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Problem-Based Learning Associated by Action-Process-Object-Schema (APOS) Theory to Enhance Students' High Order Mathematical Thinking Ability

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Abstract

The research has shown a model of learning activities that can be used to stimulate reflective abstraction in students. Reflective abstraction as a method of constructing knowledge in the Action-Process-Object-Schema theory, and is expected to occur when students are in learning activities, will be able to encourage students to make the process of formation of new mental objects, new processes and new schemes through the construction process in the form of generalization, interiorization, encapsulation, coordination and reversal. Problem-based learning that is presented through eight steps of learning has been able to enhance the mental action in students even though there is no doubt that it cannot possibly know the whole picture of a person's mental activity. All steps in the problem-based learning approach can reflect on the problems of mental action in students. Problem-based learning is appropriate to be used to improve students' high order mathematical thinking ability because of it has been able to condition the reflective abstraction related mental actions, mental processes, mental objects and schemes in students. Computer assistance and scaffolding techniques can be further stimulus for students to take place in their mental action which corresponded to expectations.

Key words: Problem-based learning; APOS theory; High order mathematical thinking

Introduction

Learning mathematics in the view of recent development of mathematics education must create processes and goals that lead to the achievement of competencies that make students able to make a conjecture, to communicate, to solve problems, reasoning logically and having positive attitude towards mathematics. This is because of learning mathematics is not only learning it as a fixed and unchanging collection of facts and skills, but it must be an emphasis on the importance of conjecturing, communicating, problem solving and logical reasoning (Conway & Sloane, 2005). High School Subjects' Mathematics Standards Content Document (Depdiknas, 2006: 388) reinforces this view of mathematics by stating that mathematics intended that learners have the ability to understand mathematical concepts, using reasoning the pattern and nature, solving problems, communicating ideas, and have respect for the usefulness of mathematics in life. This assertion is related to senior high schools students' ability of high order level mathematical thinking ability and respect for the usefulness of mathematics known as mathematical disposition after learning of mathematics in schools and termed as competency standards. Since it is considered as a standard, then the whole effort is worth the ability to be shared by all students after they completed their education at high school level.

Resnick (Mc Curry, 2005: 2) suggests the characteristics of high-order thinking as thinking and non-algorithmic complex that includes:

a. many solutions;
b. nuanced decide and interpret;
c. application of the employed several criteria;
d. lack of stipulations;
e. self-regulation of the process of thinking;
f. determination of meaning, the discovery of the lack of uniform structure, and business.

Mathematical problem solving is a part of high-order mathematical thinking ability. Resnick [2] suggests the characteristics of high-order thinking as thinking and non-algorithmic complex that includes:

a) multiple solutions;
b) nuanced judgment and interpretation;

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c) application of the employed multiple criteria;
d) often involves uncertainty;
e) self-regulation of the thinking process;
f) Involve imposing meaning.

Activities that classified to the mathematical problem solving according to Sumarmo (1993) include:
a) Declare a situation, figure, diagram, or tangible objects into the language, symbols, ideas, or mathematical models
b) Explain the idea, situation, and relations math orally or in writing.
c) Listening, discussing, and writing about mathematics.
d) To read with understanding a written mathematical representations.
e) Disclose back one paragraph description or math in own language.

Mathematical Communication ability is also a part of high-order mathematical thinking ability. This ability can be classified at the high-order mathematical thinking ability but could also be classified in the low-order mathematical thinking ability, depending on the complexity of communication involved. Mathematical Communication according to NCTM (2000) is an important part of mathematics and mathematics education and a way of sharing ideas and clarifying understanding. When students are challenged to think about mathematics and communicating it to others either orally or in writing, they learn to be a person who can explain and convince others. Given this mathematical communication, students will be able to listen to the explanations of others so that give them the opportunity to develop their understanding. Efforts to share ideas and clarify mutual understanding can occur in mathematics classroom when the discussion is in progress both in the classical and in groups.

The students involved in the discussion according to Hatano and Inagaki (NCTM, 2000) will gain a better understanding of mathematics when trying to explain their different point of view to convince their group discussion. Activities that involve mathematical communication skills thereby: helping students to develop a language that expresses mathematical ideas and an appreciation of the need for precision in the use of language. The students who have the opportunity, encouragement, and support for speaking, writing, reading and listening in mathematics classes will gain a double advantage in the form of communicating to learn mathematics and learning mathematics to obtain mathematical communication skills.

