of the MTCs could give definitions which qualify prism and comprise some critical characteristics. It was determined that MTCs gave concept narrowing definitions which comprised right prism and did not comprise other prisms and also, they gave definitions which comprised cylinder and similar objects but did not comprise all objects. Nearly half of MTCs gave definitions which can be categorized as general answer without any critical characteristics and having expressions such as "geometric figures having a certain height and volume". These findings were parallel to Gökbulut and Ubuz's (2013) investigation which determines that MTCs cannot give any special definition about prism concept. According to Gökbulut and Ubuz, when definitions of four MTCs about prism were investigated from generalization point of view, one MTC gave a general definition based on "having volume and height" characteristics comprising all objects; two MTCs gave quasi-special answer comprising objects with identical bases and one MTC tried to find a definition through daily-life usage samples and used the most general expressions. This can be because MTCs remember concept structures in their minds and the samples they had seen before and through the samples they tried to give definition. It was also thought that textbooks and lessons play important roles in formation of definitions for MTCs. Therefore, the expressions used by teachers in teaching geometric concepts and provided in textbooks should be given attention (Horzum & Ertekin, 2018). So, it is very crucial to design proper learning environments where MTCs can learn geometric concepts and definitions correctly and where they can express themselves. Besides, some learning activities where especially critical characteristics are emphasized can be organized.

When correctness criterion was taken into consideration for pyramid, it was determined that some MTCs (14.13%) could define pyramid correctly. In these definitions MTCs mentioned about polygon bases, triangle lateral faces and the existence of an apex. Çakmak and her colleagues (2014) also found out that only 12% of MTCs gave sufficient definitions. On the other hand, most definitions of MTCs were insufficient answers as they comprised one or a few critical characteristics. The definition of MTCs with insufficient answers was based on the criteria "polygon bases and the existence of an apex". Contrary to this finding in literature, it was determined that MTCs did not know the base of a pyramid must be polygon (Çakmak et al., 2014; Ubuz & Gökbulut, 2015) but they have a misconception as "base must be triangle and square" (Alkış-Küçükaydın & Gökbulut, 2013; Gökkurt, Şahin, Soylu et al., 2015; Marchis, 2012; Türnüklü & Ergin, 2016; Ubuz & Gökbulut, 2015). The insufficient answers most mentioned by MTCs were the criteria of "the existence of an apex" and "triangular lateral faces and the existence of an apex". Similarly, Türnüklü and Ergin (2016) indicate that 8th grade students define the pyramid through personal expressions such as "perpendicular", "ascending", "pointed" to touch upon critical characteristics such as lateral faces being triangular areas and the existence of an apex. As a result, the definitions of MTCs on pyramid were shaped from the most to the least by critical characteristics “existence of an apex”, “polygon base” and “triangular lateral faces” respectively. This showed that MTCs first look for an apex and then, look at the bases of objects as for prism to determine whether they were pyramids or not. The reason for this situation may be the fact that MTCs met prototype samples of pyramid concept from their early lives and critical characteristics were not emphasized enough in their geometry classes. Almost on fifth of MTCs try to define pyramid concept through indicating any critical characteristics and define it as “a three dimensional object with height, volume, side, edge” and so these were in wrong answer category. According to this, it is concluded that there are misconceptions as a pyramid has general characteristics like other geometric objects. The same results can be found in the study of Altaylı and her colleagues (2014). In order to eliminate misconceptions, individuals should think upon the definition and make drawings regarding the critical characteristics of the definition while they are taught geometry.

In addition, according to the generalization criterion of pyramid, it was determined that only a part of MTCs could give a special answer which has all critical characteristics of a pyramid and emphasized a pyramid may be a cone. Most MTCs gave definitions which define just pyramid and have some critical characteristics and so, these were evaluated in quasi-special answer category. It was determined here that MTCs gave narrowing definitions comprising the pyramids whose bases are only square or only rectangle (Alkış-Küçükaydın & Gökbulut, 2013; Türnüklü & Ergin, 2016; Ubuz & Gökbulut, 2015) or regular geometric figures (Ubuz & Gökbulut, 2015), but not other pyramids as in the literature. Although one fifth of MTCs gave definitions with any critical characteristics which can be general answers. It was also seen here that MTCs gave general definitions which comprise all objects and is a characteristics of pyramid but not a critical one as “having volume and height” and “formed by a base and lateral faces”.

Only one of the MTCs could explain the differences between prism and pyramid. However, she limited the prism with right prism as indicating square or rectangular lateral faces of prism. A big part of the MTCs mentioned about one or a few critical characteristics about the differences between prism and pyramid. It is worthy to note here that the prism definition of MTCs cannot pass beyond right prism too. Also they mentioned about triangle, square and rectangle bases. This shows that they have a narrow definition about the characteristic of the base. On the other hand, one tenth of MTCs did not mention about any critical characteristics related to the differences between prism and pyramid.

In general, it is concluded that MTCs are unsuccessful in defining prism and pyramid concepts and have difficulty in expressing the differences between these two concepts. Therefore, it was determined that MTCs had insufficient subject matter knowledge. However MTCs should possess sufficient subject matter knowledge. According to the Zaskis and Leikin (2008) definitions of mathematical concepts, the structures under these definitions and definition processes are the fundamental components of subject matter knowledge of mathematics teachers. Because many teaching activities during teaching process such as teacher’s selecting proper teaching activities, asking productive questions depend on teachers’ having sufficient content knowledge (Ball & McDiarmid, 1990). Besides, the teachers’ deficiencies in subject matter knowledge affect students’ learning negatively. For that reasons MTCs should deeply know the concept definitions they are going to teach (Türnüklü, 2005).

In addition, expression of definitions are important to teach concepts academically (Türnüklü & Ergin, 2016). Another salient point in definitions of MTCs is that they cannot use mathematical language effectively as in the study of Tsamir and her colleagues (2015). Effective usage of mathematical language should be paid attention in geometry teaching from childhood. Oberdorf and Taylor-Cox (1999) indicate that the characteristics of a concept should be emphasized, sample and non-sample situations should be presented to individuals and language usage should be paid attention in geometry teaching. Regarding the studies about geometric concepts of different education levels in literature have similar results, as Bozkurt and Koç (2012) also indicate, students at all school levels should be given opportunities to express concept definitions they learn orally and written in order to use mathematical language and express the concepts.

Recommendations

Although the current study includes limited number of MTCs, it includes ideas about the structures related to the prism and pyramid, and the differences between them in the minds of MTCs. Similar structures and misconceptions can be possible for other individuals as well (Türnüklü & Ergin, 2016), and therefore, the current study may direct teachers about the problems which can occur during teaching process and about how to prevent them. In this context, concrete models and computer software programs, such as Geogebra, can be used to improve subject matter knowledge of MTCs about the prism and pyramid concepts. According to this findings, it is possible to conduct researches in which learning environments are designed to improve the knowledge of MTCs and their progress is assessed. In addition in the highlights of this research, elective courses about basic geometric concepts and their definitions can be given to MTCs. Besides, investigations to develop mathematical language skills about prism and pyramid of MTCs can be suggested. Furthermore, in order to overcome the deficiencies of the MTCs' concept definitions, action researches can be designed.

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Author Information

Melihan Ünlü
Aksaray University
Faculty of Education
Turkey
Contact e-mail: *melihanunlu@yahoo.com*

Tuğba Horzum
Necmettin Erbakan University
Ereğli Faculty of Education
Turkey