Communication skills of high school students in mathematics at NCTM (2000) must appear in the form of the ability to construct a logical sequence of thought, express themselves in a clear and reasonable, to hear others ideas, and think about the people who pay attention to writings or their words. Senior high school students must be capable of being a good critic on the opinion of others and himself. Thus, high school students should be able to build a variety of explanations, formulate various questions, and write a variety of arguments that can be logically correct and reasonable by the teachers, their colleagues or the mathematicians. All of that can be done if the students have the ability to use language and mathematical symbols and correct both when communicating it using descriptions, geometric diagrams, ordinary language or the algebraic symbols. Students also must be able to work just as well in the sense that it can work effectively with colleagues.

Learning mathematics in accordance with the present views about mathematics have to pay attention to students' learning processes through the construction of the knowledge that they are doing by their own based on their cognitive development. Piaget's ideas about learning by means of cognitive development have developed by von Glaserfeld (1995) as a view of constructivism, a knowledge philosophy which emphasis knowledge that one acquire is a construction from him or herself and known as radical constructivism. This view of learning formulated its fundamental principles as follows:
1. Knowledge is not passively received either through the sense or by way of problem solving;
2. Knowledge is actively built up by the cognizing subject.
3. The function is adaptive, in the biological sense of the term, tending towards fit or viability;
4. Cognition serves the subject's organization of the experiential world, not the discovery of an objective ontological reality

Related to the constructivist view of learning mathematics, Meel (2003) states that construction of knowledge as a result of the understanding of mathematical concepts developed through the formation of mental objects and the association between it. Thus, when constructivism learning theory will be applied in mathematical learning, it will be needed to pay attention to the meaning of the process of formation of new knowledge and its main elements to support the learning process that is expected. Theories about constructing new mathematical knowledge that is part of the constructivist view begins with what Piaget stated (Dubinsky, 2002) about the process of reflective abstraction which are part of the three types of abstraction processes as well as empirical abstraction and pseudo-empirical abstraction. Reflective abstraction as a method of knowledge construction is the core theory of APOS (Action - Process - Object - Schema) from Dubinsky. According to Dubinsky (Dubinsky, E & Tall, D., 2002) five types of important Piaget's construction process explain how the new objects, new processes and new schemes can be constructed in an effort to develop an abstract mathematical concepts consist of: generalization, interiorization, encapsulation, coordination and reversal. A Person's response to a situation that allows the emergence of a mathematical concept through the construction of a mental process in his thoughts related to the concept is the meaning of interiorization.

The composition of two or more processes to construct a new process called coordination. Psychological description given by Ayres and Dubinsky (Dubinsky, 2002) pointed to the formation of the composition of two functions. The composition of function is a binary operation that has meaning to take action on the two objects to form a third object. Two functions are considered as two objects must be "disassembled" by a student to be reflected on the related processes and being interiorized. Construction stage which may be the most important and most difficult math for students called to convert a dynamic process into a static object and is referred to as encapsulation. When a student learns to apply an existing scheme on a wider set of phenomena, then we say that the scheme has been generalized. Meanwhile, when a process appears internally, allowing for students to think about in the opposite form, and can be interpreted as constructing a new process using the reverse process of the origin, then the ability to this kind of thinking is called a reversal.

In an effort to try to develop this idea of reflective abstraction, Dubinsky (2002) isolated what appears to be a significant feature of these ideas, reflect on their role in high-level mathematics, and reorganize or reconstruct to form a theory related to mathematical knowledge and its construction. Reflective Abstraction by Dubinsky will be the construction of mental objects and mental actions on these objects. In an attempt to elaborate his theory and relate to certain concepts in mathematics, Dubinsky then uses the idea of the schema. A schema is a collection of objects and processes that can be highly inter-related or less related. The tendency of students to involve a schema in an effort to understand, treat, organize, or interpret a problem situation has been estimated as a conceptual knowledge of mathematics itself. Thus a mathematical concept will have a unity of some vast scheme. There will be some schema for situations that include numbers, arithmetic, form the set, function, proposition, calculation, verification by mathematical induction, and so on through the whole of one's mathematical knowledge. These schemas should be completely intertwined in a large number of complex organizations.

Mathematics teachers do not only have to teach their students how to solve problems, but to learn mathematics through problem solving too. When many students develop procedural fluency, they often lack understanding of the concepts needed to solve new problems or make connections between mathematical ideas. This presents a challenge for teachers, and Problem-Based Learning (PBL) provides opportunities for teachers to face this challenge. PBL emerged as a teaching approach based on constructivism learning and the ideal student-centered learning. When using PBL, teachers help students to focus on solving problems with a real life context, encouraging them to consider the situation that posed by a problem when trying to find solutions.
Computer-assisted instruction with regard to teaching or learning presented by means of a computer device. Computer programs are interactive and can illustrate a concept through animation, sound and attractive demonstrations. The programs provide flexibility for students to develop through their own pace and work individually or in groups to solve problems. Computers can provide immediate feedback that allows students to know their correct answers. If the answer is incorrect the program shows students how to answer the question correctly. Computers offer a different type of activity and a change of pace through the guidance of a teacher or teaching groups.

MathXpert (Beeson, M., 2003) is a system that allows a person to do mathematics on a computer screen in much the same way as it is done with pencil and paper, but with some important differences:
(1) It is not possible to make a mistake.
(2) If you do not know what to do, MathXpert can help you.

These features of MathXpert should be compared to the two main difficulties that people experience in their attempts to learn mathematics: you have to be very careful in mathematics, because a slight error can throw you completely off track; and mathematics is cumulative, so you must master each part of the subject before moving on to the next part. Benefits of using mathxpert software are happens because this program can help show all steps; its computer algebra is true: it can serve as a teacher, it draws the graph correctly, and can easily make the graphs students need to see.

Computer-assisted problem-based learning (CAPBL) by using MathXpert software program can be expected to maximize the benefits of Problem Based Learning. The goals of mathematics learning which allow students to have an important role in the process of learning mathematics and mathematical abilities obtained in accordance with that goals, are expected to be more easily realized. Another important thing to note by the teacher is an attempt to make the appropriate teaching materials through the use of CAPBL.

Method

The study used a 3 × 2 and 3 × 3 factorial designs and each were varying in learning approaches and student settings (3 × 3 factorial design for students' prior mathematical ability (PMA) and 3 × 2 factorial design for school category or cluster). The two dichotomous variables in 3 × 3 factorial design generated six experimental groups: (1) CAPBL in high level of PMA; (2) CAPBL in medium level of PMA; (3) CAPBL in low level of PMA; (4) PBL in high level of PMA; (5) PBL in medium level of PMA; (6) PBL in low level of PMA; and generated three control groups: (1) conventional learning in high level of PMA; (2) conventional learning in medium level of PMA; and (3) conventional learning in low level of PMA. The two dichotomous variables in 3 × 2 factorial design generated four experimental groups: (1) CAPBL in high category school; (2) CAPBL in middle category school; (3) PBL in high category school; (4) PBL in middle category school; and generated two control groups: (1) conventional learning in high category school; and (2) conventional learning in middle category school.

The participants were 209 Year 11 students in six mathematics classes of two Indonesian senior high schools with two categories. Each school category consisted of 3 classes students. 3 classes in high category school consisted 1 class for CAPBL setting (31 students); 1 class for PBL setting (34 students); and 1 class for conventional setting (34 students). 3 classes in medium category school consisted 1 class for CAPBL setting (38 students); 1 class for PBL setting (36 students); and 1 class for conventional setting (36 students). The students had graduated from junior high schools that used a National Curriculum, and therefore, it can be assumed that all students had fairly similar mathematical experiences. The participant school used a competency-based curriculum (Depdiknas, 2006) that was part of the Indonesian National Curriculum for senior high schools. Mathematical problem solving ability developed through small group learning, such as the ability to demonstrate and interpret mathematical ideas in written or oral form, were one of the outcomes. At the time the research was carried out, students had been taught by the same mathematics teacher.

Results and Discussion

Most of all of the research results are the inferences of statistical test. Statistical result of mathematical problem solving is on the Table 1, Table 2, and Table 3. The tables show that learning factor tends to give higher mathematical problem solving achievement and gain than school category and PMA.

Table 1. CAPBL students' mathematical problem solving according to learning approach, school categories and PMA

School Category	PMA	CAPBL					n
		Pre Test		Post Test		<g>	
		r	s	**r**	s	**r**	
High	High	8.17	1.94	47.17	4.07	0.71	6
	Medium	7.43	2.11	36.35	4.13	0.58	23
	Low	5.00	4.18	31.80	2.95	0.46	5
	Sub Total	**7.21**	**2.57**	**37.59**	**6.15**	**0.55**	**34**
Middle	High	7.67	3.06	34.00	8.19	0.47	3
	Medium	4.71	3.53	35.47	6.75	0.53	17
	Low	7.89	2.32	29.67	6.65	0.40	18
	Sub Total	**6.45**	**3.30**	**32.61**	**7.19**	**0.46**	**38**
Total	High	8.00	2.18	42.78	8.39	0.63	9
	Medium	6.28	3.08	35.98	5.34	0.52	40
	Low	7.26	2.97	30.13	6.05	0.41	23
	Total	**6.81**	**2.`98**	**34.96**	**7.13**	**0.50**	**72**

Max. Score = 68

<g> = Normalized gain = $\frac{Post\,test - pretest}{Maximum\ Score - pretest}$

CAPBL: Computer-Assisted Problem Based Learning Approach

Learning factor has gave higher mathematical problem solving achievement and gain than school category because of medium category school of CAPBL setting get similar post test score and gain (32.61 and 0.46) than high category school of PBL (32.00 and 0.45) and get higher than conventional setting (27.09 and 0.35). Learning factor has gave higher mathematical problem solving achievement and gain than PMA level because of medium level of PMA from students that acquire CAPBL setting get similar post test score and gain (35.98 and 0.52) than high level of PMA from PBL (35.00 and 0.49) and get higher post test score and gain from conventional setting (27.28 and 0.36).

Table 2. PBL students' mathematical problem solving according to learning approach, school categories and PMA

School Category	**PMA**	**PBL**					N
		Pre Test		Post Test		<g>	
		r	s	**r**	s	**r**	
High	High	6.75	1.89	38.75	2.22	0.57	4
	Medium	7.47	2.37	32.94	5.46	0.46	17
	Low	6.50	3.06	27.70	4.92	0.38	10
	Sub Total	**7.06**	**2.53**	**32.00**	**6.03**	**0.45**	**31**
Middle	High	8.00	6.93	30.00	4.35	0.39	3
	Medium	8.06	3.42	32.38	6.57	0.44	16
	Low	6.71	2.89	22.53	3.59	0.24	17
	Sub Total	**7.42**	**3.47**	**27.53**	**6.99**	**0.36**	**36**
Total	High	7.29	4.27	35.00	5.54	0.49	7
	Medium	7.74	2.89	32.67	5.94	0.45	33
	Low	6.63	2.87	24.17	9.97	0.31	27
	Total	**7.25**	**3.05**	**29.60**	**6.89**	**0.40**	**67**

Max. Score = 68

<g> = Normalized gain = $\frac{Post\,test - pretest}{Maximum\ Score - pretest}$

PBL = Problem-Based Learning Approach

According to learning factor, the enhancement and the achievement of students' mathematical problem solving that acquire learning in CAPBL setting tend to get higher than the students that acquire PBL and conventional learning. So it is with PBL setting tends to get higher than the students that acquire conventional learning.

Table 3. Conventional students' mathematical problem solving according to learning approach, school categories, and PMA

School Category	**PMA**	**Conventional**					n
		Pre Test		Post Test		<g>	
		r	s	**r**	s	**r**	
High	High	9.40	2.85	27.09	4.77	0.35	5
	Medium	8.00	2.24	28.63	6.10	0.37	19
	Low	5.00	2.54	22.50	4.33	0.30	10
	Sub Total	**7.32**	**2.85**	**27.09**	**6.13**	**0.35**	**34**
Middle	High	11.50	3.53	30.50	0.71	0.37	2
	Medium	7.85	3.08	19.65	5.73	0.21	26
	Low	6.13	2.8	19.75	3.61	0.24	8
	Sub Total	**7.67**	**3.18**	**20.28**	**5.70**	**0.23**	**36**
Total	High	10.00	2.99	27.28	6.01	0.36	7
	Medium	7.91	2.73	23.44	7.35	0.28	45
	Low	5.50	2.64	21.28	4.16	0.27	18
	Total	**7.50**	**3.01**	**23.59**	**6.80**	**0.29**	**70**

Max. Score = 68

<g> = Normalized gain = $\frac{Post\,test - pretest}{Maximum\ Score - pretest}$

According to school category, the enhancement and the achievement of students' mathematical problem solving whose acquire CAPBL from high category school tend to get higher than the students whose acquire CAPBL from medium school category. So it is with PBL setting and conventional setting. According to PMA, the enhancement and the achievement of students' mathematical problem solving whose acquire CAPBL from high level tend to get higher than the students whose acquire CAPBL from medium and low level of PMA. So it is with PBL setting and conventional setting. Statistical result of mathematical communication ability is on the Table 4, Table 5, and Table 6. The Tables show that learning factor tends to give higher mathematical communication achievement and gain than school category and PMA.

Table4. CAPBL Students' Mathematical Communication According To Learning Approach, School Categories, And PMA

School Category	PMA	CAPBL					n
		Pre Test		Post Test		<g>	
		r	s	**r**	s	**r**	
High	High	16.00	2.53	56.67	3.88	0.69	6
	Medium	13.39	3.54	48.87	4.54	0.57	23
	Low	12.20	4.32	40.00	3.54	0.44	5
	Sub Total	**13.68**	**3.60**	**48.94**	**6.36**	**0.57**	**34**
Middle	High	13.33	4.62	46.00	8.19	0.53	3
	Medium	12.18	5.95	45.71	7.88	0.53	17
	Low	12.78	3.61	43.06	6.07	0.49	18
	Sub Total	**12.55**	**4.75**	**44.47**	**7.02**	**0.51**	**38**
Total	High	15.11	3.33	53.11	7.39	0.64	9
	Medium	12.88	4.69	47.53	6.29	0.56	40
	Low	12.65	3.68	42.39	5.69	0.48	23
	Total	**13.08**	**4.26**	**46.58**	**7.04**	**0.54**	**72**

Max. Score = 75

<g> = Normalized gain = $\frac{Post\,test - pretest}{Maximum\ Score - pretest}$

CAPBL: Computer-Assisted Problem Based Learning Approach

Learning factor has given higher mathematical communication achievement and gain than school category because of medium category school of CAPBL setting get higher post test score and gain (44.47 and 0.51) than high category school of PBL (42.84 and 0.47) and conventional setting (36.62 and 0.36). Learning factor has given higher mathematical communication achievement and gain than PMA level because of medium level of PMA from students that acquire CAPBL setting get higher post test score and similar gain (47.53 and 0.56) than high level of PMA from PBL (42.57 and 0.57) and get higher post test score and gain from conventional setting (43.86 and 0.21).

Table5. PBL Students' Mathematical Communication According to Learning Approach, School Categories, And PMA

School Category	PMA	PBL					N
		Pre Test		Post Test		<g>	
		r	s	**r**	s	**r**	
High	High	14.25	0.96	48.25	5.50	0.56	4
	Medium	15.00	4.53	42.82	6.14	0.47	17
	Low	13.90	3.84	40.70	4.64	0.43	10
	Sub Total	**14.55**	**3.97**	**42.84**	**5.92**	**0.47**	**31**
Middle	High	7.33	4.04	48.33	4.73	0.60	3
	Medium	12.94	4.14	43.50	5.13	0.49	16
	Low	14.24	3.80	34.18	4.90	0.32	17
	Sub Total	**13.08**	**4.29**	**39.50**	**7.16**	**0.42**	**36**
Total	High	11.29	4.42	42.57	12.83	0.57	7
	Medium	14.00	4.40	34.70	9.40	0.45	33
	Low	14.11	3.75	16.85	4.75	0.20	27
	Total	**13.76**	**4.17**	**41.04**	**6.78**	**0.44**	**67**

Max. Score = 75

<g> = Normalized gain = $\frac{Post\,test - pretest}{Maximum\ Score - pretest}$

PBL = Problem-Based Learning Approach

Table6. Conventional Students' Mathematical Communication According to Learning Approach, School Categories, And PMA

School Category	PMA	Conventional					n
		Pre Test		Post Test		<g>	
		r	s	**r**	s	**r**	
High	High	15.60	2.97	47.80	10.11	0.54	5
	Medium	15.00	4.18	36.05	3.63	0.35	19
	Low	15.00	3.20	32.10	3.75	0.28	10
	Sub Total	**15.09**	**3.66**	**36.62**	**6.98**	**0.36**	**34**
Middle	High	21.50	0.707	34.00	1.41	0.23	2
	Medium	12.73	3.157	27.08	6.324	0.23	26
	Low	12.25	2.188	24.38	4.984	0.19	8
	Sub Total	**13.11**	3.52	**26.86**	6.16	**0.22**	**36**
Total	High	17.29	3.77	43.86	10.67	0.21	7
	Medium	13.69	3.75	30.87	6.94	0.11	45
	Low	13.78	3.06	28.67	5.77	0.09	18
	Total	**14.07**	**3.70**	**31.60**	**8.17**	**0.13**	**70**

Max. Score = 75

<g> = Normalized gain = $\frac{Post\,test - pretest}{Maximum\ Score - pretest}$

According to learning factor, the enhancement and the achievement of students' mathematical communication that acquire learning in CAPBL setting tend to get higher than the students that acquire PBL and conventional learning. So it is with PBL setting tends to get higher than the students that acquire conventional learning. According to school category, the enhancement and the achievement of students' mathematical communication whose acquire CAPBL from high category school tend to get higher than the students whose acquire CAPBL from medium school category. So it is with PBL setting and conventional setting. According to PMA, the enhancement and the achievement of students' mathematical communication whose acquire CAPBL from high level tend to get higher than the students whose acquire CAPBL from medium and low level of PMA. So it is with PBL setting and conventional setting.

Conclusion

This study contrasted the influence of a learning approach in either problem-based learning (CAPBL and PBL) or conventional settings. Four experimental groups and two control groups were formed varying in learning approach (CAPBL, PBL or conventional) and student setting (School categories and students' PMA) during acquisition of multi-step calculus concepts (limit and differential). All students were then tested individually on problems that were very similar in nature to those provided during acquisition. Both mathematical problem solving and mathematical communication were assessed during these two tests.

The results based on mathematical problem solving ability test in both the experimental class of school categories have been able to show that students mathematical problem solving from high category school are higher than students in experiments middle school category. Meanwhile, students of the high PMA experimental class have been able to demonstrate higher mathematical problem solving than students from the medium and low PMA experimental class. Similarly, students of the medium PMA experimental class were able to demonstrate higher mathematical problem solving than students from low PMA experimental class. This indicates that problem-based learning with all supporting components for facilitating students can improve mathematical problem solving skills in accordance with the mathematical potential with respect to both categories of schools and PMA. Based on the results of mathematical problem solving test (MPSA Test), the students who received problem-based learning without and with computer assisted instruction show improved mathematical problem solving significantly better than students who received conventional learning. This suggests that, in the process of problem-based learning both computer-aided and unaided computer with all supporting components have contributed to the improvement of students' mathematical problem solving.

Identify the adequacy of the data for problem solving is one of the indicators of the mathematical problem-solving abilities are enhanced in this study. Problem-based learning process that places more emphasis on the activities of students in constructing their own mathematical knowledge through problem solving activities that provided by the teacher has been able to make the students accustomed to identifying the problem. Identification data through problem solving in groups to train the students to learn to challenge the adequacy of the data needed to solve the problem. Computer-assisted experimental class students have more opportunities to perform activities related to the ability of these indicators for the identification of the adequacy of the data have been trained in problem solving is better than the experimental class students without computer assistance. The information obtained by students through the existing facilities at MathXpert program makes students feel more independent and this is quite a positive influence in efforts to solve the problems they confront. Meanwhile, students in the control class have less opportunity to practice identifying the adequacy of the data needed to solve the problem because all of the proposed solutions are available or have been given by the teacher.

The results based on mathematical communication ability test in both the experimental class of school categories have been able to show that students mathematical communication ability from high category school are higher than students in experiments middle school category. Meanwhile, students of the high PMA experimental class have been able to demonstrate higher mathematical communication ability than students from the medium and low PMA experimental class. Similarly, students of the medium PMA experimental class were able to demonstrate higher mathematical communication ability than students from low PMA experimental class. This indicates that problem-based learning with all supporting components for facilitating students can improve mathematical communication skills in accordance with the mathematical potential with respect to both categories of schools and PMA. Based on the results of Mathematical Communication Ability Test (MCA Test), the students who received problem-based learning without and with computer-assisted instruction show improved mathematical communication significantly better than students who received conventional learning. This suggests that, in the process of problem-based learning both computer-aided and unaided computer with all supporting components have contributed to the improvement of students' mathematical communication ability.

The ability of students to express a situation, drawing, diagram, or tangible objects into the language, symbols, ideas, or mathematical models is the ability related to mathematical concepts relevant knowledge and true-owned to be used according to the needs in this ability. In terms of ability, the second class of experiments through learning activities that emphasize problem solving by the teacher, students have the flexibility to better understand the concept of the teacher. These activities can make students have higher ability to declare a situation into the language, symbols, ideas, or mathematical models. Especially for computer-assisted experimental class, students are better trained to be able to have the skills mentioned above. MathXpert Program used as a tool in learning the information obtained independently and students need to be able to declare a situation into the language, symbols, ideas or mathematical models. The students as well as to test the accuracy of the mathematical models they use to help MathXpert program tailored to the problems they face. Meanwhile, the control class students' ability to express a situation is not so well trained in conventional classroom learning because despite the discussion in the face of problems, but done in the classical style that students tend to immediately get an answer.

The ability of students to be able to listen, discuss, and write about mathematics can also be obtained during the learning process in the classroom experiment. The learning process conducted by the students through a discussion of each small group of students by challenging problems given on the student worksheet, make the students accustomed to doing things related to this capability. On a class of computer-aided experiments, the ability of students to be able to listen, discuss, and write about math as well as obtained through student worksheets are also available from the computer display MathXpert program that stimulates the curiosity of students. All of it is less found in the control class because the student has the opportunity to discuss at the time the teacher asked a question in the classical style. The problems submitted by teachers also tend to problems directly lead to the concept being taught.

Based on the above, it is concluded that the cause of the increase in mathematical communication ability of students in the experimental class better than students in the control class is differences used of learning approach in the experimental class and control class. Similarly, the advantage of computer-assisted problem-based learning compared to the problem without the help of computers in improving students' mathematical communication skills more on the wider opportunities provided MathXpert program to provide independence in doing the student worksheet. However, an increase in the better class of experimental than control class does not mean that students in the experimental class have mastered well all the components of mathematical communication skills.

The mean score of students’ posttest mathematical communication ability in the class computer-assisted experimentation, namely 46.58 and the class of experiments without the aid of a computer is 41.04 out of the ideal score 75 shows that this ability is less than optimal. Although the implementation of this is done with the time for half the semester, it seems that the time frame was too little compared to the custom of students over the years following the conventional learning since they were sitting on a bench elementary school through high school class. This leads to the difficulty of providing adjustments to the students. Adjustment is one of them is in their ability to demonstrate higher-order thinking, because the problem-based learning students' willingness to do the work that requires a high level of mathematical thinking skills are the main assets in constructing knowledge. This is consistent with the character-based constructivist learning which requires the construction of knowledge by students themselves.

Creating a mathematical model of a situation or daily life problem and solve it is also an indicator of the ability of solving mathematical problems which seek improved in this study. Serving teaching materials that emphasize the experimental class that each student in each group to discuss issues related to daily life problem as a mathematical problem can train students' ability to create mathematical models of the problems that confront. On a class of computer-assisted experiments, presentation of teaching materials that include a duty to utilize MathXpert program can help students to examine the accuracy and relevance of mathematical models they have acquired to the problems they are facing. Meanwhile, students in the control class have less opportunity to practice making mathematical models needed to solve the problem because all of them are presented by the teacher at the beginning of the concept being taught.

Selecting and implementing a strategy to solve mathematical problems and or outside of mathematics is one more indicator of the ability of solving mathematical problems that are the focus to be improved in this study. Serving experimental class teaching materials in the form of delivery problems containing linkages with other mathematical concepts and the relation to others can provide wider opportunities for students to understand mathematical concepts by making learning material connections in the context of other math concepts or other subjects. It is thus not surprising that the application of problem-based learning makes students accustomed to solve problems that arise in mathematics and other fields. Learning on the computer-assisted experiments class are able to present a variety of alternative problem-solving strategies for program MathXpert provide a menu that provides a complete selection of problem-solving strategies with explanations and rationale of each step is selected. Meanwhile, students in the class have less control the opportunity to perform the above activities such as learning characteristics cannot facilitate the condition.

Explain or interpret the results as concerns the origin and verify the results or answers, is an indicator of other mathematical problem-solving skills that are the focus to be improved in this study. In this connection, the problem-based learning through problem solving by the teacher, students have ample opportunity to think, to express an assortment of solutions or approaches to problem resolution, filed opinions or ideas, ask questions, consider the completion of the other students, featuring ideas in solving the problem. In problem-based learning students are conditioned to prepare or add details of an idea to counter criticism of other students. This is what seems to make the students in the experimental class used to describe anything in detail, making connections, and enrich and develop an idea to solve the problem. Various habits lead students to explain and interpret the results as concerns the origin at the time of mathematical problem solving process. In the computer-assisted classroom experiment, students attempt to interpret the results and check the correctness of the result or response can be directed more accurately through the menu and look at the program MathXpert. Meanwhile, students in the class that have less control opportunities as experienced by the students in the experimental class to do the explanation and interpretation, because the characteristics of conventional learning does not lead to the ability of students to have a better ability to do the explanation and interpretation of results.

Applying mathematics is a significant indicator of the ability of solving mathematical problems which are also the focus to be improved in this study. In this connection, the problem-based learning through solving everyday problems set by the teacher, students are encouraged to be able to apply mathematical concepts that they have based their understanding of the issues being addressed. With such a process, the significance becomes an important thing to be a foundation for students to solve problems. Students in the class have less control the opportunity to apply mathematics in meaningful because of the problems given in the learning process tends to abstract and unrelated to everyday life.

Based on the result above, it is concluded that the cause of the increase in mathematical problem solving of students in the experimental class better than students in the control class is differences used of learning approach in the experimental class and control class. Similarly, the advantage of computer-assisted problem-based learning compared to the problem-based learning without the help of computers in improving students' mathematical problem solving skills more on the wider opportunities provided MathXpert program to provide independence in doing the student worksheet. However, an increase in the better class of experimental than control class does not mean that students in the experimental class has mastered well all the components of mathematical problem solving skills.

If related to the theory of Action-Process-Object-Schema (APOS), this research has generally demonstrated a model of learning activities that can be used effectively to stimulate reflective abstraction in students. Reflective abstraction as a method of constructing knowledge on APOS theory and are expected to occur when students are learning activities, students will be able to push the process of forming a new mental objects, new processes and new schemes through the construction process in the form of generalizations, interiorization, encapsulation, coordination and reversal. Problem-based learning that is presented through an eight-step lesson has been able to push the mental action in students. The whole lesson on problem-based learning are to reflect the occurrence of mental action in students although there is no doubt that they could not possibly know the whole picture of a person's mental activity. Presentation of the problem at any given learning materials and require the student to realize that he found the problem to be understood through the steps of defining the problem is the driving force for the occurrence of mental action in students.

At the moment students are trying to gather the facts, make up provisional estimates, and investigate, they are taking advantage of the scheme is a prerequisite that they have the APOS theory is the object that will be given to action. Perfecting the problems that have been defined in a problem-based learning can be considered as interiorization of the object, in this case an existing scheme to be used as a new mental process. This is reasonable because when students are involved in this step, then they're responding to the situation by trying to construct mental processes as a way of understanding the phenomena that have their perceptions.

Perfecting these problems can be seen as the construction of the student in the form of coordination and or reversal. Students are conducting mental constructs such as coordination that compose two or more mental processes that have been generated by interiorization on previous activities. In the meantime, students can also be said to be doing mental constructs shaped reversal when perfecting this issue takes the form of constructing a new process using the inverse of the original. When students do the step that concludes the alternatives solutions collaboratively, students might still doing construction coordination and mental form reversal when they are still trying to find a new mental process. But when the student has begun steps to convert a dynamic process resulting from interiorization, coordination and reversal, into a static object, forming a new concept scheme in their mind, then the student has committed a mental constructs shaped encapsulation.

The last step of the problem-based learning is a test solution to problems step. In this step the student can be said to still be in the process of mental construction in the form of encapsulation or may have been in the form of generalized mental construction. Testing solutions to problems will still shaped encapsulation when this step is used as an attempt to strengthen students' beliefs about the mental processes they have to be subjected construction resulting scheme. Testing solutions to problems step will be considered a mental construction of the student in the form of generalizations when students step is used to apply a set of objects and the mental processes by which has been owned and Dubinsky (2000) referred to as a schema.

Related computer-assisted learning theory applied in this study are very supportive as well. This is consistent with the role of the computer is capable of making abstract ideas can be implemented or appear to be something concrete in mind, most do not even appear in the form of impressions (Dubinsky, E. Tall, D., 2002). The ideas include mathematical concepts that will be constructed by a student tends to be more concrete for them so much easier to understand. Even according to Dubinsky & Tall (2002) construction not only computers can only be used to show the processes represented by abstract ideas, but it also can be manipulated. Furthermore, when the various constructions appeared on the computer, it would be useful to reflect on its meaning in terms of how to make computers and any processes that may be involved. Thus, the computer was able to make abstract ideas more concrete, especially for students who are constructing them.

When students encounter in constructing mental processes in themselves as a result of overly complex problems they faced, there would be the possibility that the learning process will stop just on students or some construction cannot be implemented as changing mental processes into a new object (encapsulation). The role of the teacher in this case the application of scaffolding techniques to students is very important in solving the learning problem. This is in line with what is delivered by Hmelo-Silver, Duncan, & Chin (2007), that state that the use of scaffolding techniques to reduce cognitive load, providing expert guidance and help students gain disciplinary ways of thinking and activity. The description has been presented, computer-assisted problem-based learning very appropriate to be used as an alternative to appropriate learning in improving mathematical problem solving ability because of the approach has been able to condition the reflective abstraction related to mental actions, process- mental processes, mental objects and schemes in students. Computer assistance and scaffolding techniques can be further stimulus for students to be in her mental action occurs after the hopes.

Recommendations

The mean score of students' posttest mathematical problem solving in the computer assisted problem-based learning class, namely 34,96 and the problem-based learning without computer assistance is 29.60 out of the ideal score 68; and the mean score of students' posttest mathematical communication in the computer assisted problem-based learning class, namely 46.58 and the problem-based learning without computer assistance is 41.04 out of the ideal score 75 show that mathematical problem solving and mathematical communication abilities are less than optimal. Although the implementation of this is done with the time for half the semester, it seems that the time frame was too little compared to the custom of students over the years following the conventional learning since they were sitting on a bench elementary school through high school class. This leads to the difficulty of providing adjustments to the students. Adjustment is one of them is in their ability to demonstrate higher-order thinking, because the problem-based learning students' willingness to do the work that requires a high level of mathematical thinking skills are the main assets in constructing knowledge. This is consistent with the character-based constructivist learning which requires the construction of knowledge by students themselves.

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